Black-Derman-Toy interest rate tree model

Maturity (T)		1	2	3	4	5	6
Spot rates (EAR)		0.300%	0.834%	1.345%	1.826%	2.322%	2.731%
Actual price		0.9970	0.9835	0.9607	0.9302	0.8916	0.8507
Implied yield volatility in year 1		0.0%	8.0%	12.0%	14.0%	16.0%	13.0%
σ (Solver)			8.00%	14.37%	16.38%	19.51%	5.94%
R (Solver)		0.30%	1.26%	1.75%	1.94%	1.86%	3.55%
Model price			0.2134	0.2134	0.2134	0.2134	0.2134
Difference from actual			(0.7701)	(0.7473)	(0.7167)	(0.6781)	(0.6373)
Model yield volatility			0.1530548	0.1118525	0.0996916	9.4%	9.1%
Difference from actual			0.0731	(0.0081)	(0.0403)	(0.0661)	(0.0394)

Maturity 29 years from year 1

Prices P(1,M,j) P(2,M,j)

Year # of downs	0	1	2	3	4	5
0	0.2134	0.1884	0.1662	0.1474	0.1323	0.1229
1		0.2397	0.2162	0.1954	0.1778	0.1650
2			0.2693	0.2471	0.2276	0.2118
3				0.3009	0.2800	0.2618
4					0.3334	0.3135
5						0.3656
6						
7						
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1-year forward rates

Year # of downs	0	1	2	3	4	5
0	0.30%	1.48%	3.10%	5.18%	8.85%	6.43%
1		1.26%	2.33%	3.73%	5.99%	5.71%
2			1.75%	2.69%	4.06%	5.07%

3	1.94%	2.75%	4.50%
4		1.86%	4.00%
5			3.55%
6			
7			
8			
9			
10			
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	7	8	9	10	11	12	13	14	15
	3.154%	3.373%	3.599%	3.832%	3.877%	3.925%	3.976%	4.030%	4.086%
	0.8046	0.7669	0.7275	0.6866	0.6581	0.6300	0.6024	0.5751	0.5484
	10.0%	10.7%	11.3%	12.0%	11.8%	11.6%	11.4%	11.2%	11.0%
	1.48%	13.85%	15.00%	16.21%	10.40%	10.09%	9.78%	9.47%	9.17%
	5.25%	1.78%	1.53%	1.29%	1.50%	1.44%	1.40%	1.36%	1.34%
	0.2134	0.2134	0.2134	0.2134	0.2134	0.2134	0.2134	0.2134	0.2134
	(0.5912)	(0.5535)	(0.5140)	(0.4731)	(0.4447)	(0.4166)	(0.3889)	(0.3617)	(0.3350)
	8.8%	8.7%	8.6%	8.5%	8.4%	8.4%	8.3%	8.3%	8.2%
	(0.0116)	(0.0198)	(0.0277)	(0.0352)	(0.0339)	(0.0325)	(0.0310)	(0.0294)	(0.0277)

6	7	8	9	10	11	12	13	14
0.1104	0.0973	0.0904	0.0879	0.0923	0.0875	0.0840	0.0816	0.0805
0.1513	0.1373	0.1282	0.1234	0.1254	0.1191	0.1143	0.1108	0.1088
0.1975	0.1837	0.1722	0.1650	0.1640	0.1561	0.1498	0.1452	0.1421
0.2475	0.2347	0.2212	0.2114	0.2071	0.1977	0.1901	0.1841	0.1800
0.2997	0.2887	0.2735	0.2614	0.2538	0.2431	0.2341	0.2269	0.2216
0.3525	0.3440	0.3275	0.3134	0.3030	0.2911	0.2809	0.2725	0.2661
0.4047	0.3991	0.3818	0.3662	0.3534	0.3406	0.3294	0.3201	0.3127
	0.4528	0.4351	0.4186	0.4040	0.3906	0.3787	0.3685	0.3603
		0.4866	0.4696	0.4538	0.4400	0.4277	0.4169	0.4080
			0.5185	0.5021	0.4882	0.4756	0.4645	0.4551
				0.5482	0.5345	0.5219	0.5106	0.5009
					0.5783	0.5659	0.5547	0.5449
						0.6074	0.5963	0.5866
							0.6353	0.6257
								0.6623

