Quantitative Analysis HW6

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1.

(a)

Given PDF of exponential distribution and $\{y_i\}_{i=1}^n$ independent variables.

The joint log-likelihood function is then given by:

$$egin{aligned} L_n(heta) &= \ln\left(\prod_{i=1}^n rac{1}{ heta} e^{-rac{1}{ heta}y_i}
ight) = \sum_{i=1}^n \ln\left(rac{1}{ heta} e^{-rac{1}{ heta}y_i}
ight) \ &= \sum_{i=1}^n -\ln(heta) - \left(rac{1}{ heta}y_i
ight) \ &= -n\ln heta - rac{1}{ heta}\sum_{i=1}^n y_i \end{aligned}$$

To obtain $\tilde{\theta}$, we need to solve θ that maximize the log-likelihood function. Thus it must satisfy the FOC given by:

$$egin{aligned} rac{\partial L_n(heta)}{\partial heta} &= 0 = -rac{n}{ heta} + rac{1}{ heta^2} \sum_{i=1}^n y_i \ &\Longrightarrow \ heta n = \sum_{i=1}^n y_i \ &\Longrightarrow \ heta &= rac{1}{n} \sum_{i=1}^n y_i \end{aligned}$$

Finally, the maximum likelihood estimator $ilde{ heta}=rac{1}{n}\sum_{i=1}^n y_i$ which is the mean of y_i .

(b)

Let

$$H(heta) = \mathbb{E}\left[rac{\partial^2 L_n(heta)}{\partial heta^2}
ight]$$

$$= \mathbb{E}\left[\frac{n}{\theta^2} - \frac{2}{\theta^3} \sum_{i=1}^n y_i\right]$$
$$= \frac{n}{\theta^2} - \frac{2n}{\theta^2}$$
$$= \frac{-n}{\theta^2}$$

if the information equality holds,

$$egin{aligned} \sqrt{n}(ilde{ heta}- heta) & \stackrel{D}{
ightarrow} \mathcal{N}(0,-H(heta)^{-1}) \ \Longrightarrow & \lim_{n
ightarrow\infty} var(\sqrt{n}(ilde{ heta}- heta)) & \stackrel{p}{
ightarrow} -H(heta)^{-1} = rac{ heta^2}{n}
ightarrow 0 \end{aligned}$$