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Problem Set 6

Due: 4/4

Part One: Hand-Written Exercise

1. Verify the statement on slide 26, Lecture 5. That is, the NLS estimator and the MLEs are not the same under both probit and logit models.

2. Verify the statement on slide 34, Lecture 5. That is, for the probit model, show that the information equality holds.

3. Assume conditional normality for y_i : $y_i|\mathbf{x_i} \sim \mathcal{N}(\mathbf{x_i}'\beta, \sigma^2)$ as in slide 14, Lecture 5. Define the following notation:

$$\hat{\sigma}^2 = \frac{1}{n-k-1} \sum_{i=1}^{n} \tilde{u}_i^2$$

$$\tilde{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \tilde{u}_i^2,$$

where $\tilde{u}_i^2$ are the MLE (OLS) residuals. Also, the following facts are given:

$$\frac{(n-k-1)\hat{\sigma}^2}{\sigma^2}|\mathbf{X} \sim \chi^2(n-k-1)$$

$$\mathrm{var}(Q) = 2q \text{ if } Q \sim \chi^2(q).$$

(a) Find $\mathrm{var}(\sqrt{n}\hat{\sigma}^2|\mathbf{X})$.

(b) Find $\mathrm{var}(\sqrt{n}\tilde{\sigma}^2|\mathbf{X})$.

(c) Compare between $\mathrm{var}(\sqrt{n}\hat{\sigma}^2|\mathbf{X})$, $\mathrm{var}(\sqrt{n}\tilde{\sigma}^2|\mathbf{X})$ and $2\sigma_o^4$. Is it true that $\mathrm{var}(\sqrt{n}\tilde{\sigma}^2|\mathbf{X}) < 2\sigma_o^4 < \mathrm{var}(\sqrt{n}\hat{\sigma}^2|\mathbf{X})$?

(d) Does the result in (c) violate the Cramér-Rao lower bound? Explain it.

Part Two: Computer Exercise

1. Please load the dataset HMDA in R, which is a cross-sectional data on the Home Mortgage Disclosure Act, containing 2,380 observations on 14 variables. The variable we are interested in modelling is "deny", an indicator for whether an applicant's mortgage application has been accepted (deny = no) or denied (deny = yes). A regressor that ought to have

power in explaining whether a mortgage application has been denied is "`hirat`", the size of the anticipated total monthly loan payments relative to the the applicant's income.

(a) Construct a Logit model with `deny` as the dependent variable y, `hirat` as the independent variable x. Show the estimated coefficients and the corresponding robust standard error.

(b) What is the probability of facing a denial on mortgage application when `hirat` = 0.2?

(c) What is the probability of facing a denial on mortgage application when `hirat` = 0.8?

(d) Now construct a Logit model with `deny` as the dependent variable y, `hirat` and `afam` as the independent variables x. (Note that the variable "afam" equals 1 if the applicant is an African American and equals 0 otherwise.)

(e) Fixing `hirat` = 0.2, please estimate the difference in `deny` between African American and non-African American.

(f) Fixing `hirat` = 0.8, please estimate the difference in `deny` between African American and non-African American.