In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import math
import seaborn as sns
import statsmodels.api as sm
from sklearn.model_selection import train_test_split
from scipy import stats
```

In [2]:
```
## Reading the data from CSV file.
data = pd.read_csv('input_data.csv')
data
```

Out[2]:

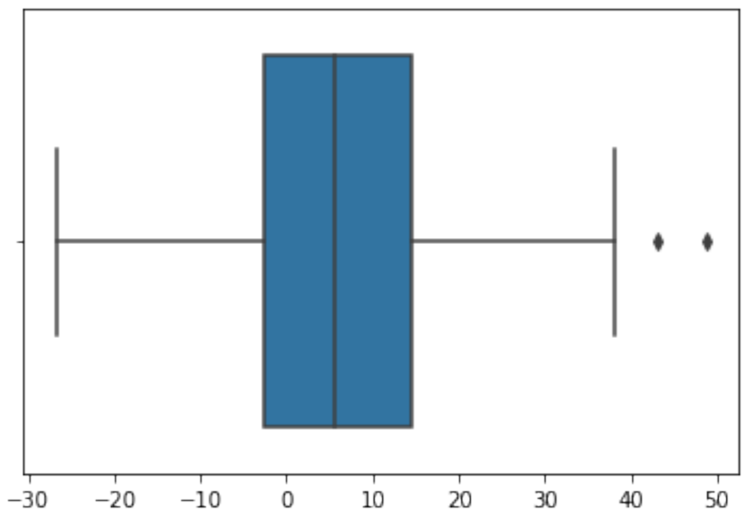
	X1	X2	X3	X4	X5	Y
0	14.47200	23.5520	67.921	76.212	88.292	1583.2
1	8.77720	23.4560	45.202	74.720	95.002	1367.7
2	13.00300	17.8300	67.540	64.430	114.630	1655.2
3	-6.35780	30.4270	46.573	93.653	67.497	1239.7
4	25.58100	7.4073	30.896	72.519	98.709	2010.4
...	...	...	...	...	...	...
295	9.24530	10.1590	80.555	72.726	99.786	1503.8
296	0.38753	44.1610	80.662	72.227	115.950	1639.8
297	11.52900	-12.7620	57.399	71.702	116.500	1524.2
298	-4.37280	18.8140	65.591	99.102	118.160	1699.5
299	-20.80500	30.3240	57.158	66.992	103.660	1918.0

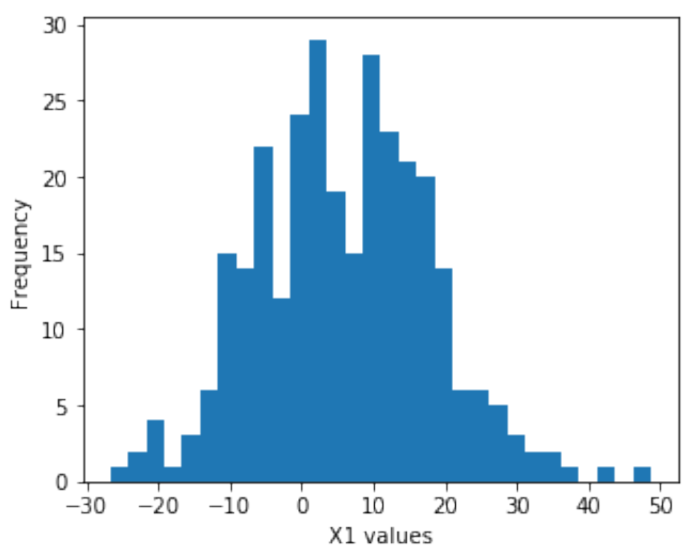
300 rows × 6 columns

In [3]:
```
## Separating data for independent and dependent variable.
x1 = data['X1'].to_numpy()
x2 = data['X2'].to_numpy()
x3 = data['X3'].to_numpy()
x4 = data['X4'].to_numpy()
x5 = data['X5'].to_numpy()
y = data['Y'].to_numpy()
```

In [4]:
```
## Calculating mean, variance, histogram, box-plot for X1.
n_bins = 30
sns.boxplot(x1)
x1_mean = np.mean(x1)
x1_var = np.var(x1)
fig, ax = plt.subplots(figsize =(5, 4))
ax.hist(x1, bins = n_bins)
plt.xlabel("X1 values")
plt.ylabel("Frequency")
x1_mean, x1_var
```

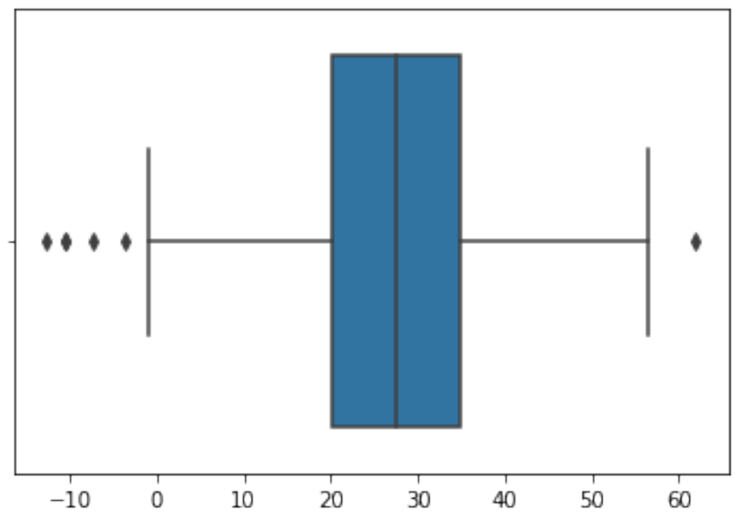
Out[4]: (6.033567975333334, 148.51399372523088)

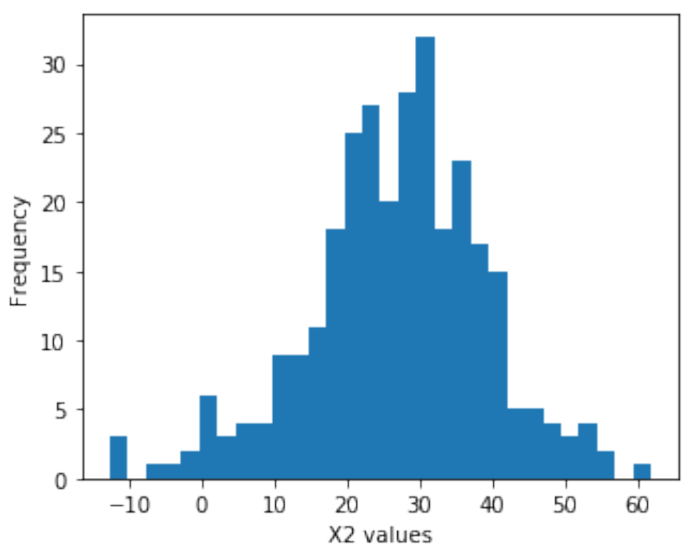




In [5]:
