L_1-REGULARIZED FUNCTIONAL SUPPORT VECTOR MACHINE

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ABSTRACT

In functional data analysis, binary classification with one functional covariate has been extensively studied. We aim to fill in the gap of considering multivariate functional covariates in classification. In particular, we propose an L_1-regularized functional support vector machine for binary classification. An accompanying algorithm is developed to fit the classifier. By imposing an L_1 penalty, the algorithm enables us to identify relevant functional covariates of the binary response. Numerical results from simulations and one real-world application demonstrate that the proposed classifier enjoys good performance in both prediction and feature selection.

Keywords B-splines · Feature selection · Functional data classification · More · Multivariate functional features · Optimization

1 Introduction

In many real-world applications, data are collected in the form of repeated signals over a continuum such as space or time for each individual subject. Functional data analysis (FDA), as a collection of powerful statistical tools to deal with such data, has been extensively studied over the past few decades. In FDA, each trajectory with repeated measurements is treated as a sample path of a continuous stochastic process, though only discretized observations are available due to practical limitations. When exploring the relationship between a functional covariate and a scalar response, the effect of the functional covariate is typically modeled as a coefficient function, rather than a finite-dimensional vector as in multivariate data analysis. The issue of the curse of dimensionality can thus be successfully circumvented by incorporating regularization on the function. This strategy has been widely applied to fields like neuroscience [1], spectrometry [4], and speech analysis [2].

Binary classification has been extensively studied in the literature on machine learning. Among binary classifiers, support vector machines (SVM) have received considerable attention in both theory and applications; see [5], [22], [28], [21], [23] and references therein.

In the context of FDA, [3] applied an SVM to binary classification where only one univariate trajectory was considered for each subject. In particular, each univariate trajectory is first projected onto a finite-dimensional subspace spanned by pre-determined basis functions, e.g. B-spline or Fourier basis functions. After the projection, the infinite-dimensional classification problem is converted to a finite-dimensional one. Classification can then be conducted through the classical SVM on the finite-dimensional projection scores.

In the aforementioned work, only one functional covariate is accounted for in classification. Nevertheless, it is quite common that multiple trajectories are accessible for each subject in practice. For example, [24] considered a large

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number of flight parameters such as aircraft speed, accelerations and warning signals, each of which was treated as a functional covariate for every flight, to identify important ones that are associated with the landing risk. [11] proposed generalized functional linear models, as well as a penalized likelihood method, and applied this framework to six magnetic resonance imaging indices, each of which was treated as a functional covariate, to identify risk factors for multiple sclerosis, which is a neurological disease affecting the central nervous system. To classify the label of each subject and select relevant functional covariates, we propose an L_1-regularized functional support vector machine (L_1-fSVM) for a binary response with multiple functional covariates. Instead of projecting each trajectory to a pre-specified set of basis functions, this classifier estimates a coefficient function for each trajectory via regression splines. The effect of each functional covariate is represented by its projection onto the coefficient function. Then we build an SVM classifier on these projection scores. Instead of the commonly used L_2 penalty, we impose an L_1 penalty in the SVM to achieve feature selection. To fit L_1-fSVM, we develop an iterative updating algorithm to update the coefficient functions and the vector in the SVM separately. The performance of our classifier in prediction accuracy and feature selection is examined under various simulation settings. Simulation studies demonstrate that L_1-fSVM slightly outperforms some existing functional classifiers in terms of both prediction accuracy and feature selection.

The rest of the article is organized as follows. In Section 2, we propose the L_1-fSVM classifier and develop an algorithm to fit this classifier. In Section 3, empirical studies are conducted to demonstrate the performance of the proposed classifier. Section 4 concludes the paper.

2 Methodology

2.1 Review of L_1-regularized Support Vector Machine with Scalar Covariates

An SVM separates different classes of samples using a hyper-plane that maximizes the margin from the hyper-plane to the support vectors [5]. Suppose a training set consists of n pairs of binary observations. In particular, for $i = 1, \ldots, n$ let $x_i = (x_{i1}, ..., x_{ip})^{\mathrm{T}} \in \mathbb{R}^p$ and $y_i \in \{-1, 1\}$ denote a p-dimensional feature and a binary class label of the ith subject, respectively. The canonical form of the hard-margin SVM problem can be formulated as:

$$\min_{\alpha_0, \alpha} \frac{1}{2} \|\alpha\|_2^2 \text{ such that } y_i(\alpha_0 + x_i^{\mathrm{T}} \alpha) \geq 1, \ i = 1, ..., n,$$

