

Quotient Ring Operation Tables

Given n and $p(x) \in \mathbb{Z}_n[x]$, calculate operation tables for $\mathbb{Z}_n[x] / \langle p(x) \rangle$.

In[]:= (*Setting a degree n*)

n = 3;

(*Defined Polynomial*)

p[x_] := x^2 + 1;

(*Creating a table from 0 to n-1 (digits in \mathbb{Z}_n *)

zn = Table[k, {k, 0, n-1}];

(*Calculates the degree of polynomial p[x]*)

deg = Length[CoefficientList[p[x], x]] - 1;

(*Finds tuples of length deg with elements from zn*)

Tuples[zn, deg];

In[]:= (*Turns a Tuple to a polynomial*)

tuple2poly[t_] := Module[{l,

l = Length[t];

Return[Sum[t[[i]] x^(i-1), {i, 1, l}]]

]

In[]:= (*Polynomial form of tuples*)

polys = Sort[Map[tuple2poly, Tuples[zn, deg]]]

Out[]:= {0, 1, 2, x, 2 x, 1 + x, 2 + x, 1 + 2 x, 2 + 2 x}

In[]:= add = Table[PolynomialMod[PolynomialRemainder[polys[[i]] + polys[[j]], p[x], x], n

{i, 1, Length[polys]}, {j, 1, Length[polys]}];

TableForm[add, TableHeadings -> {polys, polys}]

*Out[]:=*TableForm=

	0	1	2	x	2 x	1 + x	2 + x	1 + 2 x
0	0	1	2	x	2 x	1 + x	2 + x	1 + 2 x
1	1	2	0	1 + x	1 + 2 x	2 + x	x	2 + 2 x
2	2	0	1	2 + x	2 + 2 x	x	1 + x	2 x
x	x	1 + x	2 + x	2 x	0	1 + 2 x	2 + 2 x	1
2 x	2 x	1 + 2 x	2 + 2 x	0	x	1	2	1 + x
1 + x	1 + x	2 + x	x	1 + 2 x	1	2 + 2 x	2 x	2
2 + x	2 + x	x	1 + x	2 + 2 x	2	2 x	1 + 2 x	0
1 + 2 x	1 + 2 x	2 + 2 x	2 x	1	1 + x	2	0	2 + x
2 + 2 x	2 + 2 x	2 x	1 + 2 x	2	2 + x	0	1	x

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In[ ]:= polys1 = DeleteCases[polys, 0]
mult =
  Table[PolynomialMod[PolynomialRemainder[polys1[[i]] * polys1[[j]], p[x], x], n]
    {i, 1, Length[polys1]}, {j, 1, Length[polys1]}};
TableForm[mult, TableHeadings -> {polys1, polys1}]

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Out[ ]:= {1, 2, x, 2 x, 1 + x, 2 + x, 1 + 2 x, 2 + 2 x}
Out[ ]//TableForm=

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	1	2	x	2 x	1 + x	2 + x	1 + 2 x	2 + 2 x
1	1	2	x	2 x	1 + x	2 + x	1 + 2 x	2 + 2 x
2	2	1	2 x	x	2 + 2 x	1 + 2 x	2 + x	1 + x
x	x	2 x	2	1	2 + x	2 + 2 x	1 + x	1 + 2 x
2 x	2 x	x	1	2	1 + 2 x	1 + x	2 + 2 x	2 + x
1 + x	1 + x	2 + 2 x	2 + x	1 + 2 x	2 x	1	2	x
2 + x	2 + x	1 + 2 x	2 + 2 x	1 + x	1	x	2 x	2
1 + 2 x	1 + 2 x	2 + x	1 + x	2 + 2 x	2	2 x	x	1
2 + 2 x	2 + 2 x	1 + x	1 + 2 x	2 + x	x	2	1	2 x