

Inequalities

An inequality is an equation where instead of an equal sign, we have either a:

- Less than sign ($<$)
- Greater than sign ($>$)
- Less than or equal to sign (\leq)
- Greater than or equal to sign (\geq)

How do we interpret each sign?

- $<$ means the left hand side must always be less than the right hand side
- $>$ means the left hand side must always be more than the right hand side
- \leq means the left side must always be less than or equal to the right hand side
- \geq means the left side must always be greater than or equal to the right hand side

An inequality reads exactly like a normal equation, except instead of saying “equals”, we say the corresponding sign.

Examples:

$2x \leq 4$: $2x$ is less than or equal to 4

$5x < 6$: $5x$ is less than 6

$4x + 2 \geq 9$: $4x + 2$ is greater than or equal to 9

$3x - 2 > 4$: $3x - 2$ is greater than 4

Inequalities can be balanced/simplified exactly like a normal equation, except for 1 twist: if we multiply or divide by a negative number, the sign must be flipped. This means that the greater than sign becomes less than, and vice-versa. In addition, the greater than or equal to sign becomes a less than or equal to sign, and vice-versa.

Example:

$$-3y \geq 15$$

Let's say we only want y on the left side with no -3 . Naturally we divide both sides by -3 , and we will end up with:

$$y \leq -5$$

If we didn't invert the sign, would the answer still be correct? Let's check.

We would have $y \geq -5$ instead.

If we plug -5 into the above equal and the original equation, we get $-5 \geq -5$ and $-15 \geq -15$ respectively, so it seems to be correct as of right now.

But what if we plug in $y = -7$, which is a solution to the original equation? (If we plug in -7 into the original equation, we obtain: $21 \geq 15$, which is valid)

Plugging -7 into $y \geq -5$, we get $-7 \geq -5$, which is not a correct statement.