where $\alpha_0 \in \mathbb{R}$ and $\alpha = (\alpha_1, \ldots, \alpha_p)^{\mathrm{T}} \in \mathbb{R}^p$ are used to define the hyper-plane. By introducing a set of slack variables $\{\xi_i\}_{i=1}^n$, the SVM classifier can be adapted to classify non-linearly separable classes [14]. The corresponding optimization problem becomes

$$\min_{\alpha_0, \alpha} \frac{1}{2} \|\alpha\|_2^2 + \lambda \sum_{i=1}^n \xi_i \tag{1}$$

$$\text{such that} \quad y_i(\alpha_0 + x_i^T \alpha) \geq 1 - \xi_i, \ \xi_i \geq 0, \ i = 1, ..., n,$$

where λ denotes a cost parameter.

In practice, not all features are predictive of the binary response. Thus it is desirable to identify relevant features for better interpretation and to enhance prediction accuracy. To achieve this goal, [8] proposed to replace the term $\|\alpha\|_2^2$ with $\|\alpha\|_1$ in (1) in order to obtain a sparse solution. Consequently, the minimization problem of (1) is converted to

$$\min_{\alpha_0, \alpha} \|\alpha\|_1 + \lambda \sum_{i=1}^n \xi_i$$

$$\text{such that} \quad y_i(\alpha_0 + x_i^{\mathrm{T}} \alpha) \geq 1 - \xi_i, \qquad\qquad \xi_i \geq 0, \ i = 1, \ldots, n.$$

By the same arguments in Chapter 12 of [20], we can show that it is equivalent to the following optimization problem:

$$\min_{\alpha_0, \alpha} \|\alpha\|_1 + \lambda \sum_{i=1}^n \{1 - y_i f(x_i)\}_+ \tag{2}$$

where $f(x) = \alpha_0 + \sum_{j=1}^p \alpha_j x_j$ denotes the decision function and $(1 - yf)_+ = \max(0, 1 - yf)$ denotes the hinge loss function. Given an estimate of α_0 and α, a new observation x will be assigned the label of $\text{sign}(\hat{f}(x))$. Various algorithms have been proposed in the literature to solve this minimization problem; see [8], [6] and [7] for example.

Due to the non-smoothness of the hinge loss function, some extensions such as the squared hinge loss function [18] or the Huberized version of the squared hinge loss function [17] were proposed. According to [17], compared with the

squared hinge loss, the Huberized version of the squared hinge loss leads to a more robust classifier. In this paper, we consider using the squared hinge loss when solving the minimization problem at (2):

$$\min_{\alpha_0, \alpha} \|\alpha\|_1 + \lambda \sum_{i=1}^{n} [\{1 - y_i f(x_i)\}_+]^2, \tag{3}$$

for simplicity. This loss was also adopted in [16], [12], [15] and references therein.

2.2 L_1-regularized Support Vector Machine with Functional Covariates

In this article, the problem of primary interest is to discriminate multiple functional data, where the label of each subject is still binary, but the covariates become multivariate random functions. Let $X(t) = (X_1(t_1), \ldots, X_p(t_p))^{\mathrm{T}}$ denote a vector of p random functions evaluated at $t = (t_1, \ldots, t_p) \in \prod_{j=1}^{p} \mathbb{I}_j$, and $Y \in \{-1, 1\}$ denote a binary response. Note that $\mathbb{I}_j$'s could vary from one functional covariate to another. Let $\{(x_i(t), y_i) : i = 1, \ldots, n\}$ denote n i.i.d observations on $(X(t), Y)$. We aim to build a classifier that can discriminate different samples by leveraging the p functional covariates and identify which functional covariates are relevant to the response.

By replacing the linear combination with a suitable form for functional covariates, we obtain a functional version of the L_1-regularized SVM:

$$\min_{\alpha_0, \alpha} \|\alpha\|_1 + \lambda \sum_{i=1}^{n} \left[\{1 - y_i f(x_i)\}_+ \right]^2, \tag{4}$$

where the decision function is

$$f(x_i(t)) = \alpha_0 + \sum_{j=1}^{p} \alpha_j \int_{\mathbb{I}_j} \beta_j(t) x_{ij}(t) \mathrm{d}t. \tag{5}$$

Note that we utilize $\alpha_j \int_{\mathbb{I}_j} \beta_j(t) x_{ij}(t) \mathrm{d}t$ to characterize the effect of the jth functional covariate of the ith subject. In practice, we can only observe discrete observations for each feature rather than its whole sample path. However, by assuming that the sample path of each functional covariate is continuous, information on the whole trajectory can be inferred from dense observations. This illustrates the essential difference between FDA and high-dimensional methodologies. With denser observations, more data can be leveraged to recover the whole trajectory, and thus the estimation accuracy can be enhanced. In contrast, high-dimensional approaches may suffer from the curse of dimensionality due to denser observations. In addition, compared with the expression of f in (3), we have an extra multiplier α_j to identify relevant functional covariates in f to avoid directly penalizing β_j's. But this strategy would lead to the issue of identifiability of α_j and β_j. To address this issue, without loss of generality, we assume that $\mathbb{I}_j = [0, 1]$ and $\beta_j(0) = 1$ for $j = 1, \ldots, p$. Consequently, feature selection can be achieved by estimating α_j's from the minimization problem of (4).