```
## Calculating mean, variance, histogram, box-plot for X2.
n_bins = 30
sns.boxplot(x2)
x2_mean = np.mean(x2)
x2_var = np.var(x2)
fig, ax = plt.subplots(figsize =(5,4))
ax.hist(x2, bins = n_bins)
plt.xlabel("X2 values")
plt.ylabel("Frequency")
x2_mean, x2_var
```

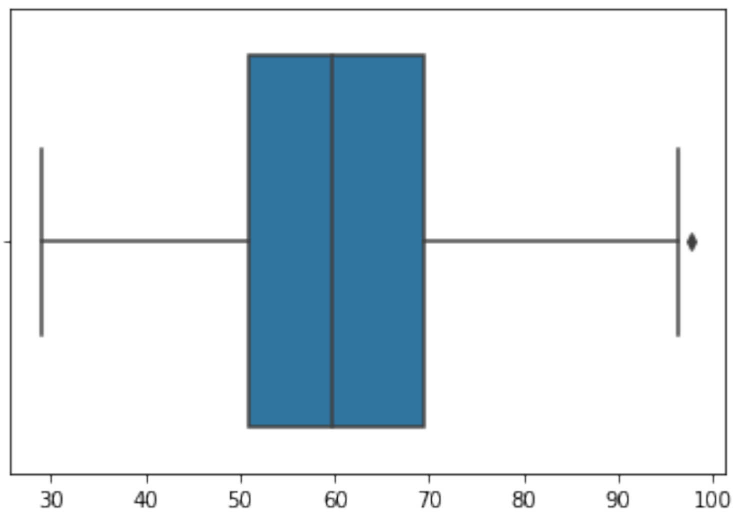
Out[5]:
```
(26.919584733333334, 151.17199258958894)
```

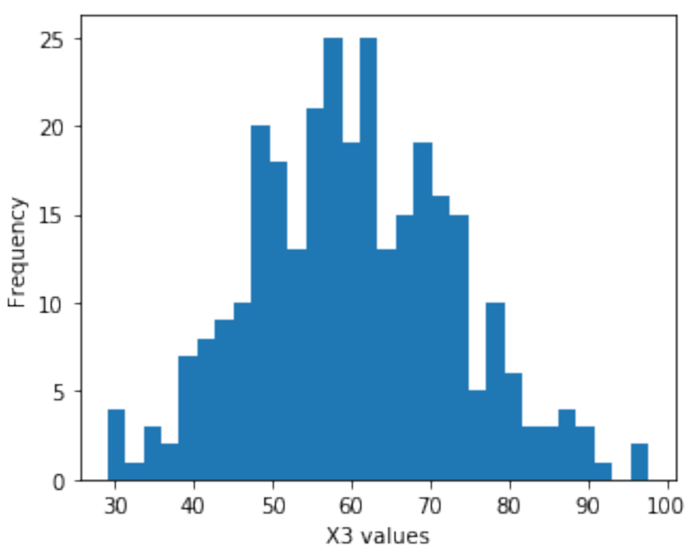




In [6]:
```
## Calculating mean, variance, histogram, box-plot for X3.
n_bins = 30
sns.boxplot(x3)
x3_mean = np.mean(x3)
x3_var = np.var(x3)
fig, ax = plt.subplots(figsize =(5,4))
ax.hist(x3, bins = n_bins)
plt.xlabel("X3 values")
plt.ylabel("Frequency")
x3_mean, x3_var
```

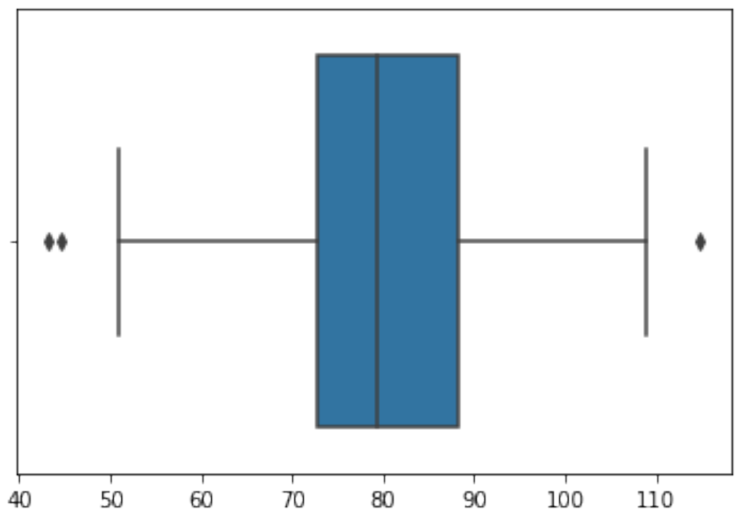
Out[6]:
```
(60.4127, 167.09651360333334)
```

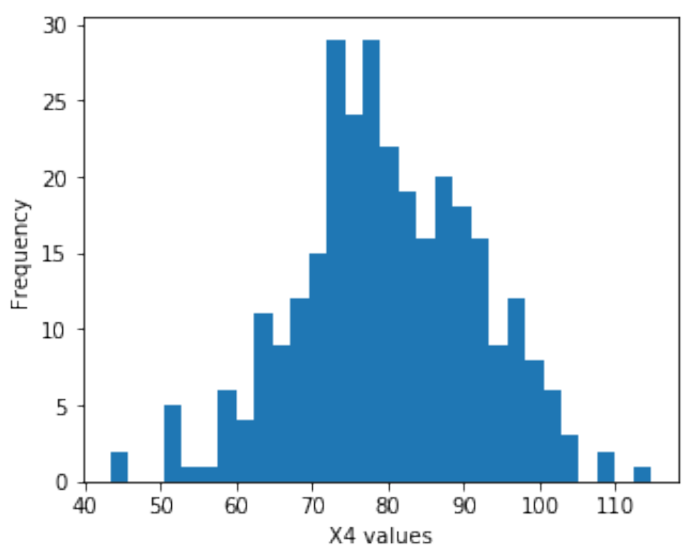




In [7]:
```
## Calculating mean, variance, histogram, box-plot for X4.
n_bins = 30
sns.boxplot(x4)
x4_mean = np.mean(x4)
x4_var = np.var(x4)
fig, ax = plt.subplots(figsize =(5, 4))
ax.hist(x4, bins = n_bins)
plt.xlabel("X4 values")
plt.ylabel("Frequency")
x4_mean, x4_var
```

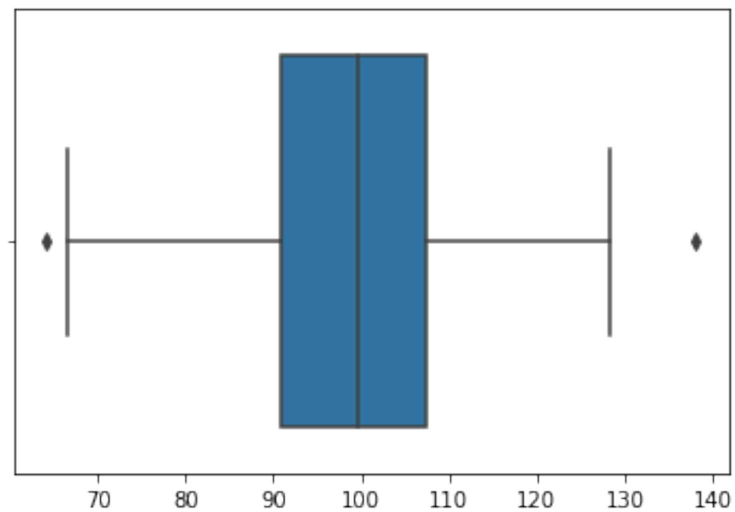
Out[7]: (80.02675333333333, 142.09622641915556)

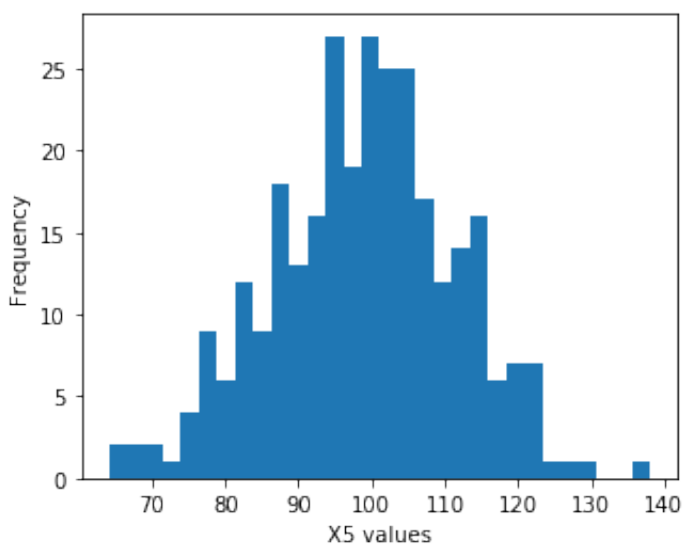




In [8]:
```python
## Calculating mean, variance, histogram, box-plot for X5.
n_bins = 30
sns.boxplot(x5)
x5_mean = np.mean(x5)
x5_var = np.var(x5)
fig, ax = plt.subplots(figsize =(5, 4))
ax.hist(x5, bins = n_bins)
plt.xlabel("X5 values")
plt.ylabel("Frequency")
x5_mean, x5_var
```

Out[8]: (98.92907666666667, 157.46047649745557)





In [9]:
```
corr_matrix = data.corr()
corr_matrix
```

Out[9]:

	X1	X2	X3	X4	X5	Y
X1	1.000000	0.022148	-0.026745	0.039889	0.002104	0.568159
X2	0.022148	1.000000	-0.016273	0.108075	-0.064093	0.241326
X3	-0.026745	-0.016273	1.000000	-0.037981	-0.022852	0.139072
X4	0.039889	0.108075	-0.037981	1.000000	0.004115	0.264501
X5	0.002104	-0.064093	-0.022852	0.004115	1.000000	0.233390
Y	0.568159	0.241326	0.139072	0.264501	0.233390	1.000000

Task 1 comments: I have calculate the mean, variance, box plots for each of the independent variable X1, X2, X3, X4, X5 and also calculate the histogram for each of the variable with bin size of 30. The box plot is used to find out the outliers present in individual independent variable. As shown in above correlation matrix the correlation value for each of individual variable including the dependent variable is calculate. From the correlation matrix we can clearly see that all the independent variables are uncorrelated with each other. While all the the independent variables are correlated to the dependent variabl Y with some value with X1 having highest correlation with Y.

In [10]:
```
## Getting X1, Y for linear regression
data_task2 = data[['X1','Y']]
data_task2
```

Out[10]:

	X1	Y
0	14.47200	1583.2
1	8.77720	1367.7
2	13.00300	1655.2
3	-6.35780	1239.7
4	25.58100	2010.4
...	...	...
295	9.24530	1503.8
296	0.38753	1639.8
297	11.52900	1524.2
298	-4.37280	1699.5
299	-20.80500	1918.0

300 rows × 2 columns

In [11]:
```
## Removing outliers by calculating z-score for the data fitting linea
r regression model showing model summary.
z = np.abs(stats.zscore(data_task2))
data_task2 = data_task2[(z<3).all(axis=1)]
x1, y = data_task2['X1'].to_numpy(), data_task2['Y'].to_numpy()
x = sm.add_constant(x1)
model = sm.OLS(y, x).fit()
y_pred = model.predict(x)
model.summary()
```

Out[11]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.204
Model:	OLS	**Adj. R-squared:**	0.201
Method:	Least Squares	**F-statistic:**	74.89
Date:	Wed, 07 Oct 2020	**Prob (F-statistic):**	3.41e-16
Time:	18:31:40	**Log-Likelihood:**	-2046.7
No. Observations:	294	**AIC:**	4097.
Df Residuals:	292	**BIC:**	4105.