6	7	8	9	10	11	12	13	14
6.26%	12.34%	16.92%	23.82%	11.97%	13.27%	14.60%	15.98%	17.41%
6.08%	9.36%	12.53%	17.22%	9.72%	10.84%	12.01%	13.22%	14.49%
5.90%	7.09%	9.29%	12.45%	7.90%	8.86%	9.87%	10.94%	12.07%

5.73%	5.38%	6.88%	9.01%	6.41%	7.24%	8.12%	9.05%	10.05%
5.57%	4.08%	5.10%	6.51%	5.21%	5.92%	6.68%	7.49%	8.36%
5.40%	3.09%	3.77%	4.71%	4.23%	4.84%	5.49%	6.20%	6.96%
5.25%	2.34%	2.80%	3.41%	3.44%	3.95%	4.52%	5.13%	5.80%
	1.78%	2.07%	2.46%	2.79%	3.23%	3.71%	4.24%	4.82%
		1.53%	1.78%	2.27%	2.64%	3.05%	3.51%	4.02%
			1.29%	1.84%	2.16%	2.51%	2.91%	3.34%
				1.50%	1.76%	2.07%	2.40%	2.78%
					1.44%	1.70%	1.99%	2.32%
						1.40%	1.65%	1.93%
							1.36%	1.61%
								1.34%

16	17	18	19	20	21	22	23	24
4.14%	4.21%	4.27%	4.33%	4.40%	4.47%	4.55%	4.62%	4.70%
0.5222	0.4964	0.4712	0.4466	0.4225	0.3990	0.3760	0.3537	0.3319
10.8%	10.6%	10.4%	10.2%	10.0%	9.8%	9.6%	9.4%	9.2%
8.87%	8.57%	8.27%	7.98%	7.68%	7.40%	7.11%	6.83%	6.55%
1.32%	1.31%	1.31%	1.32%	1.34%	1.37%	1.40%	1.45%	1.50%
0.2134	0.2134	0.2134	0.2134	0.2134	0.2134	0.2134	0.2134	0.2134
(0.3087)	(0.2830)	(0.2578)	(0.2332)	(0.2091)	(0.1855)	(0.1626)	(0.1402)	(0.1184)
8.2%	8.2%	8.2%	8.1%	8.1%	8.1%	8.1%	8.1%	8.1%
(0.0260)	(0.0243)	(0.0225)	(0.0207)	(0.0189)	(0.0170)	(0.0152)	(0.0133)	(0.0114)

15	16	17	18	19	20	21	22	23
0.0806	0.0822	0.0853	0.0904	0.0976	0.1077	0.1215	0.1401	0.1651
0.1084	0.1095	0.1125	0.1177	0.1253	0.1360	0.1507	0.1704	0.1967
0.1409	0.1414	0.1441	0.1491	0.1569	0.1680	0.1832	0.2036	0.2307
0.1777	0.1776	0.1797	0.1844	0.1920	0.2031	0.2185	0.2393	0.2668
0.2183	0.2173	0.2187	0.2228	0.2301	0.2410	0.2563	0.2770	0.3044
0.2619	0.2599	0.2605	0.2639	0.2706	0.2810	0.2959	0.3162	0.3431
0.3075	0.3046	0.3043	0.3069	0.3128	0.3226	0.3368	0.3563	0.3824
0.3542	0.3504	0.3492	0.3510	0.3560	0.3649	0.3782	0.3968	0.4218
0.4012	0.3966	0.3946	0.3955	0.3996	0.4075	0.4198	0.4372	0.4608
0.4477	0.4424	0.4396	0.4396	0.4429	0.4498	0.4609	0.4771	0.4992
0.4930	0.4872	0.4837	0.4830	0.4853	0.4912	0.5011	0.5159	0.5364
0.5367	0.5305	0.5264	0.5250	0.5264	0.5313	0.5401	0.5535	0.5723
0.5783	0.5717	0.5673	0.5652	0.5659	0.5698	0.5775	0.5895	0.6065
0.6175	0.6108	0.6060	0.6034	0.6035	0.6065	0.6130	0.6237	0.6391
0.6541	0.6474	0.6424	0.6394	0.6389	0.6411	0.6466	0.6560	0.6698
0.6881	0.6815	0.6763	0.6731	0.6720	0.6736	0.6781	0.6863	0.6986
	0.7129	0.7078	0.7043	0.7029	0.7038	0.7075	0.7146	0.7255
		0.7368	0.7332	0.7314	0.7318	0.7348	0.7409	0.7505
			0.7597	0.7577	0.7577	0.7600	0.7652	0.7736
				0.7818	0.7815	0.7832	0.7875	0.7949
					0.8032	0.8044	0.8081	0.8144
						0.8238	0.8268	0.8323
							0.8440	0.8487
								0.8636

