HW3

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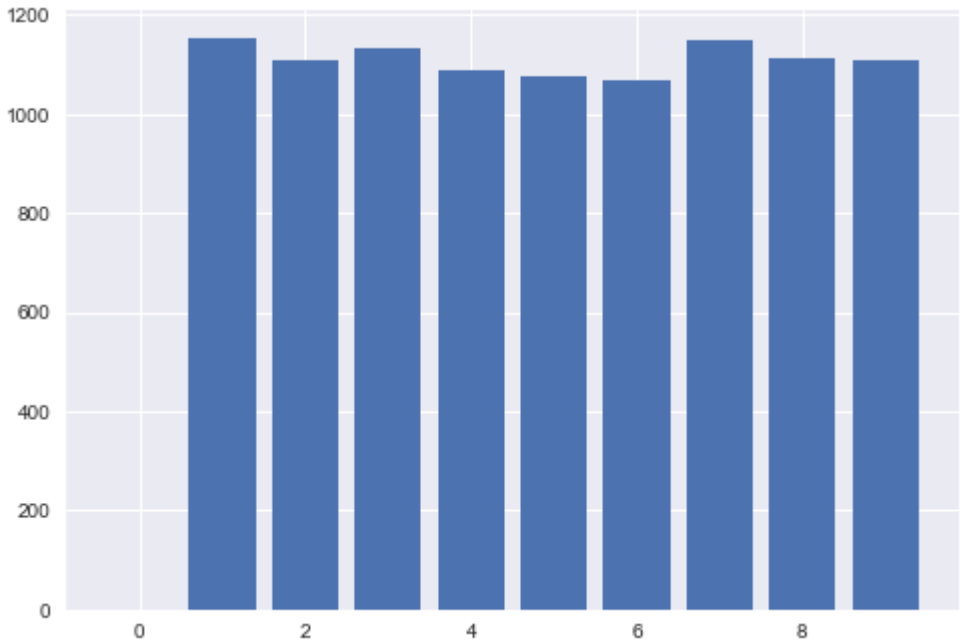
Problem 1

After 10000 trials, we can see that the simulated distrubution is asymptotically uniform distrubution.



Problem 2

From bayesian's rule we know that

$$P(node\ i\ is\ last) = P(from\ i-1) * P(i\ last|from\ i-1) + \\ P(from\ i+1) * P(i\ last|from\ i+1)$$

and if the visiting order is $i+1 \rightarrow i$, that means we must visit all the `m-1` nodes that are on the left hand side of $node\ i+1$ except $node\ i-1$

e.g

$$node\ i+1, i+2....m, 0, 1, 2....i-2$$

Therefore,

$$P(i\ last|from\ i+1) = P(visiting\ m-1\ left\ nodes\ first)$$

and by symmetry of both direction

$$P(i\ last|from\ i+1) = P(visiting\ m-1\ left\ nodes\ first) \\ = P(i\ last|from\ i-1) = p$$

and also the symmetry of each node (except for the starting node 0)

$$\forall\ node\ i \neq 0,\ P(node\ i\ is\ last) = p$$

We can conclude that

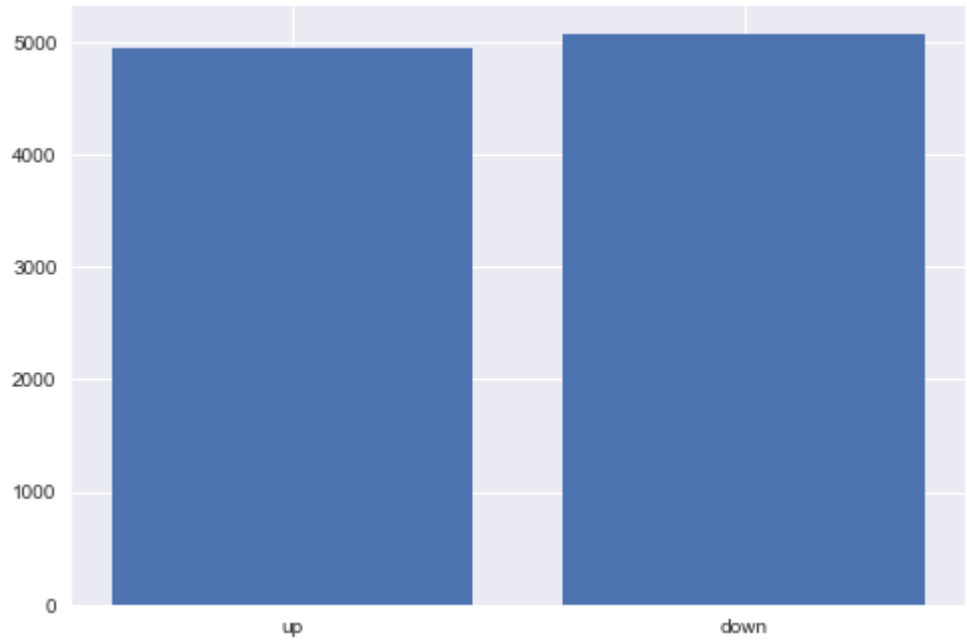
$$P(node\ i\ is\ last) = p = \frac{1}{m}$$

