Quantitative Analysis HW4

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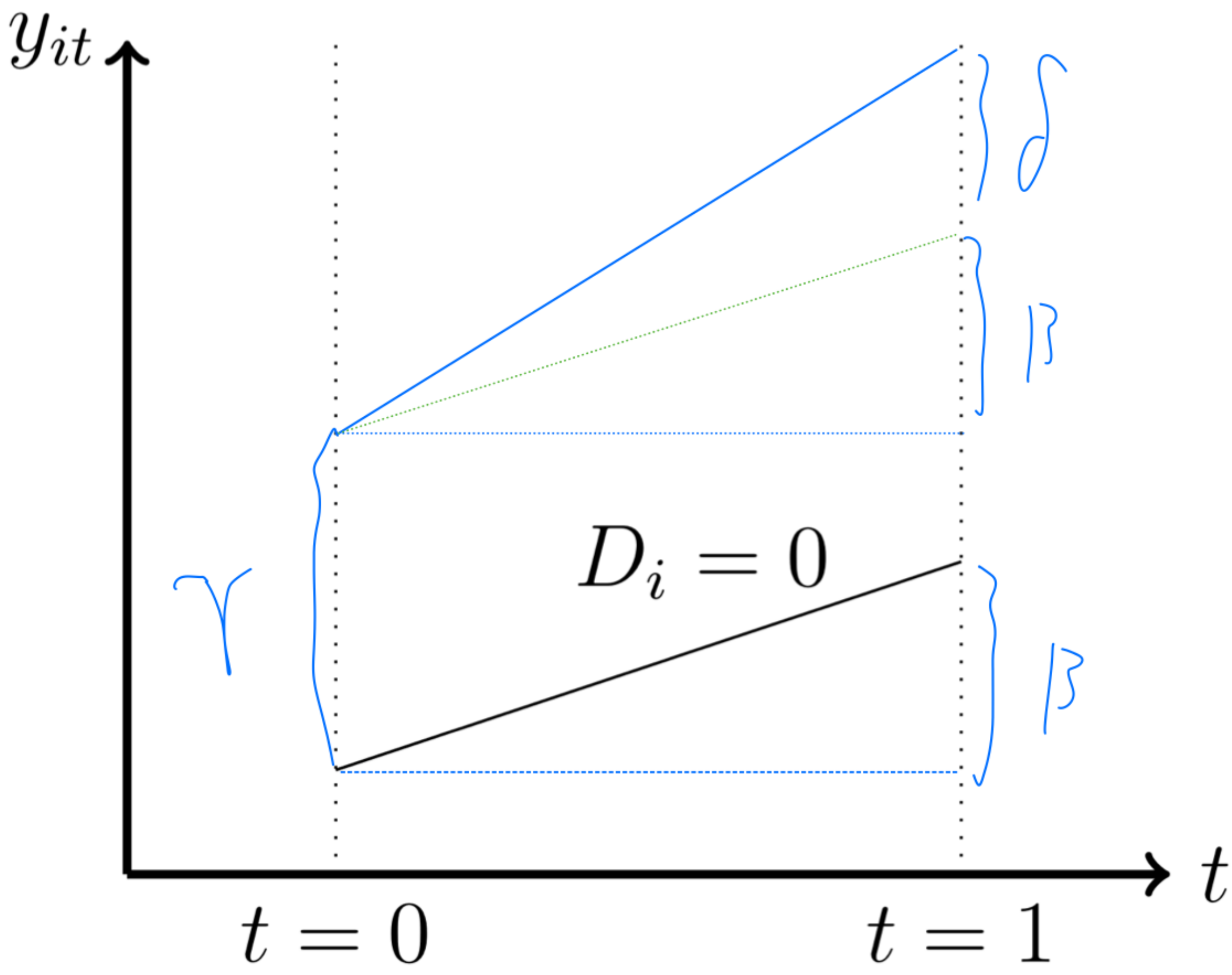
1.

The above blue solid line is the line of $D_i = 1$



2.

