Inequalities

An inequality is an equation where instead of an equal sign, we have either a:
-Less than sign ($<$)
-Greater than sign ($>$)
-Less than or equal to sign ($\leq$)
-Greater than or equal to sign ($\geq$)

How do we interpret each sign?
- $<$ means the left hand side must always be less than the right hand side
- $>$ means the left hand side must always be more than the right hand side
- $\leq$ means the left side must always be less than or equal to the right hand side
- $\geq$ means the left side must always be greater than or equal to the right hand side

An inequality reads exactly like a normal equation, except instead of saying "equals", we say the corresponding sign.
Examples:
$2x \leq 4$: 2x is less than or equal to 4
$5x < 6$: 5x is less than 6
$4x + 2 \geq 9$: 4x + 2 is greater than or equal to 9
$3x – 2 > 4$: 3x – 2 is greater than 4

Inequalities can be balanced/simplified exactly like a normal equation, except for 1 twist: if we multiply or divide by a negative number, the sign must be flipped. This means that the greater than sign becomes less than, and vice-versa. In addition, the greater than or equal to sign becomes a less than or equal to sign, and vice-versa.

Example:
$-3y \geq 15$
Let's say we only want y on the left side with no -3. Naturally we divide both sides by -3, and we will end up with:
$y \leq -5$
If we didn't invert the sign, would the answer still be correct? Let's check.
We would have **$y \geq -5$** instead.
If we plug -5 into the above equal and the original equation, we get $-5 \geq -5$ and $-15 \geq -15$ respectively, so it seems to be correct as of right now.
But what if we plug in y = -7, which is a solution to the original equation? (If we plug in -7 into the original equation, we obtain: $21 \geq 15$, which is valid)
Plugging -7 into $y \geq -5$, we get $-7 \geq -5$, which is not a correct statement.

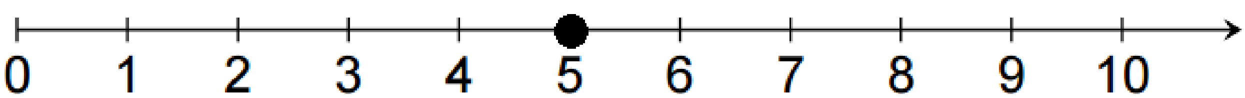
Like equations, you can swap the left side and the right side, but you have to reverse the inequality sign as well.
Simple example: $5 < x$ can also be expressed as $x > 5$

Frequently in the topic of inequalities, you will be asked to plot an inequality on a number line. Although doing this is tedious, it may help you obtain a better visualization on imagining and thinking about inequalities.

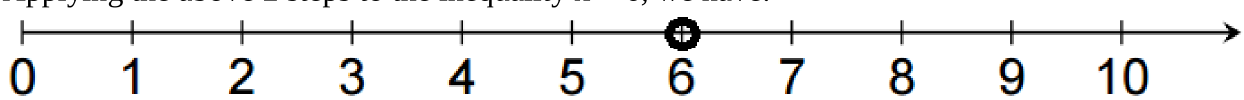
The first thing part of an inequality that should be observed when you begin plotting it onto a number line if the inequality sign allows both sides to be each to each other or not. If the inequality includes equal to, we will have a closed dot (dot that's bubbled in). Else, we will have an open dot (dot that is not bubbled in).

The next part that should be notice is the side that only has a number on it (after we have balanced and simplified the inequality). This number will tell us exactly where to place our dot to signify the boundary of the inequality on the number line. A closed dot signifies that we include the boundary number, while an open dot signifies that we do not.

Examples:
Applying the above 2 steps to the inequality $x \geq 5$, we have:

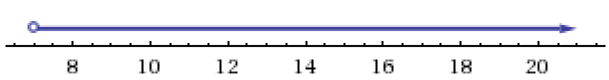


Applying the above 2 steps to the inequality $x < 6$, we have:



The third and final part is that we check what part of the inequality sign is facing the variable. If the open part of the sign (the not-pointy end) is facing the variable, then we will draw an arrow starting from the dot to the right end of the number line. If the closed part (the pointy-end) is facing the variable, then we draw an arrow starting from the dot to the end of the number line.

Examples:
$7 < x$



$x \leq 8$