Problem 3

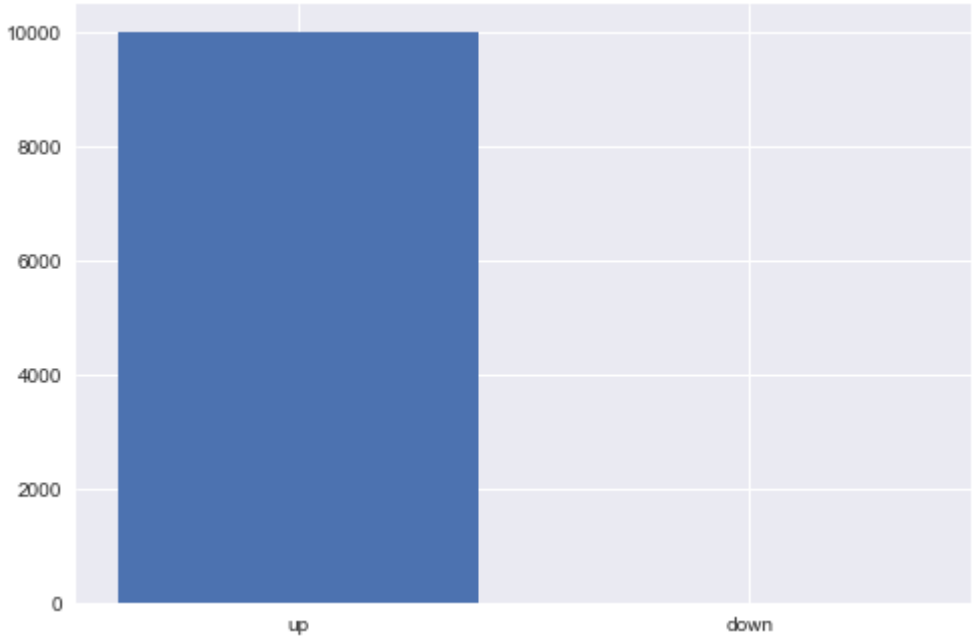
Choose `n=100` and `k=0`

if `p` is set to 0.5, the probability of reaching upper band and lower band are equal which is

$$\frac{n}{n+k} = \frac{50}{100} = \frac{1}{2}$$



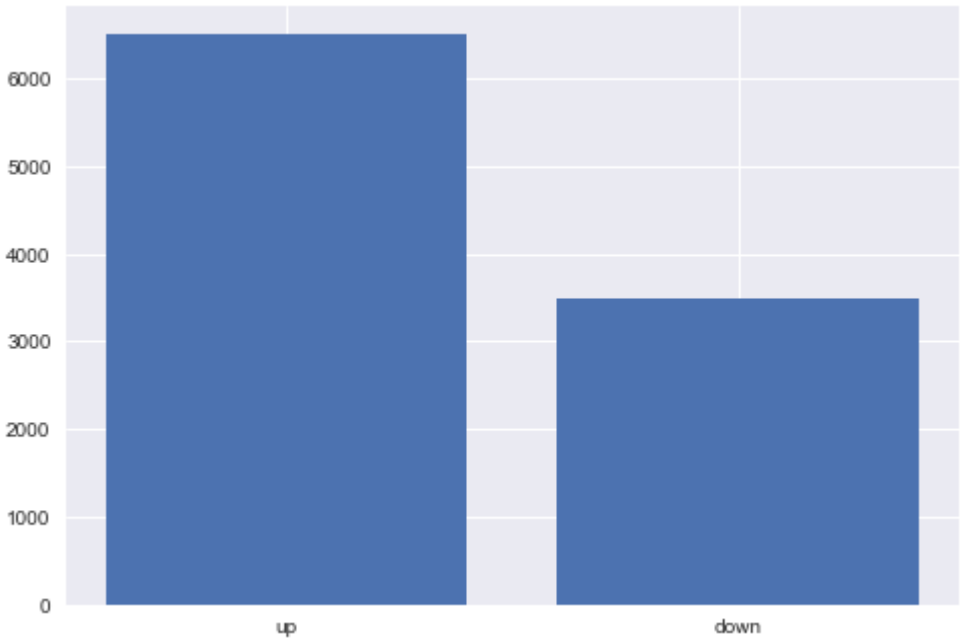
but slightly tweaking `p` to 0.6, the probability of reaching upper band and lower band change dramatically with zero trials hit the lower band.