Df Model:	1		
Covariance Type:	nonrobust		

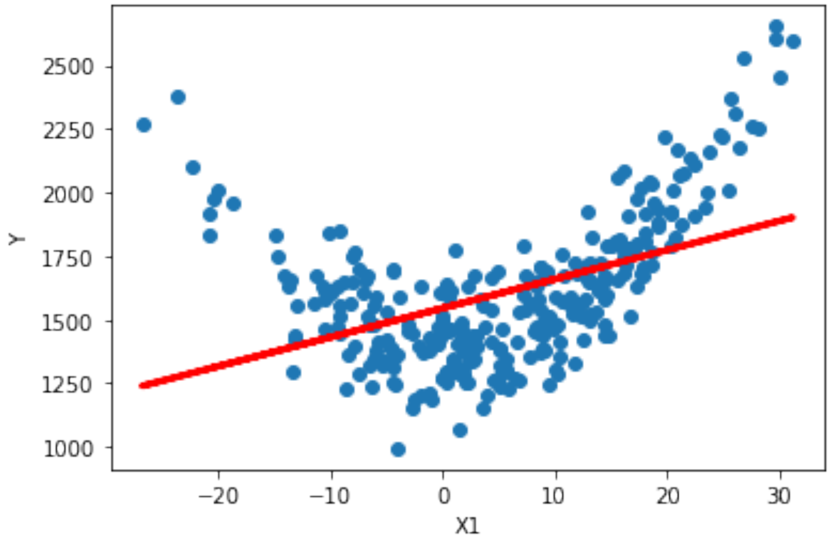
	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	1544.9989	16.533	93.449	0.000	1512.460	1577.538
x1	11.4074	1.318	8.654	0.000	8.813	14.002

Omnibus:	61.379	**Durbin-Watson:**	2.122
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	111.031
Skew:	1.126	**Prob(JB):**	7.76e-25
Kurtosis:	4.998	**Cond. No.**	13.9

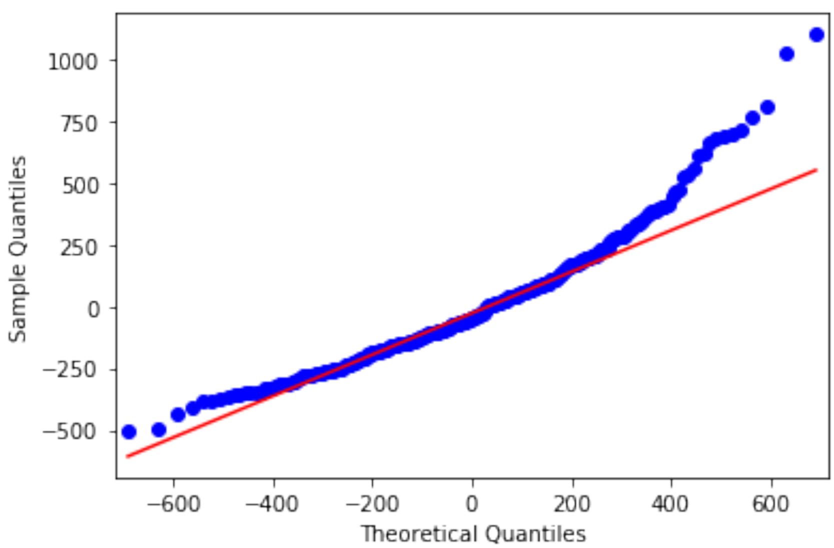
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [12]:
```
## Scatter plot for the predicted values and X1.
plt.scatter(x1,y)
plt.plot(x1, y_pred, color = 'red', linewidth = 3 )
plt.xlabel('X1')
plt.ylabel('Y')
```

Out[12]: Text(0, 0.5, 'Y')

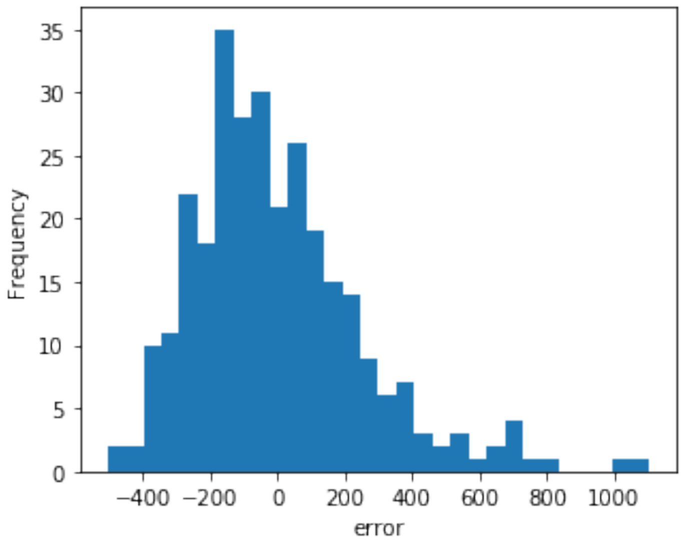


In [13]:
```
## Plotting QQ plot for residuals.
error = np.subtract(y, y_pred)
error_var = np.var(error)
fig = sm.qqplot(error, loc = 0, scale = np.sqrt(error_var), line='q')
plt.show()
```



In [14]:
```
## Plotting histogram of residuals.
n_bins = 30
fig, ax = plt.subplots(figsize =(5, 4))
ax.hist(error, bins = n_bins)
plt.xlabel("error")
plt.ylabel("Frequency")
```

Out[14]:
```
Text(0, 0.5, 'Frequency')
```

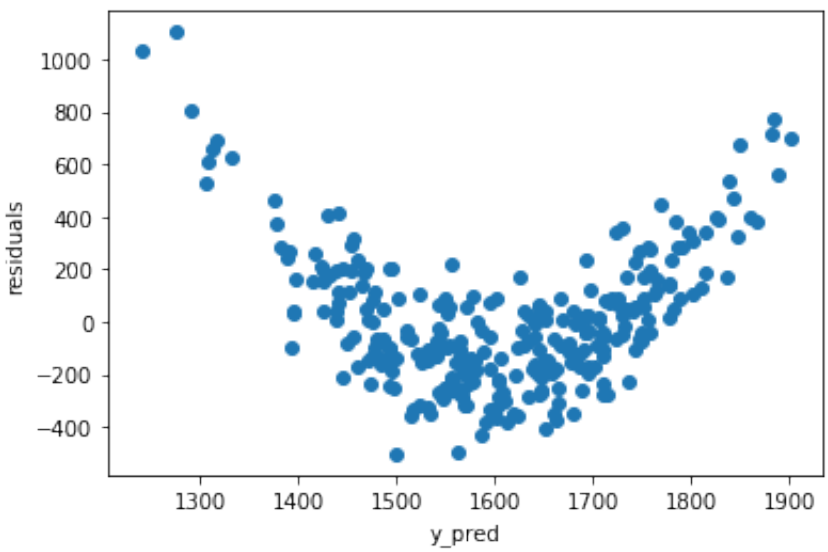


In [15]:
```
## Chi-squared test hypothesis.
k2, prob = stats.normaltest(error)
if prob < 0.05:
    print("Chi squared test doesnt satisfy hypothesis not accepted")
else:
    print('Chi squared test passed hypothesis accepted')

plt.scatter(y_pred, error)
plt.xlabel('y_pred')
plt.ylabel('residuals')
```

```
Chi squared test doesnt satisfy hypothesis not accepted
```

Out[15]:
```
Text(0, 0.5, 'residuals')
```



In [16]:
```
## Implementation for polynomial regression using X1.
data_task2 = data[['X1','Y']]
z = np.abs(stats.zscore(data_task2))
data_task2 = data_task2[(z<3).all(axis=1)]
data_task2['X1^2'] = data_task2['X1'] ** 2
x1, y = data_task2[['X1', 'X1^2']], data_task2['Y'].to_numpy()
x = sm.add_constant(x1)
model = sm.OLS(y, x).fit()
y_pred = model.predict(x)
model.summary()
```

Out[16]:

OLS Regression Results

Dep. Variable:	y	**R-squared:**	0.757
Model:	OLS	**Adj. R-squared:**	0.755
Method:	Least Squares	**F-statistic:**	452.5
Date:	Wed, 07 Oct 2020	**Prob (F-statistic):**	4.83e-90
Time:	18:31:47	**Log-Likelihood:**	-1872.5
No. Observations:	294	**AIC:**	3751.
Df Residuals:	291	**BIC:**	3762.
Df Model:	2		
Covariance Type:	nonrobust		

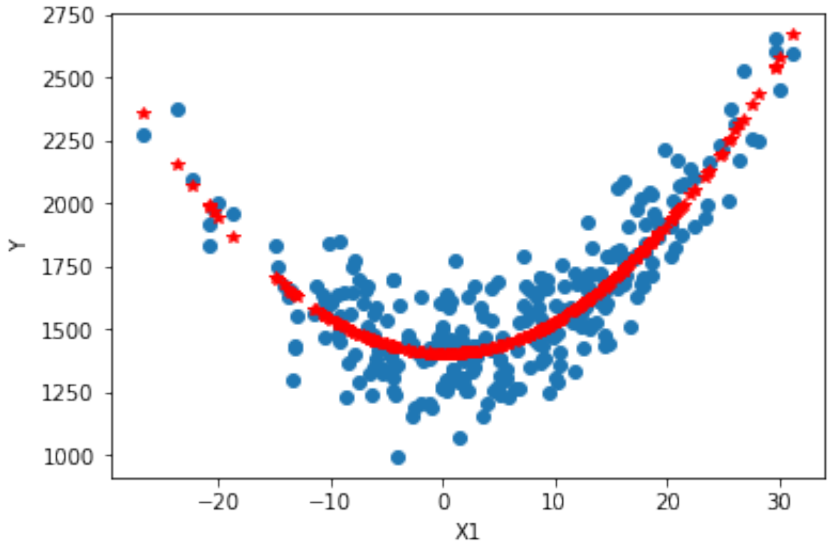
	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	1402.1781	10.710	130.919	0.000	1381.099	1423.257
X1	-0.5828	0.866	-0.673	0.502	-2.288	1.122
X1^2	1.3168	0.051	25.708	0.000	1.216	1.418

Omnibus:	0.776	**Durbin-Watson:**	2.038
Prob(Omnibus):	0.678	**Jarque-Bera (JB):**	0.869
Skew:	0.115	**Prob(JB):**	0.647
Kurtosis:	2.867	**Cond. No.**	321.

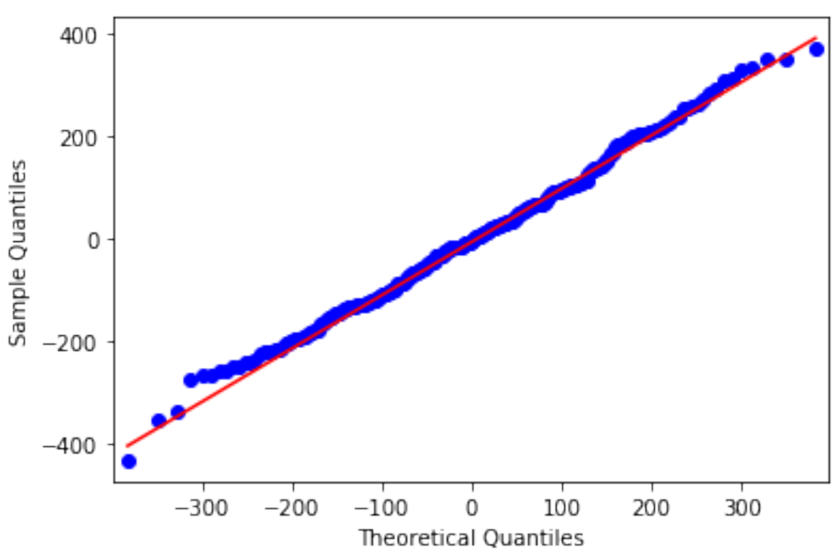
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [17]:
```
## Scatter plot for the predicted values and X1.
plt.scatter(x1['X1'],y)
plt.plot(x1['X1'], y_pred, 'r*', linewidth = 3 )
plt.xlabel('X1')
plt.ylabel('Y')
```

Out[17]: Text(0, 0.5, 'Y')

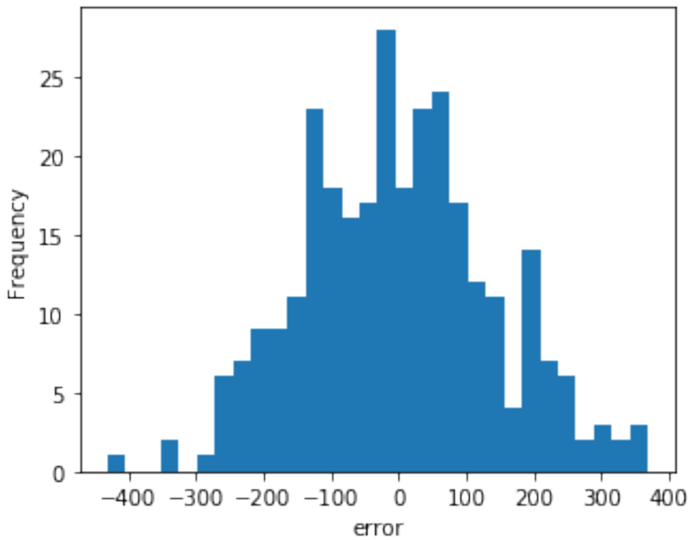


In [18]:
```
## Plotting QQ plot for residuals.
error = np.subtract(y, y_pred)
error_var = np.var(error)
fig = sm.qqplot(error, loc = 0, scale = np.sqrt(error_var), line='q')
plt.show()
```



In [19]:
```
## Plotting histogram for residuals.
n_bins = 30
fig, ax = plt.subplots(figsize =(5, 4))
ax.hist(error, bins = n_bins)
plt.xlabel("error")
plt.ylabel("Frequency")
```

Out[19]: `Text(0, 0.5, 'Frequency')`

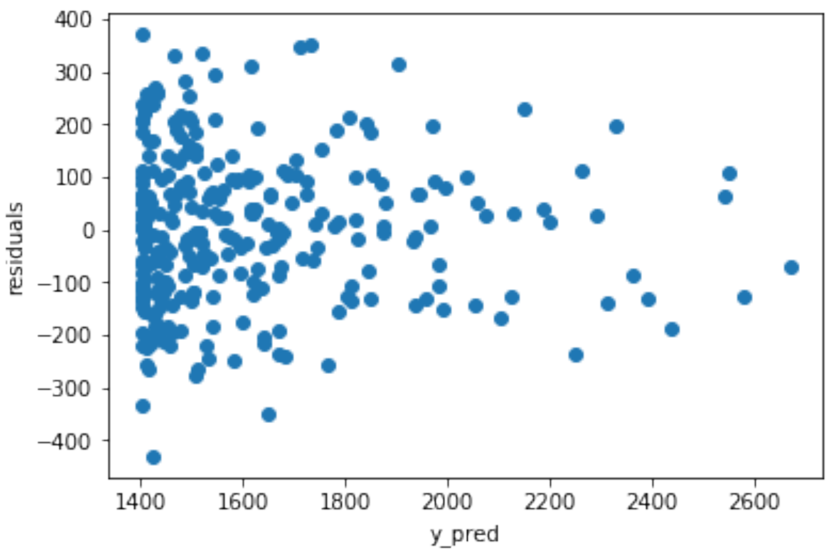


In [20]:
```
## Chi-squared test hypothesis.
k2, prob = stats.normaltest(error)
if prob < 0.05:
    print("Chi squared test doesnt satisfy hypothesis not accepted wit
h prob = ", prob)
else:
    print('Chi squared test passed hypothesis accepted with prob = ',
prob)

plt.scatter(y_pred, error)
plt.xlabel('y_pred')
plt.ylabel('residuals')
```

```
Chi squared test passed hypothesis accepted with prob =  0.678326003
0766237
```

Out[20]:
```
Text(0, 0.5, 'residuals')
```



Task 2 comments To remove the ouliers from the data i have used zscore for every data point and removed the data points which are 3standar deviation apart from the mean value. For this task I have only considered the independent variable X1. first we have used simply linear regression with Y = a0 + a1X1 + e. For the linear regression i have used statsmodel as it gives easy summary and the parameters for the model. For the linear regression we can see from the regreesion line that it is just a line which deivide the data and from the r^2 value which is 0.165 which is very low shows that this model doesnt noot fit the data very well. Also from the residual analysis we can see that QQ plot is signifcantly close to the line but the histogram for residuals with bin size of 30 shows it is similar to normal distribution but not exactly the nomal distribution. And from scatter plot for residuals we can see that the residuals are uncorrelated as there are no trends in between them. Now when we used the polynomial regression with independent variable X1. Y = a0 + a1X1 + a2X1^2 + e. From the model summary and the regression line plot we can clearly see that the model fits the data very well. Also the R^2 value which is 0.75 which is much improved that the linear regression. The QQ plot also matches the line for Normal distribution and the Chi-squared test hypothesis is also expceted. From the histogram of residuals we can see that the do follow normal distribution and the scatter plot do not show any kind of correlation od trend between them.

In [21]:
```
## Getting X1, X2, X3, X4, X5, Y for linear regression.
data_task3 = data[['X1','X2','X3','X4','X5','Y']]
data_task3
```

Out[21]:

	X1	X2	X3	X4	X5	Y
0	14.47200	23.5520	67.921	76.212	88.292	1583.2
1	8.77720	23.4560	45.202	74.720	95.002	1367.7
2	13.00300	17.8300	67.540	64.430	114.630	1655.2
3	-6.35780	30.4270	46.573	93.653	67.497	1239.7
4	25.58100	7.4073	30.896	72.519	98.709	2010.4
...	...	...	...	...	...	...
295	9.24530	10.1590	80.555	72.726	99.786	1503.8
296	0.38753	44.1610	80.662	72.227	115.950	1639.8
297	11.52900	-12.7620	57.399	71.702	116.500	1524.2
298	-4.37280	18.8140	65.591	99.102	118.160	1699.5
299	-20.80500	30.3240	57.158	66.992	103.660	1918.0

300 rows × 6 columns

In [22]:
```
## Implementation for linear regression using X1, X2, X3, X4, X5.
z = np.abs(stats.zscore(data_task3))
data_task3 = data_task3[(z<3).all(axis=1)]
x1, y = data_task3[['X1','X2','X3','X4','X5']], data_task3['Y'].to_num
py()
x = sm.add_constant(x1)
model = sm.OLS(y, x).fit()
y_pred = model.predict(x)
model.summary()
```

Out[22]:

OLS Regression Results

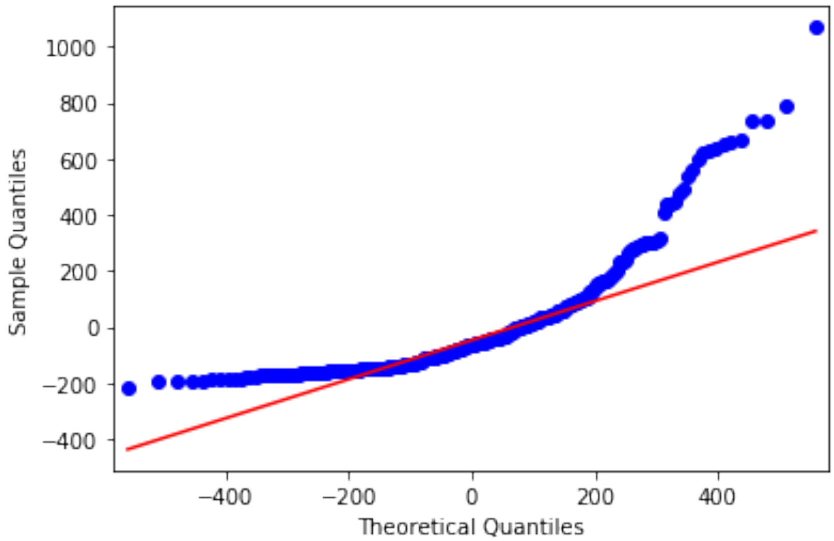
Dep. Variable:	y	**R-squared:**	0.483
Model:	OLS	**Adj. R-squared:**	0.474
Method:	Least Squares	**F-statistic:**	52.88
Date:	Wed, 07 Oct 2020	**Prob (F-statistic):**	1.24e-38
Time:	18:32:03	**Log-Likelihood:**	-1950.7
No. Observations:	289	**AIC:**	3913.
Df Residuals:	283	**BIC:**	3935.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-193.5905	145.594	-1.330	0.185	-480.175	92.994
X1	11.6203	1.081	10.749	0.000	9.492	13.748
X2	5.0868	1.057	4.813	0.000	3.007	7.167
X3	4.2968	0.942	4.559	0.000	2.442	6.152
X4	6.8171	1.044	6.527	0.000	4.761	8.873
X5	8.0684	0.986	8.184	0.000	6.128	10.009

Omnibus:	133.475	**Durbin-Watson:**	2.030
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	474.828
Skew:	2.061	**Prob(JB):**	7.81e-104
Kurtosis:	7.737	**Cond. No.**	1.71e+03

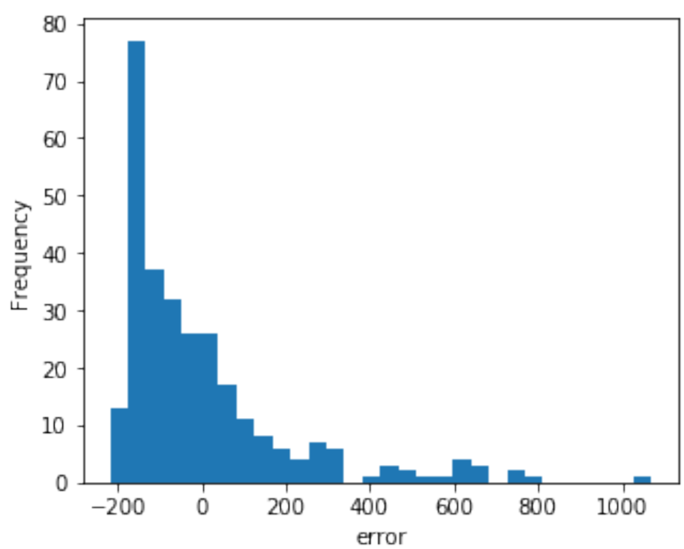
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.71e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

In [23]:
```
## Plotting QQ plot for residuals.
error = np.subtract(y, y_pred)
error_var = np.var(error)
fig = sm.qqplot(error, loc = 0, scale = np.sqrt(error_var), line='q')
plt.show()
```



In [24]:
```
## Plotting histogram for residuals.
n_bins = 30
fig, ax = plt.subplots(figsize =(5, 4))
ax.hist(error, bins = n_bins)
plt.xlabel("error")
plt.ylabel("Frequency")
```

Out[24]:
```
Text(0, 0.5, 'Frequency')
```



In [25]:
```
## Getting X1, X2, X4, Y for linear regression.
final_data = data_task3 = data[['X1','X2','X4','Y']]
final_data
```

Out[25]:

	X1	X2	X4	Y
0	14.47200	23.5520	76.212	1583.2
1	8.77720	23.4560	74.720	1367.7
2	13.00300	17.8300	64.430	1655.2
3	-6.35780	30.4270	93.653	1239.7
4	25.58100	7.4073	72.519	2010.4
...	...	...	...	...
295	9.24530	10.1590	72.726	1503.8
296	0.38753	44.1610	72.227	1639.8
297	11.52900	-12.7620	71.702	1524.2
298	-4.37280	18.8140	99.102	1699.5
299	-20.80500	30.3240	66.992	1918.0

300 rows × 4 columns

In [26]:
```
## Implementation for linear regression using X1, X2, X4.
z = np.abs(stats.zscore(data_task3))
data_task3 = data_task3[(z<3).all(axis=1)]
x1, y = data_task3[['X1','X2','X4']], data_task3['Y'].to_numpy()
x = sm.add_constant(x1)
model = sm.OLS(y, x).fit()
y_pred = model.predict(x)
model.summary()
```

Out[26]:

OLS Regression Results

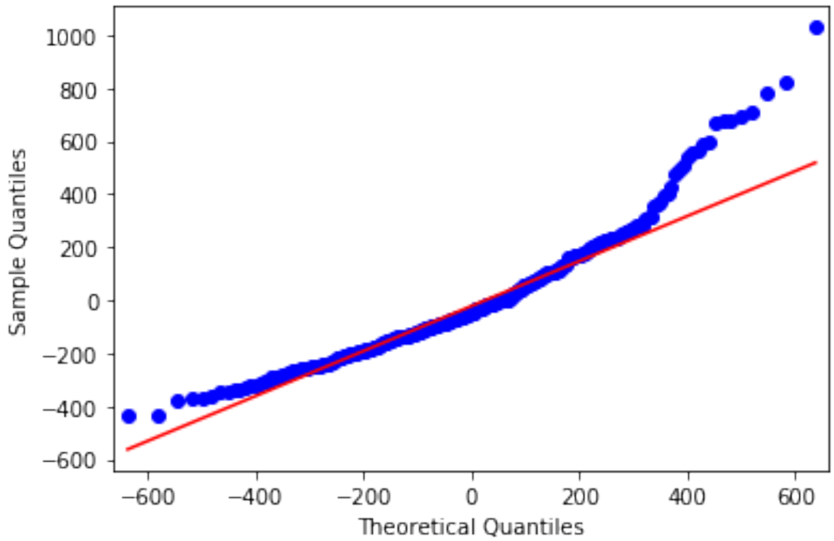
Dep. Variable:	y	**R-squared:**	0.326
Model:	OLS	**Adj. R-squared:**	0.319
Method:	Least Squares	**F-statistic:**	46.10
Date:	Wed, 07 Oct 2020	**Prob (F-statistic):**	2.48e-24
Time:	18:32:06	**Log-Likelihood:**	-1995.6
No. Observations:	290	**AIC:**	3999.
