CSE 15: Discrete Mathematics
Fall 2021
Homework #5
Solution

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1. **Question 1:** Mathematical Induction 1
   Let $P(n)$ be the statement that

   $$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

   Answer the following questions:

   (a) What is the statement $P(1)$?
   $P(1) = \left(\frac{1(1+1)}{2}\right)^2$

   (b) Basis of induction: show that the statement $P(1)$ is true.
   $P(1) = \left(\frac{1(1+1)}{2}\right)^2$
   $P(1) = \left(\frac{2}{2}\right)^2$
   $P(1) = (1)^2$
   $P(1) = 1$
   $1^3 = 1$

   (c) Inductive hypothesis for $P(n)$
   $P(k) = \left(\frac{k(k+1)}{2}\right)^2$

   (d) The inductuve step for $P(n)$
   $P(k+1) = \left(\frac{k+1(k+2)}{2}\right)^2$
   $P(1+1) = \left(\frac{1+1(1+2)}{2}\right)^2$
   $P(2) = \left(\frac{2(3)}{2}\right)^2$
   $P(2) = \left(\frac{6}{2}\right)^2$
   $P(2) = (3)^2$
   $P(2) = 9$
   $1^3 + 2^3 = 9$

2. **Question 2:** Mathematical Induction 2
   Compute the sum of the first even natural numbers and create a formula for their sum.

(a) First even number is 0: $P(1) = 0$
Second even number is 2: $P(2) = 0 + 2 = 2$
Third even number is 4: $P(3) = 0 + 2 + 4 = 6$
Fourth even number is 6: $P(4) = 0 + 2 + 4 + 6 = 12$
Formula:

$$P(n) = n(n-1)$$

(b) $P(k) = k(k-1) \rightarrow P(k+1) = k + 1(k+1-1)$
Base case: $P(1) = 1(1-1) = 0$
Induction step 1: $P(1+1) = 1 + 1(1+1-1) = 2$
$P(2) = 2(1) = 2$
Induction step 2: $P(2+1) = 2 + 1(2+1-1) = 6$
$P(3) = 3(2) = 6$
Induction step 3: $P(3+1) = 3 + 1(3+1-1) = 12$
$P(4) = 4(3) = 12$

3. **Question 3:** Mathematical Induction 3
Use Mathematical Induction to prove this inequality:

$$n! < n^n$$

(a) What is the statement $P(2)$?
$2! < 2^2$

(b) Proof for P(2):
$2! = 1 * 2$
$2^2 = 4$
$2 < 4$

(c) Inductive hypothesis:
$P(k) = k! < k^k$ where $k > 1$
$P(k+1) = (k+1)! < (k+1)^{k+1}$ where $k > 1$
$P(2) = 2! < 2^2 = 2 < 4$
$P(2+1) = (2+1)! < (2+1)^{2+1}$
$P(3) = 3! < 3^3 = 6 < 27$