CSE 15: Discrete Mathematics
Fall 2021
Homework #6
Solution

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1. **Question 1:** Recursively defined functions
   Find $f(1) - f(5)$ if $f(n)$ is defined recursively as $f(0) = 3$

   (a) $f(n+1) = -2f(n)$
   - $f(1) = -2(3) = -6$
   - $f(2) = -2(-6) = 12$
   - $f(3) = -2(12) = -24$
   - $f(4) = -2(-24) = 48$
   - $f(5) = -2(48) = -96$

   (b) $f(n+1) = 3f(n) + 7$
   - $f(1) = 3(3) + 7 = 16$
   - $f(2) = 3(16) + 7 = 55$
   - $f(3) = 3(55) + 7 = 172$
   - $f(4) = 3(172) + 7 = 523$
   - $f(5) = 3(523) + 7 = 1576$

   (c) $f(n+1) = f(n)^2 - 2f(n) - 2$
   - $f(1) = 3^2 - 2(3) - 2 = 1$
   - $f(2) = 1^2 - 2(1) - 2 = -3$
   - $f(3) = -3^2 - 2(-3) - 2 = 13$
   - $f(4) = -5^2 - 2(-3) - 2 = 141$
   - $f(5) = -17^2 - 2(-17) - 2 = 19597$

   (d) $f(n+1) = 3^{\frac{f(n)}{3}}$
   - $f(1) = 3^{\frac{3}{3}} = 3$
   - $f(2) = 3^{\frac{3}{3}} = 3$
   - $f(3) = 3^{\frac{3}{3}} = 3$
   - $f(4) = 3^{\frac{3}{3}} = 3$
   - $f(5) = 3^{\frac{3}{3}} = 3$

2. **Question 2:** Recursively defined sequences

   (a) $a_n = 4n - 2$

   - Base case: $a_1 = 4(1) - 2 = 2$
     Next 3: $a_2 = 6, a_3 = 10, a_4 = 14$
   - Recursive formula: $a_n = a_{n-1} + 4$

   (b) $a_n = 1 + (-1)^n$

   - Base case: $a_1 = 1 + (-1)^1 = 0$
     Next 3: $a_2 = 2, a_3 = 0, a_4 = 2$
   - Recursive formula: $a_n = 2 - a_{n-1}$

   (c) $a_n = n(n-1)$

   - Base case: $a_1 = 1(1-1) = 0$
     Next 3: $a_2 = 2, a_3 = 6, a_4 = 12$
   - Recursive formula: $a_n = a_{n-1} + 2n - 2$

   (d) $a_n = n^2$

   - Base case: $a_1 = 1^2 = 1$
     Next 3: $a_2 = 4, a_3 = 9, a_4 = 16$
   - Recursive formula: $a_n = a_{n-1} + (a_{n-1} - a_{n-2} + 2)$

3. **Question 3:** Recursively defined sets
   S is a set of strings made up of an equivalent amount of 0 and 1, this much is true.
   However, when aknowledging the inductive step, 0x1 is now restricting how the 0 and 1s can be ordered.x is simply part of the string, which means x is capable of being 0 or 1 respectively.
   the difference is, it must be in between 0 and 1, and neither 0 and 1 can be swapped with eachother.
   Examples of this: 001 or 011.