Df Residuals:	286	**BIC:**	4014.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	883.4184	98.816	8.940	0.000	688.920	1077.917
X1	11.3510	1.227	9.255	0.000	8.937	13.765
X2	4.7545	1.199	3.964	0.000	2.394	7.115
X4	6.6832	1.183	5.647	0.000	4.354	9.013

Omnibus:	63.850	**Durbin-Watson:**	1.982
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	115.892
Skew:	1.182	**Prob(JB):**	6.83e-26
Kurtosis:	5.000	**Cond. No.**	607.

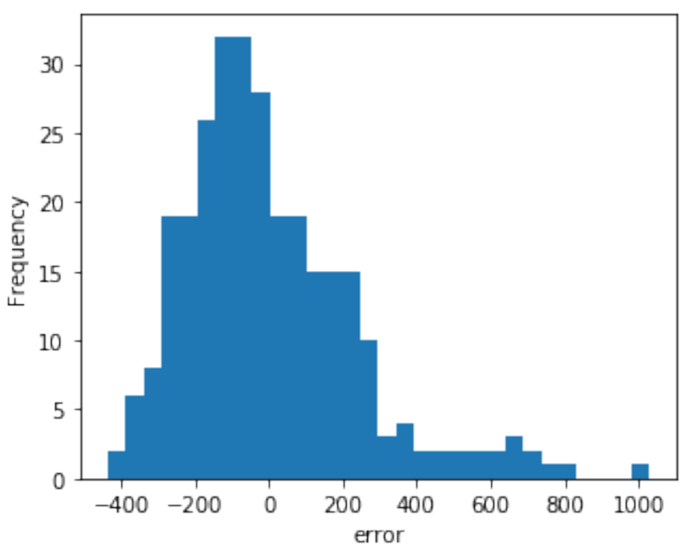
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [27]:
```
## Plotting QQ plot for residuals.
error = np.subtract(y, y_pred)
error_var = np.var(error)
fig = sm.qqplot(error, loc = 0, scale = np.sqrt(error_var), line='q')
plt.show()
```



In [28]:
```
## Plotting histogram for residuals.
n_bins = 30
fig, ax = plt.subplots(figsize =(5, 4))
ax.hist(error, bins = n_bins)
plt.xlabel("error")
plt.ylabel("Frequency")
```

Out[28]: Text(0, 0.5, 'Frequency')



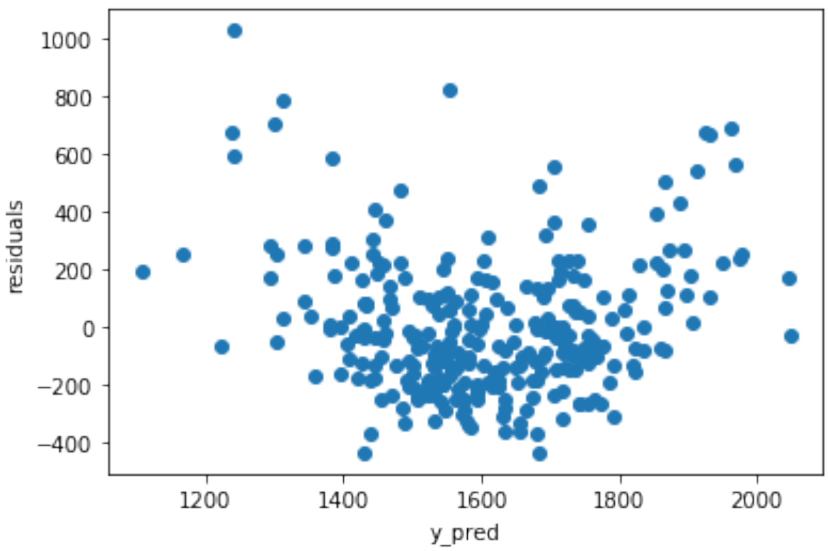
In [29]:

```
## Chi-squared test hypothesis.
k2, prob = stats.normaltest(error)
if prob < 0.05:
    print("Chi squared test doesnt satisfy hypothesis not accepted wit
h prob = ", prob)
else:
    print('Chi squared test passed hypothesis accepted with prob = ',
prob)

plt.scatter(y_pred, error)
plt.xlabel('y_pred')
plt.ylabel('residuals')
```

```
Chi squared test doesnt satisfy hypothesis not accepted with prob =
1.3653877537569556e-14
```

Out[29]: `Text(0, 0.5, 'residuals')`



Task 3 comments To remove the ouliers from the data I have used zscore for every data point and removed the data points which are 3 standard deviation apart from the mean value. For this task I have all considered the independent variable X1, X2, X3, X4, X5. First we have used simply linear regression with Y = a0 + a1X1 + a2X2 + a3X3 + a4X4 + a5X5 + e. From the r^2 value which is 0.28 which is very low shows that this model doesnt not fit the data very well. Also gives us an errort that there might be multicolinearity in the data. Also from the residual analysis we can see that QQ plot is no where close to the line and the histogram for residuals with bin size of 30 shows it is one sided and not normally distributed.
From the correlation matrix of the we can see that the independent variable X1 is more correlated to the dependent Y as compared to the other independent variables. But we if only use the X1 it will result in the same model as we have used in task 2. Therefore I have used the variables X1, X2, X4 which are top 3 indepndent variables to which Y is correlated. The 3 variables are chosen after trying out multiple variations in the number of variables. But from the QQ plot, histogram and p, R^2 values we can see that this model is better than the linear regression model but it does not match the performance for a polynomial regression model.