Quantitative Analysis
Chung-Ming Kuan
Jui-Chung Yang

Problem Set 9: Solutions

Part One: Hand-Written Exercise

1. (a)

$$f_1(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3, \text{ for all } x \leq \xi.$$
$$f_2(x) = (\beta_0 - \beta_4 \xi^3) + (\beta_1 + 3\beta_4 \xi^2)x + (\beta_2 - 3\beta_4 \xi)x^2 + (\beta_3 + \beta_4)x^3, \text{ for all } x > \xi.$$

(b)

$$f_1(\xi) = f_2(\xi) = \beta_0 + \beta_1 \xi + \beta_2 \xi^2 + \beta_3 \xi^3$$
$$f_1'(\xi) = f_2'(\xi) = \beta_1 + 2\beta_2 \xi + 3\beta_3 \xi^2$$
$$f_1''(\xi) = f_2''(\xi) = 2\beta_2 + 6\beta_3 \xi$$

(c)

$$\frac{\partial^3 f_1(\xi)}{\partial x^3} = 6\beta_3$$
$$\frac{\partial^3 f_2(\xi)}{\partial x^3} = 6\beta_3 + 6\beta_4. \ \blacksquare$$