15	16	17	18	19	20	21	22	23
18.88%	20.36%	21.86%	23.36%	24.85%	26.32%	27.76%	29.16%	30.51%
15.81%	17.16%	18.53%	19.92%	21.31%	22.70%	24.08%	25.43%	26.77%
13.24%	14.46%	15.71%	16.98%	18.28%	19.58%	20.89%	22.19%	23.48%

11.09%	12.18%	13.31%	14.48%	15.67%	16.89%	18.12%	19.36%	20.60%
9.29%	10.26%	11.28%	12.34%	13.44%	14.57%	15.72%	16.88%	18.07%
7.78%	8.65%	9.56%	10.52%	11.52%	12.56%	13.63%	14.73%	15.85%
6.51%	7.28%	8.10%	8.97%	9.88%	10.84%	11.83%	12.85%	13.90%
5.46%	6.14%	6.87%	7.65%	8.47%	9.35%	10.26%	11.21%	12.20%
4.57%	5.17%	5.82%	6.52%	7.27%	8.06%	8.90%	9.78%	10.70%
3.83%	4.36%	4.93%	5.56%	6.23%	6.95%	7.72%	8.53%	9.39%
3.21%	3.67%	4.18%	4.74%	5.34%	6.00%	6.69%	7.44%	8.23%
2.68%	3.09%	3.54%	4.04%	4.58%	5.17%	5.81%	6.49%	7.22%
2.25%	2.61%	3.00%	3.44%	3.93%	4.46%	5.04%	5.66%	6.34%
1.88%	2.20%	2.55%	2.94%	3.37%	3.85%	4.37%	4.94%	5.56%
1.58%	1.85%	2.16%	2.50%	2.89%	3.32%	3.79%	4.31%	4.88%
1.32%	1.56%	1.83%	2.13%	2.48%	2.86%	3.29%	3.76%	4.28%
	1.31%	1.55%	1.82%	2.13%	2.47%	2.85%	3.28%	3.75%
		1.31%	1.55%	1.82%	2.13%	2.47%	2.86%	3.29%
			1.32%	1.56%	1.84%	2.15%	2.50%	2.89%
				1.34%	1.58%	1.86%	2.18%	2.53%
					1.37%	1.61%	1.90%	2.22%
						1.40%	1.66%	1.95%
							1.45%	1.71%
								1.50%

25	26	27	28	29	30
4.79%	4.87%	4.97%	5.06%	5.17%	5.28%
0.3107	0.2901	0.2702	0.2508	0.2321	0.2134
9.0%	8.8%	8.6%	8.4%	8.2%	8.0%
6.28%	6.01%	5.74%	5.49%	5.24%	5.12%
1.57%	1.64%	1.73%	1.84%	1.96%	2.09%
0.2134	0.2134	0.2134	0.2134	0.2134	0.2134
(0.0973)	(0.0767)	(0.0567)	(0.0374)	(0.0187)	0.0000
8.0%	8.0%	8.0%	8.0%	8.0%	8.0%
(0.0096)	(0.0077)	(0.0058)	(0.0038)	(0.0019)	(0.0000)

Procedure for finding arbitrage-fr
Set M (cell B13) =1
Use Solver to set C9 to 0
  by changing cells C6 and C7
  subject to C11=0
Increase M by 1
Use Solver on next column