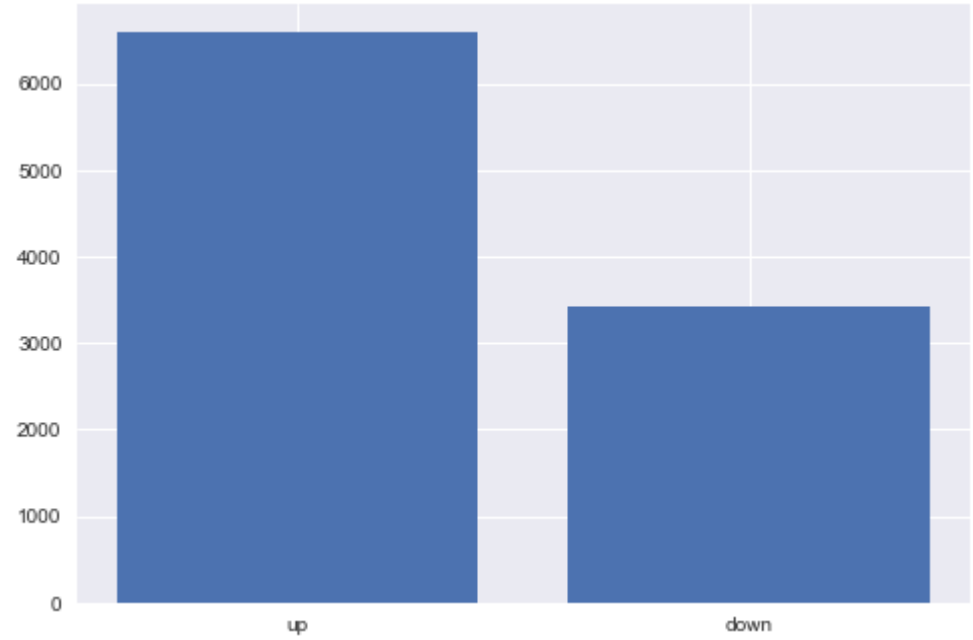
Problem 4

Given `A=1` and `B=2`

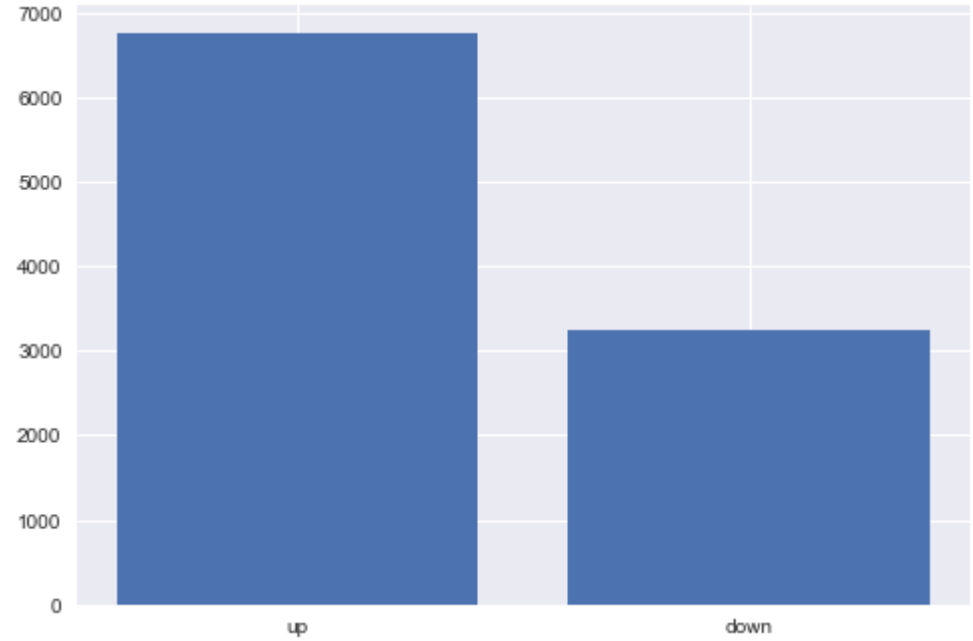
when $\Delta t = 0.1$



when $\Delta t = 0.01$



when $\Delta t = 0.001$



we can see that as Δt becomes smaller

the distribution becomes more closer to the distrubution given from the formula

$$P(up\ A\ before\ down\ B) = \frac{B}{A+B}$$

Appendix:

The experiment result and code are placed here on github.