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Quantitative Analysis
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Problem Set 7: Solution

Part One: Hand-Written Exercise

1.

$$
\begin{aligned}
& \left(\mathbf{A}-\mathbf{x_i x_i'}\right)\left(\mathbf{A^{-1}}+\frac{\mathbf{A^{-1}x_i x_i' A^{-1}}}{1-\mathbf{x_i' A^{-1} x_i}}\right) \\
= \ & \mathbf{AA^{-1}}+\frac{\mathbf{AA^{-1}x_i x_i' A^{-1}}}{1-\mathbf{x_i' A^{-1} x_i}}-\mathbf{x_i x_i' A^{-1}}-\frac{\mathbf{x_i(x_i' A^{-1} x_i) x_i' A^{-1}}}{1-\mathbf{x_i' A^{-1} x_i}} \\
= \ & \mathbf{I}+\frac{\mathbf{x_i x_i' A^{-1}}-(1-\mathbf{x_i' A^{-1} x_i})\mathbf{x_i x_i' A^{-1}}-(\mathbf{x_i' A^{-1} x_i})\mathbf{x_i x_i' A^{-1}}}{1-\mathbf{x_i' A^{-1} x_i}} \\
= \ & \mathbf{I}+\frac{\mathbf{x_i x_i' A^{-1}}-\mathbf{x_i x_i' A^{-1}}+\mathbf{x_i' A^{-1} x_i x_i x_i' A^{-1}}-\mathbf{x_i' A^{-1} x_i x_i x_i' A^{-1}}}{1-\mathbf{x_i' A^{-1} x_i}} \\
= \ & \mathbf{I}. \ \blacksquare
\end{aligned}
$$

2. (a) For this situation, the i^{th} bootstrap observation can take anyone except the j^{th} of the N original observations. Thus, the probability is $\frac{N-1}{N}$, which doesn't depend on i and j.

   (b) The j^{th} observation from original sample is not selected for N times. So the probability is $(\frac{N-1}{N})^N$

   (c) When $N=5$, $(\frac{5-1}{5})^5=0.32768$;
   When $N=5000$, $(\frac{5000-1}{5000})^{5000}=0.36784$.

   (d) $\lim_{N\to\infty}(\frac{N-1}{N})^N=e^{-1}=0.36788$ $\blacksquare$.