CS M151B - Homework 3

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Section 1B

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Output	Equation	Calculation
G_0	$a_0 \cdot b_0$	**2T**
P_0	$a_0 \oplus b_0$	**2T**
G_α	$G_3 + G_2 \cdot P_3 + G_1 \cdot P_2 \cdot P_3 + G_0 \cdot P_1 \cdot P_2 \cdot P_3$	$5T + (2T + 5T) = \textbf{12T}$
P_α	$P_0 \cdot P_1 \cdot P_2 \cdot P_3$	$2T + 5T = \textbf{7T}$
C_{12}	$G_\gamma + G_\beta \cdot P_\gamma + G_\alpha \cdot P_\beta \cdot P_\gamma + C_0 \cdot P_\alpha \cdot P_\beta \cdot P_\gamma$	$(12T + 3T) + 5T = \textbf{20T}$
C_{15}	$G_{14} + G_{13} \cdot P_{14} + G_{12} \cdot P_{13} \cdot P_{14} +$ $C_{12} \cdot P_{12} \cdot P_{13} \cdot P_{14}$	$(20T + 5T) + 5T = \textbf{30T}$
C_{16}	$G_\sigma + G_\gamma \cdot P_\sigma + G_\beta \cdot P_\gamma \cdot P_\sigma +$ $G_\alpha \cdot P_\beta \cdot P_\gamma \cdot P_\sigma + C_0 \cdot P_\alpha \cdot P_\beta \cdot P_\gamma \cdot P_\sigma$	$(12T + 5T) + 7T = \textbf{24T}$
S_{15}	$(a_{15} \oplus b_{15}) \oplus C_{15}$	$(30T + 3T) = \textbf{33T}$
C_{20}	$G_{19} + G_{18} \cdot P_{19} + G_{17} \cdot P_{18} \cdot P_{19} +$ $G_{16} \cdot P_{17} \cdot P_{18} \cdot P_{19} + (1|0) \cdot P_{16} \cdot P_{17} \cdot P_{18} \cdot P_{19}$	$(7T + 2T) + 7T = \textbf{16T}$
S_{19}	$(a_{19} \oplus b_{19}) \oplus C_{19}$ $\rightarrow C_{19} = G_{18} + G_{17} \cdot P_{18} + G_{16} \cdot P_{17} \cdot P_{18} +$ $(1|0) \cdot P_{16} \cdot P_{17} \cdot P_{18}$ $C_{19} = (5T + 2T) + 5T = 12T$	$(12T + 3T) = \textbf{15T}$
C_{24}	$G_{23} + G_{22} \cdot P_{23} + G_{21} \cdot P_{22} \cdot P_{23} +$ $G_{20} \cdot P_{21} \cdot P_{22} \cdot P_{23} + C_{20} \cdot P_{20} \cdot P_{21} \cdot P_{22} \cdot P_{23}$	$(16 + 7T) + 7T = \textbf{30T}$
C_{31}	$G_{30} + G_{29} \cdot P_{30} + G_{28} \cdot P_{29} \cdot P_{30} + C_{28} \cdot P_{28} \cdot P_{29} \cdot P_{30}$ $\rightarrow C_{28} = G_{27} + G_{26} \cdot P_{27} + G_{25} \cdot P_{26} \cdot P_{27} +$ $G_{24} \cdot P_{25} \cdot P_{26} \cdot P_{27} + C_{24} \cdot P_{24} \cdot P_{25} \cdot P_{26} \cdot P_{27}$ $C_{28} = (30T + 7T) + 7T = 44T$	$(44T + 5T) + 5T = \textbf{54T}$
C_{32} (after MUX)	$G_{31} + G_{30} \cdot P_{31} + G_{29} \cdot P_{30} \cdot P_{31} +$ $G_{28} \cdot P_{29} \cdot P_{30} \cdot P_{31} + C_{28} \cdot P_{28} \cdot P_{29} \cdot P_{30} \cdot P_{31}$	$((44T + 7T) + 7T) + 4T = \textbf{62T}$
S_{31} (after MUX)	$\max(C_{16}, (a_{31} \oplus b_{31}) \oplus C_{31}) + 4T$	$\max(24T, (54T + 3T)) + 4T = \textbf{61T}$

The table above summarizes the calculations used to get the output delays based on the maximal delay per Sum of Product Equation, which takes into account the fan-in delays as per the

problem statement. Note that the $+$ and $\cdot$ operators are used to denote the SoP when used with the Generates and Propagates, and represent the mathematical interpretation of the symbols when used with delay calculation. In the instances where we required more intermediate calculations, we denote it with the $\rightarrow$ symbol. It is also worth mentioning that the final Sum after the mux is calculated as a maximum between C_{16} and the sum before the mux because of the fact that C_{16} is the control for the mux, and if this delay is higher than the delay of the sum itself, then it would be the maximal delay for the sum. Any mention of $4T$ implies use of MUX delay. Thus we have:

$$\mathbf{Max\_Delay} = \max(C_{32}, S_{31}) = \max(62T, 60T) = \mathbf{62T}$$