Spring 2022

Quantitative Analysis
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Problem Set 1

Due: 2/21

Part One: Hand-Written Exercise

1. Verify the statement on slide 25, Lecture 1. That is, suppose $y_i = \beta_0 + \beta_1 x_{1i} + u_i$, please show that the OLS estimators are:
   (a) $\hat{\beta}_1 = \dfrac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$.
   (b) $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$.

2. (a) Show $SST = SSR + SSE$ when there is an intercept term in the regression.
   (b) Show SST need not be equal to $SSR + SSE$ when there is no intercept term.

3. Verify the statement on slide 42, Lecture 1. That is, $\text{var}(\hat{\beta}_0) = \sigma_0^2 \dfrac{\sum_{i=1}^n x_i^2/n}{\sum_{i=1}^n (x_i - \bar{x})^2}$.

4. Consider the following regression models:

$$\begin{aligned} &\text{Model A: } y_i = \beta_0 + \beta_1 x_i + u_i \\ &\text{Model B: } y_i = \alpha_0 + \alpha_1 (x_i - \bar{x}) + v_i \end{aligned}$$

   where $\bar{x} = \frac{1}{n} \sum x_i$, and $\text{Var}(y_i) = \sigma^2$.

   (a) Find the OLS estimators of β_0 and α_0. Are they identical? Are their variances identical? If not, which variance is larger?
   (b) Find the OLS estimators of β_1 and α_1. Are they identical? Are their variances identical? If not, which variance is larger?

Part Two: Computer Exercise

1. (a) Let $x = c(1:150)$
   (b) Select the number in x that is greater than 135 or smaller or equal to 5.
   (c) Select the number in x that is greater than 70 and smaller than 90.
   (d) Select the number in x that is divisible by 4 and 5

2. (a) Draw 150,000 observations from standard normal distribution and name it as “X”

(b) Evaluate the mean, median, max, min, and variance of X.

(c) Randomly select 5,000 subsamples from X without replacement, call it Y and calculate its mean and variance.

(d) Randomly select 5,000 subsamples from X with replacement, call it Z and calculate its mean and variance.

(e) Find the 45^{th} percentile in X. Also, find the number z such that $Pr(a \leq z) = 0.45$, where $a \sim N(0, 1)$.

(f) Find the probability of drawing $x \in X$ such that $x \in (-0.55, 1.25]$. Also, find the probability of drawing a, where $a \sim N(0, 1)$ such that $a \in (-0.55, 1.25]$.

3. (a) Create matrix $\mathbf{X} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 4 & 4 \\ 1 & 6 & 7 \\ 1 & 8 & 10 \\ 1 & 10 & 13 \\ 1 & 12 & 16 \end{bmatrix}$

(b) Create matrix $\mathbf{Y} = \begin{bmatrix} 1 & 9 \\ 2 & 8 \\ 3 & 7 \\ 4 & 6 \\ 5 & 5 \\ 6 & 4 \end{bmatrix}$

(c) Create matrix $\mathbf{Z}$, a $6 * 3$ matrix, where

$$Z_{ij} = \begin{cases} X_{i1} + Y_{i1}, & \text{if } j = 1 \\ X_{i2}, & \text{if } j = 2 \\ X_{i3} - 2 * Y_{i2}, & \text{if } j = 3 \end{cases} \text{,for } i = 1, 2, ..., 6$$