

Problem Set 3

Due: 03/14

Part One: Hand-Written Exercise

1. We mentioned that the F statistic is given by:

$$F = \frac{(\text{SSR}_r - \text{SSR}_{ur})/q}{\text{SSR}_{ur}/(n - k - 1)},$$

where SSR_r and SSR_{ur} are the residual sums of squares of restricted and unrestricted regressions respectively. $(\text{SSR}_r - \text{SSR}_{ur})$ and SSR_{ur} are independent of each other.

- (a) Given the fact that:

$$\frac{(n - k - 1 + q)\hat{\sigma}_r^2}{\sigma^2} - \frac{(n - k - 1)\hat{\sigma}_{ur}^2}{\sigma^2} \sim \chi^2(q),$$

where $\hat{\sigma}_r^2$ and $\hat{\sigma}_{ur}^2$ are the OLS estimators of σ^2 of the restricted and unrestricted regressions respectively. Please show that

$$\frac{(\text{SSR}_r - \text{SSR}_{ur})/q}{\text{SSR}_{ur}/(n - k - 1)} \sim F(q, n - k - 1).$$

- (b) Show that the F statistic can also be written as the R-squared form

$$F = \frac{(R_{ur}^2 - R_r^2)/q}{(1 - R_{ur}^2)/(n - k - 1)},$$

where R_r^2 and R_{ur}^2 are the R^2 s of the restricted and unrestricted regressions.

2. Consider an explanatory variable x and two dummy variables:

$D_{i1} = 1$ if the trading volume of stock i is the top one-third and $D_{i1} = 0$ otherwise;

$D_{i2} = 1$ if the trading volume of stock i is the bottom one-third and $D_{i2} = 0$ otherwise.

Assume that there are differences in the intercept and slope between each group and the reference group (stocks having middle one-third trading volume).

- (a) Please write down the regression model with the settings above.
- (b) Please write down the regression models when $(D_{i1}, D_{i2}) = (1, 0)$, $(D_{i1}, D_{i2}) = (0, 1)$ and $(D_{i1}, D_{i2}) = (0, 0)$, respectively.

3. The following model can be used to study whether campaign expenditures affect election outcomes:

$$\text{voteA} = \beta_0 + \beta_1 \ln(\text{expendA}) + \beta_2 \ln(\text{expendB}) + \beta_3 \text{prtystrA} + u,$$

where “voteA” is the percentage of the vote received by candidate A, “expendA” and “expendB” are campaign expenditures by candidates A and B, and “prtystrA” is a measure of party strength for candidate A (the percentage of the most recent presidential vote that went to A’s party).

- (a) What is the interpretation of β_1 ?
- (b) In terms of the parameters, state the null hypothesis that the effect of the increase in A’s expenditure will be offset by the increase in B’s expenditure.
- (c) Write the detailed procedure to do the hypothesis testing in (b).
- (d) If someone claims that both candidates’ expenditures do not have any effect on the outcome, how can you specify a testing null hypothesis?
- (e) Write the detailed procedure to do the hypothesis testing in (d).

Part Two: Computer Exercise

Following Question 2 of the computer exercise in Problem Set 2, consider the following model:

$$\text{drat}_i = \beta_0 + \beta_1 \text{wt}_i + \beta_2 \text{hp}_i + \beta_3 \text{qsec}_i + \beta_4 \text{vs}_i + u_i,$$

1. Test the hypothesis $H_0 : \beta_1 = 0$.
 - (a) Please construct the t statistic without the function `lm()`.
 - (b) Use the function `lm()` to directly obtain the t statistic. Verify that it’s identical to (a).
2. Test the hypothesis $H_0 : \beta_1 = \beta_2 = 0$.
 - (a) Please construct the constrained and unconstrained model, obtain R_{ur}^2 and R_r^2 and construct the F statistic.
 - (b) Instead of R_{ur}^2 and R_r^2 , please obtain SSR_{ur} and SSR_r and recalculate the F statistic. Verify that it’s identical to (a).
 - (c) Use the function `linearHypothesis()` to directly obtain the F statistic. Verify that it’s identical to (a).