CS 181 - Homework 4

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Section 1B

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Problem 1.21

The key here is to add a new start and accept state, and remove all chosen states after adding donors and receivers, adding "shortcuts" for each donor to receiver that bypasses the chosen state.

a Here we remove states 1 and 2 to get

$$a^* b\,(a \cup ba^* b)^*$$

b Here we remove states 1,2 and 3, and remember that accept states can take some ε, therefore getting

$$\varepsilon \cup \big((a \cup b)\, a^* b\, ((b \cup a\,(a \cup b))\, a^* b)^*\, (\varepsilon \cup a)\big)$$

Problem 1.28

a

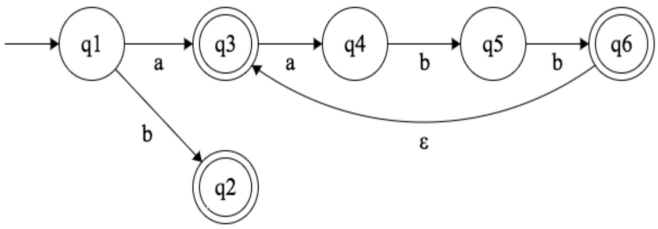

Figure 1: NFA for the regular expression $a(abb)^* \cup b$

b

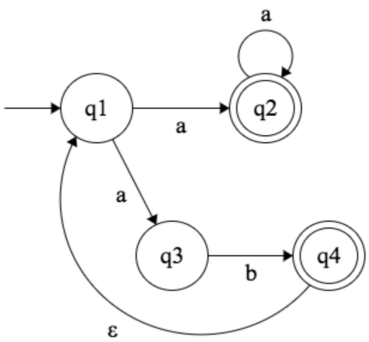

Figure 2: NFA for the regular expression $a^+ \cup (ab)^+$

c

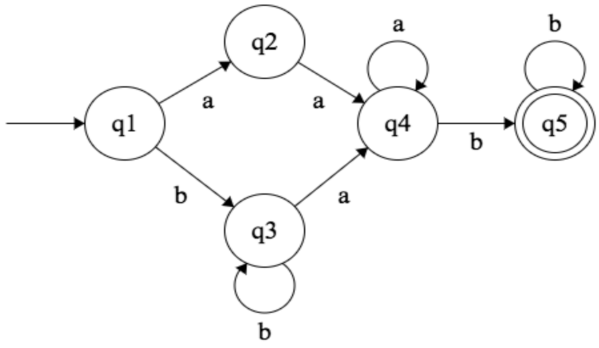

Figure 3: NFA for the regular expression $(a \cup b^{+})a^{+}b^{+}$

Problem 1.47

We can use the *pumping lemma* to prove that **Y** is not regular by contradiction. Let us start by assuming that **Y** is regular, and obtain the *pumping length* **p**. Let us say then that

$$w = 1^{p!}\#1^{2p!}$$

By the pumping lemma we can say that $w = xyz$, satisfying the conditions of the pumping lemma. Based on the condition $|xy| \leq p$, we see that y lies on the left hand side of 1's. Let

$$k = \frac{p!}{|y|}$$

We know that k is an integer, since $|y|$ is a divisor of $p!$. Therefore, if we pump k 1's then we get

$$w = xy^{1+k}z \equiv 1^{2p!}\#1^{2p!}$$

But we see that w does not exist in the language **Y** and therefore, it is not regular.

Problem 1.49

a We can just notice that

$$B = 1\Sigma^{*}1\Sigma^{*}$$

Therefore since we can produce a regular expression for the language, then **B** is regular.

b We can show that **C** is not a regular language by the pumping lemma. We begin by assuming that **C** is a regular language, that has a pumping length p. Let

$$w = 1^{p}01^{p} \equiv xyz$$

such that $y \neq \varepsilon$ and $|xy| \leq p$. This means that in our string we have $y = 1^{k}$ where $k \geq 1$. By pumping down we get the string

$$w = xz \equiv 1^{p-k}01^{p}$$

where for $xy^{i}z \notin C$ for $i \geq 0$. Since, xz is not in the language, **C** is not regular.

Problem 1.53

By the pumping lemma, let us assume ADD is regular with some pumping length p. We can then say we have the string

$$1^{p} = 0 + 1^{p}$$

as a member of ADD. Doing so, we can say that by the pumping lemma this string w can be split into

$$w = xyz$$

, where $|xy| \leq p$, where $y = 1^k$ for some $k \geq 1$ such that by pumping up to

$$xy^2z \equiv 1^{p+k} = 0 + 1^p$$

We see that this string is not in the language ADD when it should be by the pumping lemma, therefore proving that ADD is not regular.