CSE 015: Discrete Mathematics
Fall 2021
Homework #1
Solution

Joshua Chin
Lab F21-CSE 015 07L

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1. **Question 1:** Translating English Sentences
   Translate the following compound propositions to english.
   p: "Josh scored 100% in the CSE015 Final"
   q: "Josh scored at least 90% in the labs."
   r: "Josh receives an A+ in CSE015."
   t: "Josh is a CSE major.

   (a) $\neg t$ = "It is not the case that Josh is a CSE Major
   (b) $\neg q$ = "It is not the case that Josh scored at least 90% in the labs
   (c) $(p \vee q) \rightarrow r$: "If Josh scored 100% in the CSE015 final or if Josh scored 90% in the labs, then Josh gets an A+
   (d) $(p \wedge q) \rightarrow r$: "If Josh scored 100% in the CSE015 final and if Josh scored at least 90% in the labs, then Josh receives an A+ in CSE015
   (e) $\neg(t \rightarrow r)$: "It is not the case that if Josh is a CSE major then he will receive an A+ in CSE015.

2. **Question 2:** Truth Tables

   (a) Truth table for: $p \oplus (q \vee \neg r)$

p	q	r	$\neg r$	$q \vee \neg r$	$p \oplus (q \vee \neg r)$
T	T	T	F	T	F
T	T	F	T	T	F
T	F	T	F	F	T
T	F	F	T	T	F
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	T	T

(b) Truth table for: $(p \vee q) \rightarrow (\neg r \vee p)$

p	q	r	$p \vee q$	$\neg r$	$\neg r \vee p$	$(p \vee q) \rightarrow (\neg r \vee p)$
T	T	T	T	F	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	F	F	T
F	F	F	F	T	T	T

(c) Truth table for: $((p \rightarrow q) \wedge p) \rightarrow q$

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$((p \rightarrow q) \wedge p) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

3. **Question 3:** Logical Equivalencies

(a) Truth table proof for: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (Distributive Property)

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

(b) Truth table proof for: $(p \rightarrow q) \wedge (p \rightarrow r) \equiv (q \wedge r)$

p	q	r	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$	$q \wedge r$	$(p \rightarrow q) \wedge (p \rightarrow r) \equiv (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	F	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F
F	F	T	T	T	T	F	F
F	F	F	T	T	T	F	F

4. **Question 4:** Tautologies, Contingencies, and Contradictions

(a) $p \rightarrow (p \vee q)$: Is a tautology because the compound proposition is always true.
Proof:

p	$p \wedge q$	$\neg p$	$(p \wedge q) \rightarrow \neg p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

(b) $(p \wedge q) \rightarrow \neg p$: Contingency because the compound proposition is not always true or false.
Proof:

p	q	$p \wedge q$	$\neg p$	$(p \wedge q) \rightarrow \neg p$
T	T	T	F	F
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

(c) $(p \rightarrow (q \vee r)) \rightarrow (\neg q \vee p)$: Contingency because the compound proposition is not always true or false.
Proof:

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$	$\neg q$	$\neg q \vee p$	$(p \rightarrow (q \vee r)) \rightarrow (\neg q \vee p)$
T	T	T	T	T	F	T	T
T	T	F	T	T	F	T	T
T	F	T	T	T	T	T	T
T	F	F	F	F	T	T	T
F	T	T	T	T	F	F	F
F	T	F	T	T	F	F	F
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

5. **Question 5:**De Morgan's Laws
Using De Morgan's laws, rewrite the following sentences in English.

$$\neg(p \wedge q) \equiv (\neg p \vee \neg q) \tag{1}$$

$$\neg(p \vee q) \equiv (\neg p \wedge \neg q) \tag{2}$$

(a) You cannot be late and you cannot smoke.

- $\neg p$: "It is not the case that you can be late.
- $\neg q$: "It is not the case that you can smoke.
- p: You can be late.
- q: You can smoke.

By De Morgan's law, this sentence can be translated to: "It is not the case that you can be late or smoke"

(b) It is not the case that you can take an annuity and you can take a lump sum.

- $\neg p$: "You cannot take an annuity"
- $\neg q$: "You cannot take a lump sum"
- p: "You can take an annuity"

- q: "You can take a lump sum

By De Morgan's law, this sentence can be translated to: You cannot take an annuity or a lump sum"