

Equations with Zero or Infinite Solutions

So far, we've addressed how to balance an equation in order to obtain a solution and resulting form obtained when it's balanced. But sometimes, we won't always get an equation in that form. Sometimes, we might get something ugly like $7 = 6$ or $0 = 0$. When we obtain resultant forms like those, what does it mean?

When we end up with an equation where we have one number on one side and a different other on the other side, this tells us the equation has no solutions.

For example:

$$4x + 2x + 7 = 6x + 8$$

No matter what is being plugged in as x , the left side of the equation will never equal to the right side of the equation. If we plug in 0, we get $7 = 8$. If we plug in 50, we get $307 = 308$. Regardless of the number being plugged in, the right side will always have 1 more than the left side. We can observe this by removing $6x$ from each side:

$$4x + 2x + 7 = 6x + 8$$

$$6x + 7 = 6x + 8$$

$$7 = 8$$

As you can see, when we simplify the equation, we end up with two unequal numbers on both sides. The side with the larger number will always be bigger than the side with the smallest number by their difference; because one side will always be larger than the other, there will never be a solution.

When we end up with an equation where we have a simplified result of $0 = 0$, this tells us that the equation has infinite solutions. What this tells us is that both sides of the equation are exactly the same; whatever value is on one side is mirrored on the next, so any number that can be plugged in is a valid solution.

Example:

$$4x + 8 = 4x + 6 + 2$$

If we simplify the equation:

$$4x + 8 = 4x + 6 + 2$$

$$4x + 8 = 4x + 8$$

As we can see, both sides mirror each other perfectly. If we were to subtract $4x$ from each side, and then 8 our final equation is:

$$0 = 0$$