(a)

The OLS estimator of $\tilde{\beta}_1$ is calculated by:

$$
\begin{aligned}
\tilde{\beta}_1 &= \frac{\sum(x_{1i}-\bar{x}_1)y_i}{\sum(x_{1i}-\bar{x}_1)^2} \\
&= \frac{\sum(x_{1i}-\bar{x}_1)(\beta_0+\beta_1 x_{1i}+\beta_2 x_{2i}+u_i)}{\sum(x_{i1}-\bar{x}_1)^2} \\
&= 0+\beta_1+\beta_2\frac{\sum(x_{1i}-\bar{x}_1)x_{2i}}{\sum(x_{i1}-\bar{x}_1)^2}+\frac{\sum(x_{1i}-\bar{x}_1)u_i}{\sum(x_{i1}-\bar{x}_1)^2} \\
&= \beta_1+\beta_2\frac{\sum(x_{1i}-\bar{x}_1)x_{2i}}{\sum(x_{i1}-\bar{x}_1)^2}+\frac{\sum(x_{1i}-\bar{x}_1)u_i}{\sum(x_{i1}-\bar{x}_1)^2}
\end{aligned}
$$

Under Modern Assumption I

$$
\frac{1}{n}\sum(x_{1i}-\bar{x}_1)u_i = \frac{1}{n}\sum x_{1i}u_i - \frac{1}{n}\bar{x}_1\sum u_i \xrightarrow{p} \mathbb{E}(x_1 u)+\mu_{x_1}\mathbb{E}(u)=0 \quad (1)
$$

$$
\frac{1}{n}\sum(x_{1i}-\bar{x}_1)^2 \xrightarrow{p} Var(x_1)=\sigma_{x_1}^2 \quad (2)
$$

$$
\frac{1}{n}\sum(x_{1i}-\bar{x}_1)x_{2i} = \frac{1}{n}\sum(x_{1i}-\bar{x}_1)(x_{2i}-\bar{x}_2) \xrightarrow{p} Cov(x_1, x2)=\sigma_{x_1x_2} \quad (3)
$$

With `(1)(2)(3)` ,

We can derive,

$$
\tilde{\beta}_1 \xrightarrow{p} \beta_1+\frac{\sigma_{x_1x_2}}{\sigma_{x_1}^2}\beta_2
$$

Therefore, $\tilde{\beta}_1$ is not consistent.

(b)

It depends on the sign of β_2, if $\beta_2>0$, $\tilde{\beta}_1$ overestimate β_1 by $\frac{\sigma_{x_1x_2}}{\sigma_{x_1}^2}\beta_2$

as $n\to\infty$. On the other hand, if $\beta_2<0$, $\tilde{\beta}_1$ underestimates β_1 by $\frac{\sigma_{x_1x_2}}{\sigma_{x_1}^2}\beta_2$.

(c)

Since $\tilde{\beta}_1$ is not a consistent estimator of β_1 ,

$$
\tilde{\beta}_1-\beta_1 \xrightarrow{p} \frac{\sigma_{x_1x_2}}{\sigma_{x_1}^2}\beta_2 \neq 0
$$

Therefore,

$$
\sqrt{n}(\tilde{\beta}_1-\beta_1)\to\infty
$$

as $n\to\infty$. This statistic diverges and does not converge to a distribution.

3.

Let $\{x_i\}_{i=1}^n$ be i.i.d random variables with a distribution that exist finite second moment.

(a) False

we can estimate μ_x of the distribution by a biased estimator $\frac{1}{n}\sum x_i + \frac{1}{n}$

The estimator is biased clearly. However, as $n \to \infty$

$$\frac{1}{n}\sum x_i + \frac{1}{n} \xrightarrow{p} \mathbb{E}[x] + 0 = \mu_x$$

therefore the estimator is biased but consistent.

(b) False

We can estimate μ_x with an unbiased estimator given by

$$\hat{\mu}_x = \begin{cases} \mu_x + 1, \ p = 0.5 \\ \mu_x - 1, \ p = 0.5 \end{cases}$$

Though $\hat{\mu}_x$ does not depend on x_i, it is still an unbiased estimator of μ_x since

$$\mathbb{E}[\hat{\mu}_x] = \mu_x$$

but clearly, as $n \to \infty$,

$$\hat{\mu}_x \overset{p}{\nrightarrow} \mu_x$$

since it does not depend on x_i.

Therefore, the estimator is unbiased but not consitent.