Next, we introduce a B-spline representation for the decision function defined in (5). Let $\{B_1, \ldots, B_K\}$ denote K cubic B-spline basis functions on $[0, 1]$. We approximate each coefficient function by $\beta_j(t) = c_j^{\mathrm{T}} B(t)$, where $c_j = (c_{j1}, \ldots, c_{jK})^{\mathrm{T}} \in \mathbb{R}^K$ and $B(t) = (B_1(t), \ldots, B_K(t))^{\mathrm{T}}$. This treatment has been widely adopted in the literature of FDA, and interested readers can refer to [10] for more details. It should be noted that there are three main advantages of using this representation. Firstly, due to the property of locally compact support, i.e., B-spline basis functions are vanishing in most subintervals defined by the knots, calculation of the integral in (5) can be facilitated. Secondly, we only need to fix c_{j1} to be 1 to entertain the constraint $\beta_j(0) = 1$ for $j = 1, \ldots, p$. Lastly, when β_j satisfies certain smoothness conditions, e.g., the Hölder condition, the B-spline approximation can attain desirable accuracy by Lemma 5 in [19].

With the B-spline representation, the decision function in (5) is approximated by

$$f(x_i(t)) \approx \alpha_0 + \sum_{j=1}^{p} \alpha_j \int_0^1 \sum_{k=1}^{K} c_{jk} B_k(t) x_{ij}(t) \mathrm{d}t$$

$$= \alpha_0 + \sum_{j=1}^{p} \alpha_j \sum_{k=1}^{K} c_{jk} \int_0^1 B_k(t) x_{ij}(t) \mathrm{d}t$$

$$= \alpha_0 + \sum_{j=1}^{p} \alpha_j (c_j^{\mathrm{T}} u_{ij}), \tag{6}$$

where $u_{ij} = \left(\int_0^1 B_1(t) x_{ij}(t) \mathrm{d}t, \ldots, \int_0^1 B_K(t) x_{ij}(t) \mathrm{d}t \right)^{\mathrm{T}}$ and each component in u_{ij} can be approximated via a finite Riemann sum.

2.3 Algorithm

Now we present the details to solve the minimization problem at (4). Let $\alpha = (\alpha_1, \ldots, \alpha_p)^{\mathrm{T}}$ and $c = (c_1^{\mathrm{T}}, \ldots, c_p^{\mathrm{T}})^{\mathrm{T}}$. A coordinate descent algorithm is employed to iteratively update the parameters α and c.

Given an initial value of c_j, denoted by $c_j^{(0)}$ for $j = 1, \ldots, p$, we update the values of α_0 and α, denoted by $\alpha_0^{(0)}$ and $\alpha^{(0)}$, in the first step. At this step, functional features are first transformed to scalar features via (6). Hence, solving (4) is converted to solving an L_1-regularized SVM problem. We employ the coordinate descent algorithm proposed in [12] and [15] to address this optimization problem as shown in (3). Then we fix the values of α_0 and α to be $\alpha_0^{(0)}$ and $\alpha^{(0)}$, respectively, and update the value of c_j for $\{j : \alpha_j^{(0)} \neq 0, 1 \leq j \leq p\}$ using the gradient descent. These procedures are iterated until convergence, i.e., some stopping criteria are satisfied.

More specifically, define the Lagrangian function for (4) with constraint (5) as

$$\ell(c) = \|\alpha\|_1 + \lambda \sum_{i=1}^{n} \left[\left\{ 1 - y_i \left(\alpha_0 + \sum_{j=1}^{p} \alpha_j u_{ij}^{\mathrm{T}} c_j \right) \right\}_+ \right]^2,$$

and the index of the support vectors as

$$I_s = \left\{ i : 1 - y_i \left(\alpha_0 + \sum_{j=1}^{p} \alpha_j u_{ij}^{\mathrm{T}} c_j \right) > 0 \right\}.$$

