Spring 2022

Quantitative Analysis
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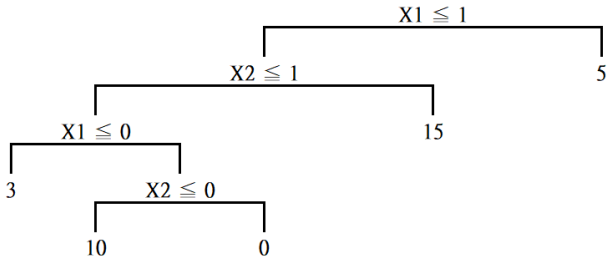
# Problem Set 10: Solutions

## Part One: Hand-Written Exercise

1.




2. (a)

$$T_1 \ : \ \sum_{j=1}^{3} N_j(1-\hat{p}_{j,m_j^*}) = 3 \cdot 300 \cdot \frac{1}{3} = 300$$

$$T_2 \ : \ \sum_{j=1}^{3} N_j(1-\hat{p}_{j,m_j^*}) = 100 \cdot 0 + 400 \cdot \frac{150}{400} + 400 \cdot \frac{200}{400} = 350.$$

As a result, $T_1$ would be more preferable according to misclassification error rate.

(b)

$$\begin{aligned} T_1 \ : \ & \sum_{j=1}^{3} N_j \left( \sum_{m=1}^{3} \hat{p}_{jm}(1-\hat{p}_{jm}) \right) \\ =& 3 \cdot 300 \cdot \left( \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{6} \cdot \frac{5}{6} + \frac{1}{6} \cdot \frac{5}{6} \right) \\ =& 450 \end{aligned}$$

$$\begin{aligned} T_2 \ : \ & \sum_{j=1}^{3} N_j \left( \sum_{m=1}^{3} \hat{p}_{jm}(1-\hat{p}_{jm}) \right) \\ =& 100 \cdot 0 + 400 \cdot \left( \frac{1}{8} \cdot \frac{7}{8} + \frac{5}{8} \cdot \frac{3}{8} + \frac{1}{4} \cdot \frac{3}{4} \right) + 400 \cdot \left( \frac{3}{8} \cdot \frac{5}{8} + \frac{1}{8} \cdot \frac{7}{8} + \frac{1}{2} \cdot \frac{1}{2} \right) \\ =& 450. \end{aligned}$$

As a result, both $T_1$ and $T_2$ are equally preferable according to Gini index.

(c) 60%. *obs*.4 and *obs*.5 are wrongly classified.

3.

| | $\hat{f}_0(\mathbf{x})$ | $r_0$ | $\hat{\varphi}_1(\mathbf{x})$ | $\hat{f}_1(\mathbf{x})$ | $r_1$ | $\hat{\varphi}_2(\mathbf{x})$ | $\hat{f}_2(\mathbf{x})$ |
|---|---|---|---|---|---|---|---|
| *obs*.1 | 0 | 6 | 9 | 5.4 | 0.6 | $0.35a - 0.3$ | $5.22 + 0.21a$ |
| *obs*.2 | 0 | 9 | 9 | 5.4 | 3.6 | $\frac{13}{3} - 0.1a$ | $8 - 0.06a$ |
| *obs*.3 | 0 | 12 | 9 | 5.4 | 6.6 | $\frac{13}{3} - 0.1a$ | $8 - 0.06a$ |
| *obs*.4 | 0 | 4 | $2 + 0.5a$ | $1.2 + 0.3a$ | $2.8 - 0.3a$ | $\frac{13}{3} - 0.1a$ | $3.8 + 0.24a$ |
| *obs*.5 | 0 | $a$ | $2 + 0.5a$ | $1.2 + 0.3a$ | $0.7a - 1.2$ | $0.35a - 0.3$ | $1.02 + 0.51a$ |