System of Equations

A system of equations is a set of two or more equations which share the same set of variables/unknowns. When we are solving a system of equations, we are trying to find the numerical values of the variables/unknowns in the system such that when we plug in those numbers, the left side of each equation will equal to the respective right side.

For example, if we're given this system of equations:
$x + 2y = 3$
$6x - y = 4$
If we put in $x = 1$ and $y = 2$, then we end up with:
$3 = 3$
$4 = 4$
Thus $x = 1$ and $y = 2$ solves this system of equation.

So how do we solve a system of equations?
The first step we do in solving a system of equations is making sure that for all equations in the system, all variables and numbers are lined up. If they are not, then you must move around parts of each equation to ensure that it. You may rearrange the equations in whatever format you may please, as long as similar variables and numbers line up (x terms like up with x terms, y terms like up with y terms, 2 lines up with 6, etc.).
For example:
$x + 7y = 10$
$2x - 5 = y$
Let's say we want the equations in the format where x and y are on the left side, with y being right after x, and all numbers will be on the right side of the equation. The first equation is already in this format, so we leave it alone. The second equation is not in this format, so we must re-arrange it so that it will be. We can start by removing from the right side by subtracting y from both sides.
$2x - 5 = y$ => **$2x - 5 - y = 0$**
As we can see from the left side, we want y to right after y so we switch – 5 and – y.
$2x - 5 - y = 0$ => **$2x - y - 5 = 0$**
We want 5 to be on the right side now; as we can see on the left side, we are subtracting 5. Naturally in order to remove it, we must add 5 to both sides.
$2x - 5 - y = 0$ => **$2x - y = 5$**
Now both variables are on the left side (with the y term being right after the x term) and all numbers on the right side.
So our system as of right now is:
$x + 7y = 10$
$2x – y = 5$

The next step is to eliminate x. We can do so by multiplying one of the equations by a number so that the x terms in both equations are exactly the same. If we look at our system of equations, we have: x and 2x as our respective x terms.
So, we can either:
-Multiply equation #1 by 2.
-Multiply equation #2 by ½. (Fractions are fairy nasty to deal with early on, so you want to always go for multiplying by a flat number for now).
Since it's easier to deal with whole numbers rather than fraction, we will multiply equation #1 by 2.
Our system of equations is now:
2x + 14y = 20
2x – y = 5

The next step to remove the x term from the first equation by subtracting the second equation from it. Because we had previously scaled the equation such that they both have the same x term, by subtracting one equation from the other we remove the x term from the first equation. You can also subtract the first equation from the second; however when you are just starting to learn algebra, it's much easier to have a set, unchanging process for solving these problems.
After subtracting the second equation from the first, we get:
(2x + 14y) – (2x – y) = 15
When we simplify we get:
(2x + 14y) – 2x + y = 15
14y + y = 15 (We have a positive 2x term – 2x, which is equal to 0!)
15y = 15
y = 1
We now know that y = 1 is the solution for the y part of our system of equations. Then what do we do with it?

We now can plug y = 1 into any one of the 2 original equations, and then solve for x because **by plugging in the solution for y, we reduce the equation to a form where there's only an x term and a number. By reformatting the equation to where there's the x term on one side and the number on the other, we can divide both sides by the number x is being multiplied by (aka a constant) in order to obtain the x term.**

So let's substitute the y term (swapping out the y variable for the number it represents) in both of the original equations and see what happens.

For original equation 1:
x + 7y = 10 => **x + 7(1) = 10** => **x + 7 = 10**
x + 7 – 7 = 10 – 7 (Remove 7 from the left side by subtracting 7 from both sides)

x = 3

For original equation 2:
2x – 5 = y **=> 2x – 5 = 1**
2x – 5 + 5 = 1 + 5 (Remove subtracting 5 from the left side by adding 5 to both sides)
2x = 6
x = 3 (Number in front of the x term is 2, so we divide both sides by 2 to remove it)

As you can see plugging in y = 1 into both equations get us the same value for x. If you do not obtain the same values from plugging in the y value you obtained, then you have probably made an arithmetic error on at least 1 of the equations

Human error is natural, so it's fine if you mess up on solving them; however, as you get more and more used to solving them, you will be making less and less human errors.

When you have gotten to the step where you are subtracting 1 equation from another to solve for 1 variable, sometimes you will get an equation with no variables at all. This does not mean you have messed up; it simply means that the system of equations either has infinitely many solutions or no solutions at all. It follows the same standards stated in the Equations with Zero or Infinite solutions lessons:
-If you end up with 0 = 0, then there are infinite solutions
-If you end with 0 = some number or some number = a different number, then there are no solutions