Problem Set 7

Due: 5/01

Part One: Hand-Written Exercise

1. Verify the statement on slide 29, Lecture 6. That is, please show that the information matrix equality holds for the probit model.

2. Verify the statement on slide 31, Lecture 6. That is, the NLS estimator and the MLEs are not the same under both probit and logit models.

3. Consider the logit model $G(\mathbf{x_i}'\theta)$ for the Bernoulli response variable y.

   (a) Write down the model's $\mathbf{L}_n(\theta)$ and $\nabla\mathbf{L}_n(\theta)$, that is, the model's log-likelihood function and the derivative of it.

   (b) Show that as long as $\mathbf{X}$ contains an intercept, then $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} G(\mathbf{x_i}'\tilde{\theta})$, where $\tilde{\theta}$ is the solution to $\nabla\mathbf{L}_n(\theta) = \mathbf{0}$.

Part Two: Computer Exercise

1. Please load the dataset `HMDA` in `R`, which is a cross-sectional data on the Home Mortgage Disclosure Act, containing 2,380 observations on 14 variables. The variable we are interested in modelling is "`deny`", an indicator for whether an applicant's mortgage application has been accepted (deny = no) or denied (deny = yes). A regressor that ought to have power in explaining whether a mortgage application has been denied is "`hirat`", the size of the anticipated total monthly loan payments relative to the the applicant's income.

   (a) Construct a Logit model with `deny` as the dependent variable y, `hirat` as the independent variable x. Show the estimated coefficients and the corresponding robust standard error.

   (b) What is the probability of facing a denial on mortgage application when `hirat` = 0.3?

   (c) What is the probability of facing a denial on mortgage application when `hirat` = 0.7?

   (d) Now construct a Logit model with `deny` as the dependent variable y, `hirat` and `afam` as the independent variables x. (Note that the variable "afam" equals 1 if the applicant is an African American and equals 0 otherwise.)

(e) Fixing `hirat` $= 0.3$, please estimate the difference in `deny` between African American and non-African American.

(f) Fixing `hirat` $= 0.7$, please estimate the difference in `deny` between African American and non-African American.