MAP for π

$$P(\pi \mid \gamma) = P(\gamma \mid \pi) P(\pi) \propto P(\gamma \mid \pi) P(\vec{a})$$

where $P(\gamma / \pi) = \prod_{i=1}^{p} (P_{\gamma_i} \mid \pi) \propto \prod_{i=1}^{p} (\pi_i^{\gamma_i} (1-\pi_i)^{1-\gamma_i})$

$$P(\vec{a}) = \cancel{\prod_{i=1}^{p}} P(\vec{a}_i) \text{ with } P(\vec{a}_i) = \frac{1}{\sqrt{2\pi\sigma_{a_i}^2}} \cdot e^{\left(\frac{-\vec{a}^2}{2\sigma_{a_i}^2}\right)}$$

$$\pi_i = \frac{e^{A_i \vec{a}_i}}{1+e^{A_i \vec{a}_i}}$$

$$l(\pi) = E_\gamma [\ln(P(\pi \mid \gamma))]$$

$$\propto \sum_{i=1}^{p} [\ln(P(\gamma_i \mid \pi_i))] + \sum_{i=1}^{p} \ln(P(\vec{a}_i))$$

$$= \sum_{i=1}^{p} [\mathrm{Prob}(\gamma_i=1) \ln(\pi_i) + \mathrm{Prob}(\gamma_i=0) \ln(1-\pi_i)] + \ln P(\vec{a}_i) + c$$

$$\propto \sum_{i=1}^{p} [\hat{\gamma}_i \ln(\pi_i) + (1-\hat{\gamma}_i) \ln(1-\pi_q)] + \ln e^{\left(\frac{-\vec{a}^2}{2\sigma_{a_i}^2}\right)} + c$$

$$= \sum_{i=1}^{p} \left[\hat{\gamma}_i \cdot \ln\left(\frac{e^{A_i \vec{a}_i}}{1+e^{A_i \vec{a}_i}}\right) + (1-\hat{\gamma}_i) \ln\left(\frac{1}{1+e^{A_i \vec{a}_i}}\right)\right] + \left(-\frac{\vec{a}_i^2}{2\sigma_{a_i}^2}\right) + c$$

$$= \sum_{i=1}^{p} [\hat{\gamma}_i \cdot A_i \vec{a}_i - \hat{\gamma}_i \cdot \ln(1+e^{A_i \vec{a}_i}) - \ln(1+e^{A_i \vec{a}_i})$$
$$+ \hat{\gamma}_i \cdot \ln(1+e^{A_i \vec{a}_i})] - \frac{1}{2\sigma_{a_i}^2} \cdot \vec{a}_i^2 + c$$

$$= \sum_{i=1}^{p} [\hat{\gamma}_i \cdot A_i \cdot \vec{a}_i - \ln(1+e^{A_i \vec{a}_i})] - \frac{1}{2\sigma_{a_i}^2} \cdot \vec{a}_i^2 + c$$

$$\frac{d l(\pi)}{d(\vec{a})} = \sum_{i=1}^{p} \left[\hat{\gamma}_i \cdot A_i - \frac{A_i e^{A_i \vec{a}_i}}{1+e^{A_i \vec{a}_i}}\right] - \frac{1}{\sigma_{a_i}^2} \vec{a}_i = 0$$

~~For ith variant~~

$$\cancel{\frac{A_i \cdot e^{A_i \vec{a}_i}}{1+e^{A_i \vec{a}_i}} + \frac{1}{\sigma_{a_i}^2} \cdot \vec{a}_i = \hat{\gamma}_i A_i}$$

MAP for σ^2

$$P(\sigma^2 \mid \beta, \gamma, \tau) \propto P(\beta \mid \gamma, \sigma^2, \tau) \boxed{P(\vec{b})}$$

where $P(\vec{b}) = \prod_{i=1}^{p} P(\vec{b_i})$ with $P(\vec{b_i}) = \frac{1}{\sqrt{2\pi\sigma_{b_i}^2}} \cdot e^{\left(\frac{-\vec{b_i}^2}{2\sigma_{b_i}^2}\right)}$

$$P(\beta_i \mid \sigma_i^2, \gamma_i, \tau) = (\gamma_i N(\beta_i; 0, \tau^{-1}\sigma_i^2) + (1-\gamma_i)\delta_0(\beta_i))$$

$$\sigma_i^2 = e^{A_i \vec{b}}$$

$$
\begin{aligned}
l(\sigma^2) &= E_\gamma[\ln(P(\sigma^2 \mid \beta, \gamma, \tau))] \\
&= E_\gamma\left[\sum_{i=1}^{p} \ln(P(\beta_i \mid \sigma_i^2, \gamma, \tau))\right] + \sum_{i=1}^{p} \ln(P(\vec{b_i})) + c \\
&= \sum_{i=1}^{p} \left[E_\gamma \ln(P(\beta_i \mid \sigma_i^2, \gamma_i, \tau)) + \ln(P(\vec{b_i}))\right] + c \\
&\approx \sum_{i=1}^{p} \left[\hat{\gamma}_i \ln(P(\beta_i \mid \gamma_i = 1, \sigma_i^2)) + (1-\hat{\gamma}_i)\ln(P(\beta_i \mid \gamma_i = 0))\right. \\
&\quad \left. + \ln\left(\frac{1}{\sqrt{2\pi\sigma_{b_i}^2}} \cdot e^{\left(\frac{-\vec{b_i}^2}{2\sigma_{b_i}^2}\right)}\right)\right] + c \\
&= \sum_{i=1}^{p} \left[\hat{\gamma}_i \cdot \left(\frac{1}{2}\ln\left(\tau \cdot \frac{1}{\sigma_i^2}\right) - \frac{\tau\beta_i^2}{2} \cdot \frac{1}{\sigma_i^2}\right) + \ln e^{\left(-\frac{\vec{b_i}^2}{2\sigma_{b_i}^2}\right)}\right] + c \\
&= \sum_{i=1}^{p} \left[\frac{\hat{\gamma}_i}{2}\ln\tau + \frac{\hat{\gamma}_i}{2}(-A_i\vec{b}) - \frac{\hat{\gamma}_i\tau\hat{\beta}_i^2}{2} \cdot \frac{1}{e^{A_i\vec{b}}} - \frac{\vec{b_i}^2}{2\sigma_{b_i}^2}\right] + c
\end{aligned}
$$

$$\frac{dl(\sigma^2)}{d\vec{b}} = \sum_{i=1}^{p}\left[-\frac{\hat{\gamma}_i A_i}{2} + \frac{\hat{\gamma}_i\tau\hat{\beta}_i^2}{2} \cdot \frac{1}{(e^{A_i\vec{b}})^2} \cdot e^{A_i\vec{b}} \cdot (A_i) - \frac{2b_i}{2\sigma_{b_i}^2}\right] = 0$$

$$\sum_{i=1}^{p}\left[\frac{\hat{\gamma}_i\tau\hat{\beta}_i^2 A_i}{2} \cdot \frac{1}{e^{A_i\vec{b}}} - \frac{1}{\sigma_{b_i}^2} \cdot \vec{b}\right] = \sum_{i=1}^{p}\frac{\hat{\gamma}_i A_i}{2}$$

For the ith variant :

$$\cancel{\frac{\tau\hat{\beta}_i^2 A_i}{2} \cdot \frac{1}{e^{A_i\vec{b}_i}} - \frac{1}{\sigma_{b_i}^2} \cdot \vec{b}_i = \frac{\hat{\gamma}_i A_i}{2}}$$

$\hat{\vec{b}}$

Newton - Rampson

A_{ij}