CS 181 - Homework 5

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Section 1B

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Problem 1

To find the equivalence classes for any language, we have to essentially consider all possible state configurations that language could get into. This basically defines the congruence classes.

a

$$\varepsilon, 0\Sigma^*, 1(0^*1)^*, 1(0^*1)^*0^+$$

b

$$0^*, 0^*10^*, 0^*10^*10^*, 0^*10^*10^*1\Sigma^*$$

c

$$(1 \cup 01 \cup 001)^*, (1 \cup 01 \cup 001)^*0, (1 \cup 01 \cup 001)^*00, \Sigma^*000\Sigma^*$$

Problem 2

a

For $i \neq j$, we have $0^i \equiv L0^i$ because we get the following equivalence classes:

$$0^i1^i2^i \in L,$$

$$0^j1^i2^i \notin L$$

This essentially means that each string $\varepsilon, 0, 00, ..., 0^n, ...$ each belongs in its own equivalence class which leads to a generation of *infinite* equivalence classes in $\equiv L$, which thereby, by the **Myhill-Nerode** theorem means L is non-regular.

b

For $i \neq j$, we have $0^i \equiv L0^i$ because we get the following equivalence classes:

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This essentially means that each string $\varepsilon, 0, 00, ..., 0^n, ...$ each belongs in its own equivalence class in $\equiv L$ which leads to a generation of *infinite* equivalence classes, which thereby, by the **Myhill-Nerode** theorem means L is non-regular.

c

For $i \neq j$, we have $0^i \equiv L0^i$ because we get the following equivalence classes:

$$0^i 2^{k-i} \in L,$$

$$0^j 2^{k-i} \notin L$$

This essentially means that each string $\varepsilon, 0, 00, ..., 0^n, ...$ each belongs in its own equivalence class in $\equiv L$ which leads to a generation of *infinite* equivalence classes, which thereby, by the **Myhill-Nerode** theorem means L is non-regular.

Problem 3

a

We see that by this language, $\varepsilon \in L$ and $0 \notin L$ which means they belong in different equivalence classes. This also means that by the **Myhill-Nerode** theorem, only 2 states need to exist for those equivalence classes. This yields to the following DFA of smallest size:

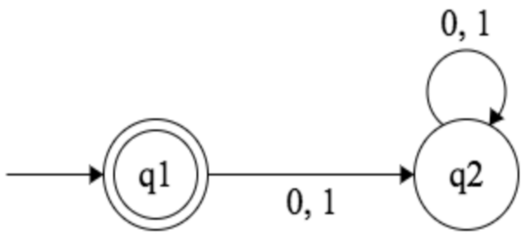

Figure 1: Smallest DFA for the language $L = \{\varepsilon\}$

b

We see by this language that for $0 < i < j \leq 2$, we have $0^i \equiv L0^i$ because

$$0^j 0^{2-i} \in L,$$

$$0^i 0^{2-j} \notin L$$

This shows us that $\varepsilon, 0, 00$ belong in their own equivalence classes. Thus by the **Myhill-Nerode** theorem, we can generate the smallest DFA as follows with just *3* states:

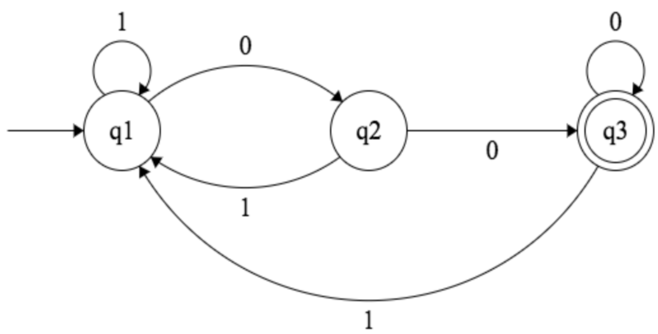

Figure 2: Smallest DFA for the language $L = \{w :$ w ends with 00$\}$