CSE 15: Discrete Mathematics
Fall 2021
Homework #7
Solution

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1. **Question 1:** Asymptotic Notation
   Inorder to determine complexity, you ignore all constants and keep track of the variable n. So if anywhere in the function there exists any n^2, the complexity is $O(n^2)$

   (a) $f(n) = 178n + 45$
   Based on the rule above, in this case there is only one power of n. Therefore, it is not $O(n^2)$.

   (b) $f(n) = nlogn + 12$
   Based on the rule above, in this case there is a single power of n and a logarithm of n. Therefore, it is not $O(n^2)$.

   (c) $f(n) = 34n^2 + 34n + 34$
   Based on the rule above, in this case we see n with a power of 2. Therefore, this function has a complexity of $O(n^2)$.

   (d) $f(n) = \sqrt{n} + 2$
   Based on the rule above, in this case there is only a square root of n. Therefore, it is not $O(n^2)$.

   (e) $f(n) = 0.001n^3 + 72n$
   Based on the rule above, in this case we see n with a power of 3 and n with a power of n. Therefore it is not $O(n^2)$.

2. **Question 2:** Asymptotic Notation 2
   This is the order of the 9 functions in from fastest Big-O complexity to slowest:

   (a) $logn$
   (b) $\sqrt{n}$
   (c) n
   (d) $nlogn$
   (e) n^2
   (f) n^2logn
   (g) n^4
   (h) 2^n

(i) 3^n

3. **Question 3:** Asymptotic Growth
For this, we find the highest value of n that Computer A and B can solve within an hour with the the inequality $f(n) \leq g(n)$ where $f(n)$ is the algorithm and $g(n)$ is the computer.
Since we are inluding all numbers up to and including the highest value, simply equating both sides of the inequality will also work.

   (a) Computer A: $3.6 * 10^9$

      (a) Algorithm 1: $5n^2 + 34n + 12$
      $n = 26829$, it is given.

      (b) Algorithm 2: $10n + 4$
      $10n + 4 = 3.9 * 10^9$
      $10n = 3600000000 - 4$
      $n = 3599999996/10$
      $n = 359999999.6$

      (c) Algorithm 2: 2^n
      $2^n = 3.9 * 10^9$
      $nln(2) = ln(3600000000)$
      $n = \frac{ln(3600000000)}{ln(2)}$
      $n \approx 31.75$

   (b) Computer B: $3.6 * 10^{11}$

      (a) Algorithm 1: $5n^2 + 34n + 12$
      $5n^2 + 34n + 12 = 3.6 * 10^{11}$
      $5n^2 + 34 + 360000000012 = 0$
      Solve with quadratic formula: $n \approx 268234.76$

      (b) Algorithm 2: $10n + 4$
      $10n + 4 = 3.6 * 10^{11}$
      $10n = 359999999996$
      $n = 359999999996/10$
      $n = 35999999999.6$

      (c) Algorithm 2: 2^n
      $2^n = 3.6 * 10^{11}$
      $nln(2) = ln(360000000000)$
      $n = \frac{ln(360000000000)}{ln(2)}$
      $n \approx 38.39$