CSE 15: Discrete Mathematics
Fall 2021
Homework #3
Solution

Joshua Chin
Lab 021-CSE 015 01L

October 15, 2021

1. **Question 1:** Rules of Inference
   Convert:

   If Jane does not fly, then she is not a bird.
   Jane is a bird.
   $\therefore$ Jane flies.

   Into argument form:

   $$\begin{array}{l} \neg q \to \neg p \\ p \\ \hline \therefore r \end{array}$$

2. **Question 2:** More Rules of Inference
   For each of the following arguments, write w hich rule of inference is used.

   (a) Bats can fly(p) and are mammals(q). Therefore bats are mammals(q).
   $$\begin{array}{l} p \wedge q \\ \hline \therefore q \end{array}$$
   This is Simplification.

   (b) Pigs are mammals(q) or birds(p). Pigs are not birds($\neg p$). Therefore pigs are mammals(q).
   $$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$
   This is Disjunctive Syllogism.

   (c) Jack is a cse major(p). Jack is a freshmen(q). Therefore Jack is a CSE major and a freshmen($p \wedge q$).
   $$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$
   This is Conjunction.

   (d) Mary is a CSE major(p). Therefore Mary is a CSE major or Mary is a History major($p \vee q$).
   $$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

This is Addition.

(e) If I go hiking, I will sweat a lot($p \to q$). If I sweat a lot, I will lose weight($q \to r$). Therefore, if I go hiking, I will lose weight($p \to r$).

$p \to q$
$q \to r$
$\therefore p \to r$

This is Hypothetical Syllogism.

3. **Question 3:** Checking arguments.
State whether the following arguments are correct or not with proof.

(a) If it is sunny, then I will go swimming. It is not sunny. Therefore I will not go swimming.
Argument form:
$p \to q$
$\neg p$
$\therefore \neg q$
This argument is valid. p needs to be true in order for q to be true. Since p is not true, q cannot be true.

(b) If it is Sunday, then I will go to the park. It is not sunday. Therefore I will not go to the park.
$p \to q$
$\neg q$
$\therefore \neg p$
This argument is invalid. p needs to be true only for q to be true. While q is not true, that does not mean p cannot be true.

(c) I will pass the class if and only if I score at least 6F percent on the final exam. I scored 55 percent on the final exam. Therefore, I will not pass the class.
$p \leftrightarrow q$
$\neg q$
$\therefore \neg p$
This argument is valid. Both p and q need eachother to be true. Since one is not true, the other cannot be true.

Question 4: Proof by Contraposition.
if n is an integer and n^2 is odd, then n is odd.
This is:

$$p \to q$$

The contrapositive of this is:

$$\neg p \to \neg q$$

This is the same thing as: if n is an integer and n^2 is even, then n is even. This is a true statement.

5. **Question 5:** Proof by Cases
Prove: $((pT \wedge p2 \wedge p3) \to q) <-> ((pT \to q) \wedge (p2 \to q) \wedge (p3 \to q))$

pT	$p2$	$p3$	q	$pT \to q$	$p2 \to q$	$p3 \to q$	$pT \wedge p2 \wedge p3 \to q$
F	F	F	F	T	T	T	F
F	F	F	T	T	T	T	F
F	F	T	F	T	T	F	F
F	F	T	T	T	T	T	F
F	T	F	F	T	F	T	F
F	T	F	T	T	T	T	F
F	T	T	F	T	F	F	F
F	T	T	T	T	T	T	F
T	F	F	F	F	T	T	F
T	F	F	T	T	T	T	F
T	F	T	F	F	T	F	F
T	F	T	T	T	T	T	F
T	T	F	F	F	F	T	T
T	T	F	T	T	T	T	T
T	T	T	F	F	F	F	F
T	T	T	T	T	T	T	T

$(pT \to q) \wedge (p2 \to q) \wedge (p3 \to q)$	$((pT \wedge p2 \wedge p3) \to q) <-> ((pT \to q) \wedge (p2 \to q) \wedge (p3 \to q))$
T	T
T	T
T	T
F	T
F	T
F	T
F	T
F	T
F	T
T	T

Judging by the final column of the truth table, this is a tautology.