24	25	26	27	28	29
0.1991	0.2455	0.3096	0.3993	0.5266	0.7103
0.2319	0.2794	0.3440	0.4327	0.5562	0.7309
0.2667	0.3147	0.3790	0.4660	0.5851	0.7506
0.3030	0.3508	0.4142	0.4989	0.6131	0.7693
0.3404	0.3876	0.4495	0.5312	0.6402	0.7870
0.3784	0.4244	0.4843	0.5628	0.6661	0.8036
0.4166	0.4610	0.5186	0.5933	0.6909	0.8193
0.4546	0.4971	0.5519	0.6227	0.7144	0.8340
0.4919	0.5323	0.5842	0.6508	0.7366	0.8477
0.5284	0.5663	0.6151	0.6776	0.7576	0.8604
0.5636	0.5991	0.6447	0.7030	0.7773	0.8723
0.5975	0.6304	0.6728	0.7269	0.7957	0.8833
0.6297	0.6601	0.6994	0.7494	0.8129	0.8934
0.6602	0.6882	0.7243	0.7704	0.8289	0.9028
0.6890	0.7145	0.7477	0.7900	0.8437	0.9114
0.7160	0.7392	0.7694	0.8082	0.8573	0.9193
0.7411	0.7621	0.7897	0.8250	0.8700	0.9266
0.7644	0.7834	0.8084	0.8406	0.8816	0.9333
0.7860	0.8031	0.8257	0.8550	0.8923	0.9394
0.8059	0.8212	0.8416	0.8682	0.9021	0.9450
0.8242	0.8378	0.8562	0.8803	0.9111	0.9500
0.8409	0.8531	0.8696	0.8914	0.9193	0.9547
0.8562	0.8671	0.8819	0.9015	0.9268	0.9589
0.8702	0.8798	0.8931	0.9107	0.9336	0.9628
0.8829	0.8914	0.9033	0.9192	0.9398	0.9663
	0.9020	0.9126	0.9269	0.9455	0.9695
		0.9210	0.9338	0.9507	0.9723
			0.9402	0.9554	0.9750
				0.9596	0.9774
					0.9795

24	25	26	27	28	29
31.83%	33.10%	34.35%	35.59%	36.84%	40.78%
28.07%	29.36%	30.63%	31.89%	33.18%	36.81%
24.76%	26.03%	27.30%	28.58%	29.88%	33.23%

21.84%	23.09%	24.34%	25.61%	26.91%	29.99%
19.27%	20.47%	21.70%	22.95%	24.23%	27.07%
16.99%	18.16%	19.34%	20.56%	21.82%	24.44%
14.99%	16.10%	17.25%	18.43%	19.65%	22.06%
13.22%	14.28%	15.37%	16.51%	17.70%	19.91%
11.66%	12.66%	13.71%	14.79%	15.94%	17.97%
10.29%	11.23%	12.22%	13.26%	14.35%	16.22%
9.07%	9.96%	10.89%	11.88%	12.92%	14.64%
8.00%	8.83%	9.71%	10.64%	11.64%	13.22%
7.06%	7.83%	8.66%	9.54%	10.48%	11.93%
6.23%	6.95%	7.72%	8.55%	9.44%	10.77%
5.49%	6.16%	6.88%	7.66%	8.50%	9.72%
4.84%	5.46%	6.13%	6.86%	7.65%	8.77%
4.27%	4.84%	5.47%	6.15%	6.89%	7.92%
3.77%	4.30%	4.87%	5.51%	6.21%	7.15%
3.32%	3.81%	4.35%	4.94%	5.59%	6.45%
2.93%	3.38%	3.87%	4.42%	5.03%	5.83%
2.59%	3.00%	3.45%	3.96%	4.53%	5.26%
2.28%	2.66%	3.08%	3.55%	4.08%	4.75%
2.01%	2.36%	2.74%	3.18%	3.68%	4.28%
1.77%	2.09%	2.45%	2.85%	3.31%	3.87%
1.57%	1.85%	2.18%	2.56%	2.98%	3.49%
	1.64%	1.94%	2.29%	2.69%	3.15%
		1.73%	2.05%	2.42%	2.84%
			1.84%	2.18%	2.57%
				1.96%	2.32%
					2.09%

ree forward rates in BDT model