

Balancing Equations

Think of equations as a scale where both sides must remain perfectly balanced no matter what. If you remove something from one side of the scale, then something of equal value must be removed from the other side. The same applies to addition. We will apply this concept to reformatting equations such that only certain terms will be on the left and the right side (i.e terms with x and y on the left side and numbers on the right side).

For example, let's take the equation $x + 7 = 10$, and we want to reformat the equations so that there are no numbers on the left side. We first look at the left side, and see that the only number on the left side is positive 7 (if the left side were $x - 7$ instead, the number would be negative 7). Naturally in order negate this positive 7, we must do the opposite of adding 7: subtracting 7. So we subtract 7 from the left side; however, because the equation is a scale that needs to always be perfectly balanced, we must also subtract 7 from the right side as well.

$$x + 7 - 7 = 10 - 7 \text{ (Subtracting 7 from both sides)}$$

$$x = 3 \text{ (Simplified)}$$

Another example: $x - 7 = 10$, we have only x on the left side and a number on the right side. As stated above, the number on the left side of the equation is negative 7. In order to remove this negative number, we must do the opposite of it: adding 7. Again, since the equation needs to be a perfectly balanced scale, we must add 7 to both sides.

$$x - 7 + 7 = 10 + 7 \text{ (Adding 7 to both sides)}$$

$$x = 17 \text{ (Simplified)}$$

When an equation is simplified to a form where there's a single variable on 1 side and a number on the other, we have obtained the solution of that equation for that 1 variable. In the case of the equations previously used, we have obtained the whole solution for the equation since the equation only has 1 variable.

Another example: $x + 5 = 10$, x on the right side and number on the left side

We start by noticing that x is on the left side and the 10 is on the right side, which is the opposite of what it should be. We can start the problem by moving either the x or the 10.

Moving x , then moving 10:

$$x + 5 - x = 10 - x \text{ (subtracting } x \text{ from both sides to remove it from the left)}$$

Because there is no x term that is opposite of subtracting x (there is no positive x term on the right side), there is nothing to negate it. This means that once we move the x term over, an x term will appear on the right side. Don't worry about the fringe cases where moving over variable terms over ends up cancelling all of them out, leaving you with no items with that variable.

5 = 10 - x (10 is still on the right side. We want constants on the left side, so the next step is subtracting 10 from both sides).

$$\mathbf{5 - 10 = 10 - 10 - x}$$

$$\mathbf{-5 = -x}$$

We have successfully reformatted the equation such that numbers are on the left side and the x-term is on the right side but we aren't done yet though. Because there is a negative sign in front of the x term, we must multiply both sides by -1 in order to make the answer look a lot cleaner. So we end up with our final answer of: **5 = x**.