Then the partial derivatives of $\ell(c)$ with respect to c_j for $j \in A_\alpha := \{j : \alpha_j \neq 0, 1 \leq j \leq p\}$ are

$$\frac{\partial \ell}{\partial c_j} = \left(0, \frac{\partial \ell}{\partial c_{j2}}, \ldots, \frac{\partial \ell}{\partial c_{jK}} \right)^{\mathrm{T}}$$

$$= \left[0, -2\lambda \sum_{i \in I_s} y_i \alpha_j u_{ij2} \left\{ 1 - y_i \left(\alpha_0 + \sum_{j=1}^{p} \alpha_j u_{ij}^{\mathrm{T}} c_j \right) \right\}, \ldots, \right.$$

$$\left. -2\lambda \sum_{i \in I_s} y_i \alpha_j u_{ijK} \left\{ 1 - y_i \left(\alpha_0 + \sum_{j=1}^{p} \alpha_j u_{ij}^{T} c_j \right) \right\} \right]^{\mathrm{T}}.$$

We update c_j for those j's in the active set A_α by the gradient descent:

$$c_j^{(r+1)} = c_j^{(r)} - \eta \left. \frac{\partial \ell}{\partial c_j} \right|_{c_j^{(r)}},$$

where η is the learning rate and $c_j^{(r)}$ denotes the value of c_j at the rth iteration. Note that at each iteration the first element of c_j is fixed to be 1.

The algorithm stops when either the maximum number of iterations, *maxiter*, is reached or

$$\max\{|\alpha_0^{(r+1)} - \hat{\alpha}_0^{(r)}|, \max_{1 \leq j \leq p} |\alpha_j^{(r+1)} - \alpha_j^{(r+1)}|,$$

$$\max_{1 \leq j \leq p} \|c_j^{(r+1)} - c_j^{(r)}\|_1\} \leq \epsilon,$$

where $\|a\|_1$ denotes the L_1-norm of a vector a and ϵ is a pre-determined tiny constant. Algorithm 1 details the implementations of the L_1-regularized functional SVM.

3 Numerical Studies

3.1 Simulation Studies

This subsection is to demonstrate the finite sample performance of the proposed classification method. We are particularly interested in the classifier's prediction accuracy and its performance in feature selection.

Algorithm 1: Coordinate Descent for solving the minimization problem (4) in L_1-fSVM

Initialization

    $\alpha_0^{(0)} = 1$ if $\#\{y_i = 1\} \geq \#\{y_i = -1\}$, else $\hat{\alpha}_0^{(0)} = -1$

    $\alpha^{(0)} = (0, ..., 0)^{\mathrm{T}}$

    $c_j^{(0)} = (1, ..., 1)^{\mathrm{T}}$ for $1 \leq j \leq p$, find u_{ij}

    η = learning rate

    ϵ = tolerance

    $maxiter$ = maximum iterations

    $r = 0$

while *True* **do**

    Step 1:

        Fix c_j to be $c_j^{(r)}$ and find the value of $u_{ij}^{\mathrm{T}} c_j$ for $j = 1, \ldots, p$. Obtain the updated coefficients $\alpha_0^{(r+1)}$ and $\alpha^{(r+1)}$ from the L_1-SVM.

    Step 2:

        Fix $\alpha_0 = \alpha_0^{(r+1)}$ and $\alpha = \alpha^{(r+1)}$, and update $c_j^{(r+1)}$ for $j \in \{j | \alpha_j^{(r+1)} \neq 0, \ j = 1, \ldots, p\}$ using the gradient descent.

$$converged = \max\{|\alpha_0^{(r+1)} - \alpha_0^{(r)}|, \max_{1 \leq j \leq p} |\alpha_j^{(r+1)} - \alpha_j^{(r)}|,$$

$$\max_{1 \leq j \leq p} \|c_j^{(r+1)} - c_j^{(r)}\|_1\} \leq \epsilon,$$

    if *converged == true* **then**

        print("Algorithm Converged.")

        break

    end

    if $r \geq maxiter$ **then**

        print("Reached maximum iterations. Algorithm does not converge.")

        break

    end

    Update:

        $r = r + 1$

end

Return $\hat{\alpha}_0$, $\hat{\alpha}$ and $\hat{c}_j$ for $j = 1, \ldots, p$.

