CSE 015: Discrete Mathematics
Fall 2021
Homework #2
Solution

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1. **Question 1:** Quantifiers
   Let $P(x) = x < x^3$.
   Determine truth values for each of the following formulas:

   (a) $P(2)$
   - True. $2 < 2^3 \rightarrow 2 < 8$

   (b) $P(-1)$
   - False. $-1 < -1^3 \rightarrow -1 = -1$

   (c) $\forall x P(x)$
   - False. This is false whenever $x \leq -1, x = 0, x = 1 \rightarrow -1 = -1^3, 0 = 0^3, 1 = 1^3$

   (d) $\exists x P(x)$
   - True. We already know this is true, as part a exhibits a value of x where P(x) is valid.

   (e) $\exists! x P(x)$
   - False. Since we know there is at least 1 valid x, in part a, and that pard d is true, this is false.

2. **Question 2:** Translating English Sentences into Formulas
   Consider the following predicates:
   $S(x)$: x is a student in CSE015.
   $M(x)$: x plays a musical instrument.
   Let domain be the set of all people. Using these predicates, translate the following sentences into formulas using hte appropriate quantifiers and operators.

   (a) Not every student in CSE015 plays a musical instrument
   - $\forall x(S(x) \rightarrow M(x))$

   (b) A person is either a student in CSE015 or plays a musical instrument, but not both.
   - $\exists x(S(x) \oplus M(x))$

   (c) There exists at least one student in CSE015 who does not play a musical instrument.
   - $\exists x(S(x) \rightarrow \neg(M(x))$

3. **Question 3:** Logical Equivalence

$$\forall x(A(x) \wedge B(x)) \equiv \forall x(A(x) \rightarrow B(x)) \tag{1}$$

- This logical equivalence is not valid. If $A(x)$ is false and $B(x)$ is true, then the left side of the equation would be false. However, the right side of the equation would be true because $A(x) \rightarrow (B(x)$ is true when $B(x)$ is true, regardless of $A(x)$ being false. Therefore, since $\forall x$ is stating that these predicates are equivalent through all the domain, this logical equivalency is not valid because they are not equivalent through all domain. The same can be noted when both $A(x)$ and $B(x)$ are false. The left is false, and the right is true.
- Truth table:

$A(x)$	$B(x)$	$A(x) \wedge B(x)$	$A(x) \rightarrow B(x)$	$A(x) \wedge B(x) \equiv A(x) \rightarrow B(x)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	F
F	F	F	T	F

4. **Question 4:** Nested Quantifiers
The following two statements are defined for real numbers:
$A(x, y)$ is the statement $xy = 0$.
$B(x, y)$ is the statement $x + y = 0$
For each of the following formulas determine their truth values.

   (a) $\exists x \forall y A(x, y)$
   - True. There is one x for all y which validates $xy = 0$, that is $x = 0$.

   (b) $\exists x \exists y B(x, y)$
   - True. There is one x and one y which validates $x + y = 0$. $x = -1$ and $y = 1$ or vice-versa. Any positive integer with its counterpart negative will result in this.

   (c) $\forall x \exists y A(x, y)$
   - True. For all x there is one y which validates $xy = 0$, that is $y = 0$

   (d) $\exists x \forall y(A(x, y) \wedge (B(x, y))$
   - False. There is one x for all y which validates $A(x, y)$, $x = 0$, but not $B(x, y)$. Because the formula states that both functions are valid with $\exists x \forall y$, this is false.

   (e) $\exists x \exists y(A(x, y) \wedge \neg B(x, y))$
   - True. There is at least one combination of x and y which validates $A(x, y)$ and does not validate $B(x, y)$. That combination is any integer x or y paired with 0 for either x or y.

5. **Question 5:** Negating formulas with Nested Quantifiers
Write the negation of the following statements so that the negation never appearse in front of a quantifier or of an expression involving logical connectives.

   (a) $\exists x \exists y(P(x) \rightarrow Q(y))$
   - $\neg \exists x \exists y(P(x) \rightarrow Q(y))$ becomes:
   - $\exists x \exists y(\neg P(x) \rightarrow \neg Q(y))$

   (b) $\exists y(\exists x A(x, y) \vee \forall x B(x, y))$
   - $\neg \exists y(\exists x A(x, y) \vee \forall x B(x, y))$ becomes:
   - $\exists y(\exists x \neg A(x, y) \vee \forall x \neg B(x, y))$