Heuristic analysis

I came up with four heuristic evaluation functions:

1) **Entropy**. It's to capture how the legal moves are distributed on the board, all on one side of the board, or on both sides. More diversity(chaos) means more flexibility, kind of like the entropy. So it's defined as -P1 * log(P1) + -P2 * log(P2), where P1 is the percentage of moves on the left half of the board, P2 is the percentage of moves on the right half.
2) **Monopoly**. Like the game of Monopoly, you are at a better position if you occupy more valuable real estate. Here valuable estate is defined as the central quadrant of the game board. So I calculate the "monopoly" factor by counting the number of cells the player occupies so far versus those occupied by the opponent
3) **Centrality**. I regard legal moves which are close to the board center are better than those moves far way from board center, as you have more chance of moving around when arrive at the central area. So I calculate the "centrality" for summing up all legal moves' distance to board center (width/2, height/2).
4) **Threat**. A player could take opponent's move before he gets to it and hence has more upper hand. So I calculate the "threat" as the number of shared legal moves between player and his opponent, as more shared moves give more chance of taking advantage of first move.

I combined the above 4 factors with the heuristic of legal moves difference used by "improved" agent with equal weights, i.e.

1) Entropy heuristic = 0.5 * entropy_difference + 0.5 * legal_moves_difference
2) Monopoly heuristic = 0.5 * monopoly_difference + 0.5 * legal_moves_difference
3) Centrality heuristic = 0.5 * centrality_difference + 0.5 * legal_moves_difference
4) Threat heuristic = 0.5 * threat + 0.5 * legal_moves_difference

Run tournament script and I got the below result:

Heuristic	Score vs Improved
Entropy	72.14%
Monopoly	76.43%
Centrality	62.14%
Threat	62.14%

It shows "Monopoly" has the best score in the 4 heuristics. While the other 3 heuristic only rely on the current legal moves without the knowledge of moving history, "Monopoly" on the other side considers the whole history up to the current state. This partially explains why it works better. Also its computing only involves counting which is faster then the "Log" calculation in "Entropy" heuristic and Euclidean distance calculation in "Centrality", so works better with "iterative deepening" algorithms. So out of the 4 heuristics, I recommend using "Monopoly" as the custom score.