We adopt the data generation mechanism in [13] to generate multiple functional covariates for n subjects. In particular, the j-th functional covariate of the i-th subject, $x_{ij}(t)$, is generated from

$$x_{ij}(t) = \frac{1}{100}\left(5 + \sum_{k=1}^{5}\left[b_{ijk}\sin\left\{2\pi(5 - b_{ijk})\frac{t}{T_j} - m_{ijk}\right\}\right]\right)$$

with

$$b_{ijk} \sim U(0,5), \ \ m_{ijk} \sim U(0,2\pi), \ \ T_j \sim U(50,100),$$

where $U(a,b)$ denotes the uniform distribution on (a,b). We fix the domain of all functional features to be the unit time interval, [0,1]. Additionally, the j-th functional feature is observed at T_j equidistant time points. The labels of subjects are simulated to be balanced such that the number of negative labels is almost equal to that of positive ones in both training and test sets. Moreover, the binary label of each subject is designed to be related to only the first five functional covariates as follows:

$$y_i = \text{sign}\left\{\sum_{j=1}^{5}\int \beta_j(t)x_{ij}(t)\mathrm{d}t\right\},$$

where the first five coefficient functions are :

$$\beta_1(t) = 1, \quad \beta_2(t) = 1 - 2t, \quad \beta_3(t) = \sin\left(5t + \frac{\pi}{2}\right),$$

$$\beta_4(t) = \exp(2t), \quad \beta_5(t) = \log(2t+1) + 1.$$

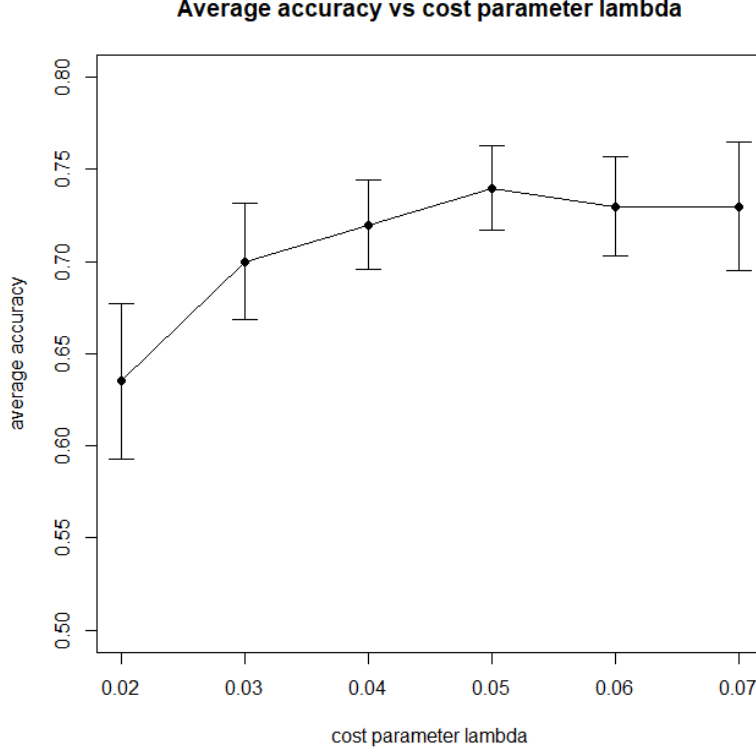**Average accuracy vs cost parameter lambda**

Figure 1: 5-fold cross-validation accuracy with one standard error under $n = 200$ and $p = 20$, given $K = 5$.

Further, random noises are added to the response such that each subject has a 0.1 probability of being mislabelled.

To demonstrate the performance of the proposed classifier in feature selection and prediction accuracy, we compare it with the group-lasso-based functional logistic regression model (grplFlogit) proposed by [11]. This alternative method approximates coefficient functions using a linear combination of basis functions. In contrast to our method, it imposes a group lasso penalty on the coefficients of basis functions to achieve feature selection. Six simulation scenarios are designed to assess their performances; the number of subjects in the training set $n \in \{100, 200\}$ and the number of functional features $p \in \{10, 20, 40\}$ are considered. Additionally, a test set of size 200 is generated to assess the prediction accuracy of these two classifiers in each simulation trial, and 100 independent simulation trials are conducted under each scenario.

There are two tuning parameters, the cost parameter λ and the number of basis functions K in L_1-fSVM. The cost parameter controls the sparsity of α or the number of selected functional covariates. A smaller λ would lead to fewer functional covariates selected in the final classifier. The number of basis functions, however, determines the smoothness of the estimated coefficient functions. A smaller K would lead to a smoother estimator of the coefficient functions.

A 5-fold cross-validation procedure is employed to select the optimal combination of tuning parameters for both classifiers. Figure 1 displays how the cross-validation accuracy changes with respect to the cost parameter λ given the number of basis functions $K = 5$. The highest accuracy is attained at $\lambda = 0.05$. Thus $\lambda = 0.04$ is selected since the corresponding accuracy value of $\lambda = 0.04$ is within one standard error of that of $\lambda = 0.05$, and it can yield a more sparse estimate of α. Consequently, fewer functional covariates are identified as relevant features of the binary outcome.

