HW3

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1.

(a)

Let $\hat{r}_{i,r}^2$ and $\hat{r}_{i,ur}^2$ be the residuals from the model restricted model and the unrestricted model respectively.

$$\hat{\sigma}_r^2 = \frac{\sum_{i=1}^n \hat{r}_{i,r}^2}{n-k-1+q} = \frac{SSR_r}{n-(k+1-q)}$$
$$\hat{\sigma}_{ur}^2 = \frac{\sum_{i=1}^n \hat{r}_{i,ur}^2}{n-k-1} = \frac{SSR_{ur}}{n-k-1}$$

Since we have,

$$\frac{(n-k-1+q)\hat{\sigma}_r^2}{\sigma^2} - \frac{(n-k-1)\hat{\sigma}_{ur}^2}{\sigma^2} \sim \chi^2(q)$$
$$\implies \frac{SSR_r - SSR_{ur}}{\sigma^2} \sim \chi^2(q)$$

and also we know that,

$$\frac{SSR_{ur}}{\sigma^2} \sim \chi^2(n-k-1)$$

Given $(SSR_r - SSR_{ur})$ and SSR_{ur} are independent. We can rewrite the given statistic

$$\frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-k-1)} = \frac{(SSR_r - SSR_{ur})/(q\sigma^2)}{SSR_{ur}/[(n-k-1)\sigma^2]}$$
$$\sim \frac{\chi^2(q)/q}{\chi^2(n-k-1)/(n-k-1)} \sim F(q, n-k-q)$$

(b)

From `(a)` we know that

$$F = \frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-k-1)} = \frac{(\frac{SSR_r}{SST} - \frac{SSR_{ur}}{SST})/q}{\frac{SSR_{ur}}{SST}/(n-k-1)}$$
$$= \frac{(1-R_r^2-1+R_{ur}^2)/q}{1-R_{ur}^2/(n-k-1)}$$

$$= \frac{(R_{ur}^2 - R_r^2)/q}{(1 - R_{ur}^2)/(n - k - 1)}$$

2.

(a)

From $E[y]$ of non-smokers, we know that

$$\begin{aligned} \beta_0 + \beta_1 0 &= \alpha_0 + \alpha_1 \\ \implies \beta_0 &= \alpha_0 + \alpha_1 \end{aligned}$$

on the other hand, from $E[y]$ of smokers,

$$\begin{aligned} \beta_0 + \beta_1 &= \alpha_0 + \alpha_1 0 \\ \implies \beta_1 + \alpha_0 + \alpha_1 &= \alpha_0 \\ \implies \beta_1 &= -\alpha_1 \end{aligned}$$

(b)

Yes, since both model minimized the residual sum of squares, they must be the same model. Given condition of smokers or non-smokers, it outputs the same $\hat{y}$.

(c)

No, his statement is not true.

Since

$$x_1 + x_2 = 1$$

for all given samples.

This causes multicollinearity in his model.

3.

(a)

Increasing $expendA$ in 1% will cause $voteA$ to increase $0.01\beta_1$ unit in average.

(b)

$$
\begin{aligned}
H_0 : \beta_1 &= -\beta_2 \\
\implies \beta_1 + \beta_2 &= 0
\end{aligned}
$$

(c)

Let $R = (0, 1, 1, 0)'$ and $\hat{\boldsymbol{\beta}} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)'$

Under the null hypothesis, we can construct our test statistics T with t distribution by:

$$T = \frac{\boldsymbol{R}\hat{\boldsymbol{\beta}}}{\sqrt{\boldsymbol{R}(X'X)^{-1}\boldsymbol{R}'}} \sim t(n-4)$$

(d)

$$H_0 : \beta_1 = \beta_2 = 0$$

(e)

Let $R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $\hat{\boldsymbol{\beta}} = (\beta_0, \beta_1, \beta_2, \beta_3)$

We can construct the test statistic F :

$$F = \frac{(\boldsymbol{R}\hat{\boldsymbol{\beta}})'[R(X'X)^{-1}R']^{-1}(\boldsymbol{R}\hat{\boldsymbol{\beta}})}{2\hat{\sigma}^2} \sim F(2, n-4)$$