COMS 4721 - Spring 2016 - HW 4
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Problem 2

(a) The reason that this does not indicate a problem has to do with the fact that the square *(n* x *n)* matrix $A^T A$ is $= I$, and therefore, for a given eigenvalue λ_i, we can say that $I\mathbf{v} = \lambda_i \mathbf{v}$, which can then be simplified to $\mathbf{v} = \lambda_i \mathbf{v}$. Therefore, since $\mathbf{v}$ is on both sides of the equation, the equation will be valid for any value of $\mathbf{v}$. This is why there is no problem with the software.

(b) This indicates a problem, because in order to compute the orthogonal projection operator to the three-dimensional subspace spanned by the eigenvectors corresponding to $\{\lambda_1, \lambda_2, \lambda_3\}$, both Alice and Bob will use $\{\lambda_1, \lambda_2, \lambda_3\}$, since the other eigenvalues $\lambda_i = 1/i$ for $i = 4, 5, ..., d$. In other words, since only the first 3 eigenvalues are equal to 1, they must both use those exact 3 eigenvalues, and therefore $\prod_{Alice}$ should $= \prod_{Bob}$.

(c) There is no problem in this case, since there are multiple ways to draw 3 distinct eigenvalues from a set of 4 equivalent eigenvalues. E.g., Alice may use $\{\lambda_1, \lambda_2, \lambda_3\}$ and Bob may use $\{\lambda_2, \lambda_3, \lambda_4\}$, etc.