Figure 2 depicts the boxplots of the prediction accuracy calculated on the test set for these two classification methods. For both classifiers, the medians of prediction accuracy are above 80% in all simulation scenarios. Moreover, when n is fixed, the prediction accuracy of both classifiers decreases as p, the number of functional covariates, increases. This finding is consistent with the curse of dimensionality in the setting of multivariate data classification. In contrast, when p is fixed, the prediction accuracy of both classifiers is enhanced when sample size increases. L_1-fSVM compares favorably with grplFlogit in terms of prediction accuracy in most simulation settings.
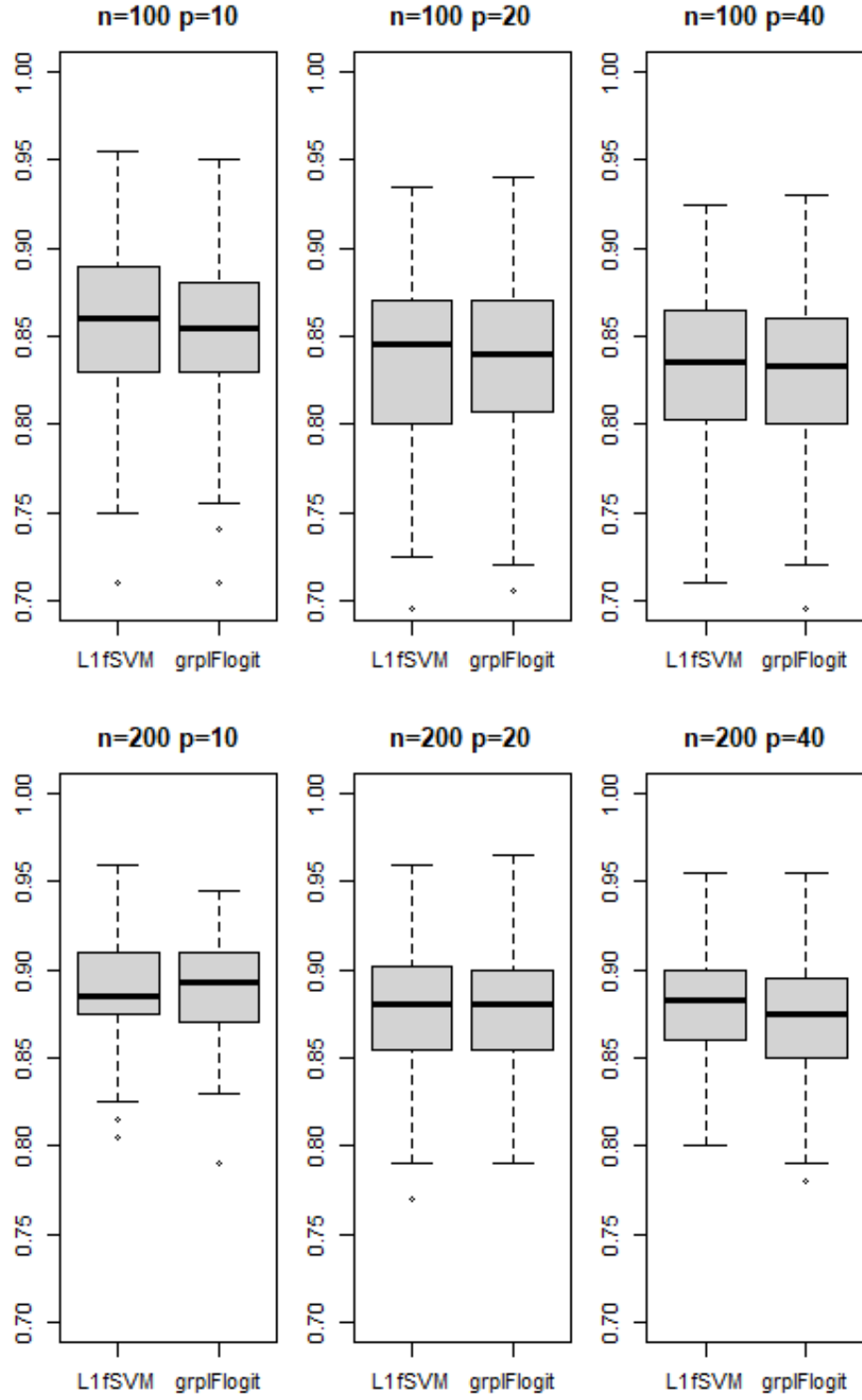
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Figure 2: Boxplots of test accuracy for L1-fSVM and grplFlogit under different combinations of n and p.

