Spring 2023

Quantitative Analysis
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Problem Set 2

Due: 3/13

Part One: Hand-Written Exercise

1. Verify the statement in slide 18, Lecture 2. That is, let $\hat{r}_{i,1}$ be the OLS residual of regressing x_1 on the constant one and $x_2, ..., x_k$. Show that $\sum_{i=1}^n \hat{r}_{i,1} x_{i,1} = \sum_{i=1}^n \hat{r}_{i,1}^2$.

2. For the multiple linear regression, the data matrix denoted as $\mathbf{X}$ is below:

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 1 & 8 \\ 2 & 4 & 5 & 7 \\ 3 & 6 & 2 & 9 \\ 4 & 8 & 2 & 2 \end{bmatrix}$$

   For this data matrix, can you calculate the OLS estimators? Why or why not? Please give a brief explanation.

3. Consider the model $y_i = \beta_0 + \beta_1 x_i + u_i$ with $\text{Var}(y_i) = \sigma^2$. Under the Classical Assumptions, the OLS estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are unbiased. Let $\tilde{\beta}_1$ be the OLS estimator of β_1 by assuming the intercept is zero. That is, $\tilde{\beta}_1$ is obtained under the assumption $\beta_0 = 0$.

   (a) Calculate $\mathbb{E}(\tilde{\beta}_1)$ in terms of x_i, β_0, and β_1.

   (b) If $\beta_0 \neq 0$, is $\tilde{\beta}_1$ unbiased?

   (c) Calculate the variance of $\tilde{\beta}_1$.

   (d) Compare between $\text{Var}(\tilde{\beta}_1)$ and $\text{Var}(\hat{\beta}_1)$. Is it true that $\text{Var}(\tilde{\beta}_1) \leq \text{Var}(\hat{\beta}_1)$ in general?

   (e) Does the result in (d) violate the Gauss-Markov Theorem, which states that $\hat{\beta}_1$ should have the smallest variance? Explain.

Part Two: Computer Exercise

1. Let $\mathbf{X} = \begin{bmatrix} 7 & 2 & 3 \\ 4 & 6 & 7 \\ 9 & 2 & 0 \\ 0 & 9 & 0 \\ 5 & 3 & 5 \end{bmatrix}$ and $\mathbf{Y} = \begin{bmatrix} 6 \\ 2 \\ 4 \\ 2 \\ 1 \end{bmatrix}$.

   (a) Please construct the OLS estimator $\hat{\beta}$. (Reminder: Don't forget the intercept term.)

   (b) Given a new observation $x^* = (0, 4, 3)'$, please calculate $\hat{y}$.

2. Please load the data set "mtcars" from R using the code `data(mtcars)`. The data was extracted from the 1974 Motor Trend US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles (1973-74 models).

   (a) Please show the data for the automobile "Duster 360".

   (b) Please show the `qsec` (1/4 mile time) for all the automobile.

   (c) Please show the data with `cyl` (number of cylinders) = 6.

   (d) Please list the automobiles with `mpg` (miles/gallon) > 15, `vs` (Engine) = 1, and `hp` (horsepower) between 50 and 150.

   (e) Suppose we have the following model:

$$\texttt{drat}_i = \beta_0 + \beta_1 \texttt{wt}_i + \beta_2 \texttt{hp}_i + \beta_3 \texttt{qsec}_i + \beta_4 \texttt{vs}_i + u_i.$$

   Please find $\beta_0, \beta_1, \beta_2, \beta_3$ and β_4 without the function `lm()`.

   (f) Following (e), find those estimators with the function `lm()`.