	FP rates		FN rates	
	L_1-fSVM	grplFlogit	L_1-fSVM	grplFlogit
$n = 100,\ p = 10$	.00 (.18)	.00 (.40)	.40 (.18)	.40 (.22)
$n = 100,\ p = 20$	.07 (.09)	.07 (.27)	.60 (.16)	.40 (.21)
$n = 100,\ p = 40$	.03 (.06)	.06 (.16)	.60 (.17)	.40 (.17)
$n = 200,\ p = 10$	.00 (.16)	.20 (.33)	.40 (.11)	.20 (.14)
$n = 200,\ p = 20$	.07 (.10)	.20 (.11)	.40 (.10)	.20 (.14)
$n = 200,\ p = 40$	.09 (.04)	.20 (.07)	.40 (.11)	.20 (.15)

Table 1: Medians of the False Positive (FP) rates and False Negative (FN) rates of feature selection for L1-fSVM and grplFlogit across the 100 simulation runs, with standard error presented in parentheses.

To compare the performance in feature selection of the two classifiers, the false positive (FP) rate and the false negative (FN) rate are calculated under each simulation scenario. Table 1 summarizes the medians of their FP rates and FN rates across 100 simulation runs. For both classifiers, when n is fixed, as p increases, the FP rate increases, while the FN rate remains almost unchanged. Moreover, the FP rate of L_1-fSVM is smaller than that of grplFlogit regardless of sample size or the number of functional covariates. In contrast, the FN rate of L_1-fSVM is larger than that of grplFlogit. One possible reason is that grplFlogit tends to select more functional covariates than L_1-fSVM. Hence the grplFlogit is less likely to commit a type II error, that is, the grplFlogit has a larger power or a smaller FN rate.

3.2 Real Data Analysis

In this subsection, both L_1-fSVM and grplFlogit were applied to an electroencephalogram (EEG) dataset obtained from the UCI Machine Learning Repository.

For each subject in the dataset, EEG signals were collected by $p = 64$ electrodes placed at the 64 specific locations on the head. As explained in [26], the entire 10/20 international montage and 41 extra sites were utilized according to the Standard Electrode Position Nomenclature proposed by American Electroencephalographic Association in 1990. Additionally, one nose electrode, channel "nd", and two bipolar deviations recording electrooculogram, channels "X" and "Y", were also utilized. Figure 3 [27] shows the locations of the electrodes, where channels "FPz" and "Oz" represent the front most channel and the back most channel on the scalp, respectively. During the experiments, each electrode collected signals with a frequency of 256 Hz for 1 second. Each subject took at most 120 trials where 3 different stimuli were presented. However, only the trials with a single stimulus (S1) were considered in our study. Signals from the multiple trials with the S1 stimulus were averaged at each frequency for each EEG channel per participant. The trials that contained missing data were removed from the study. Among all the participants included in the study, 77 participants were labeled as alcoholic and 45 participants were labeled as nonalcoholic. The main purpose of this study is to predict the alcoholism status and identify the channels that are associated with alcoholism.

To assess the prediction performance of these two classifiers, L_1-fSVM and grplFlogit, the whole dataset was randomly divided into a training set of size 98 and a test set of size 24. We employed 5-fold cross-validation with the one-standard-error rule to choose tuning parameters when training these two classifiers. The prediction accuracy was calculated on the test set for both classifiers. 100 random splits were considered to better evaluate the uncertainty in prediction. Figure 4 shows the histograms of the prediction accuracy for both classifiers, where the median values of the prediction accuracy are marked as red vertical lines in these two histograms. These two classifiers achieved almost the same median prediction accuracy across the 100 splits.

If we chose the mode of the selected tuning parameters across 100 splits, which are $\lambda = 0.05$ and $K = 5$ for L_1-fSVM, we found out that channels "FP2", "F3", "Cz", "Pz", "PO1", "O1", "X", "FT8", "AFz", "CP3", "PO7", "POz", "nd" and "Y" are identified to be associated with alcoholism. In contrast, taking the same strategy for grplFlogit, channels "F3", "Cz", "P4", "O2", "X", "AF8", "AFz", "PO7", "PO8", "nd" and "Y" were identified. These channels are mainly located in the frontal region and the occipital region of the brain, which are related to self-movement and visual function. The "FT8" channel selected by grplFlogit is located in the temporal region on the right side of the brain, which is related to the formation of memory. All these functions are associated with alcoholism.
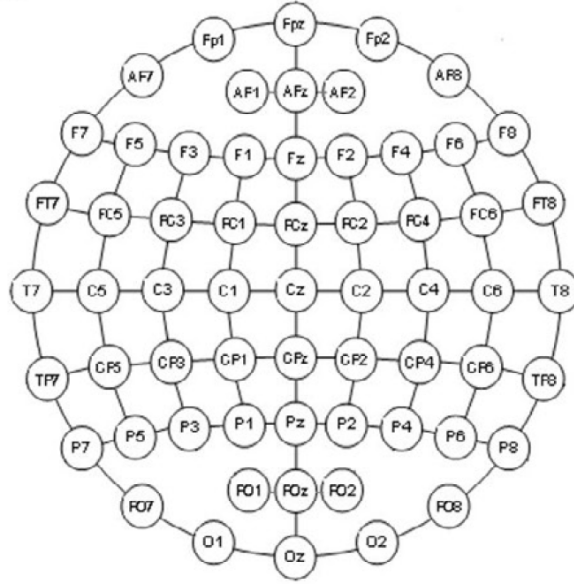
Figure 3: The 61 locations of EEG channels on the scalp according to the Standard Electrode Position Nomenclature, American Electroencephalographic Association 1990.
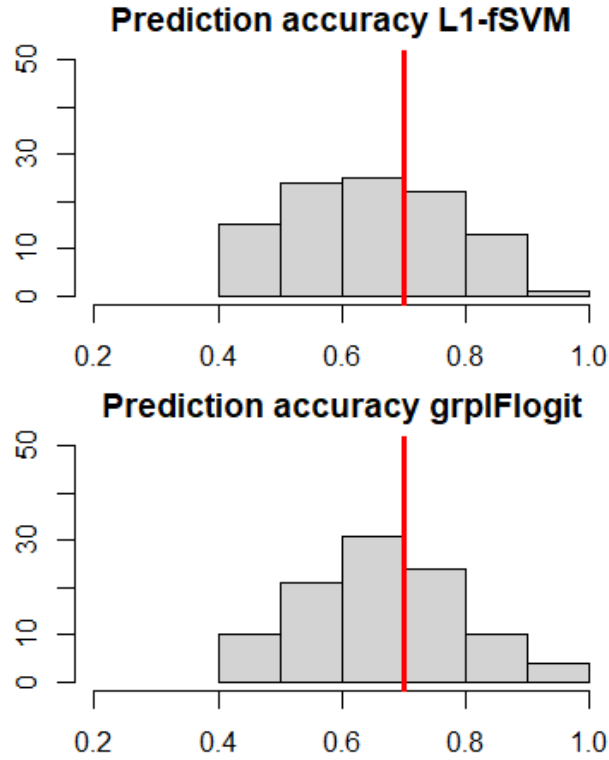

Figure 4: The histograms of prediction accuracy of L_1-SVM and grplFlogit for the UCI EEG dataset across 100 splits. The red vertical lines indicate the median of prediction accuracy.

4 Conclusion

In this paper, we propose an L_1-regularized functional support vector machine for functional data classification. Additionally, we develop a coordinate descent-based algorithm to estimate the functional coefficients and select functional covariates that are relevant to the binary response. Compared with the existing method in the literature, the proposed classifier achieves better prediction accuracy in simulation settings and shows a comparable performance in feature selection. We then apply the proposed classifier to an EEG dataset to predict the status of alcoholism. We identify several channels (regions) on the scalp that may be associated with the status of alcoholism.

[29] proposed a functional distance-weighted discrimination (DWD) classifier. Let $\hat{f}$ and f^* denote the estimated functional DWD classifier and the Bayes classifier, respectively. The paper studied the non-asymptotic upper bound on the excessive risk, which is defined as $L(\hat{f}, f^*) = E\{\ell(\hat{f}) - \ell(f^*)\}$ when the loss function ℓ satisfies mild conditions. It is worthwhile to consider the non-asymptotic bound on the excessive classification risk for this proposed classifier. Actually, the loss function used in the DWD classifier, $\ell_q(u) = 1 - u$ if $u \leq \frac{q}{1+q}$ and $\frac{1}{u^q}\frac{q^q}{(q+1)^{q+1}}$ otherwise, is pretty close to the hinge loss used in the SVM when q is sufficiently large. Thus this result can be generalized to a functional SVM classifier with a smooth penalty, $J(\beta) = \int_0^1 \{\beta''(t)\}^2 dt$. However, in our context, there are multiple functional covariates and the task of functional covariate selection is involved. Consequently, establishing a non-asymptotic error bound becomes a challenging task. It will be left for future research.

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