Sudoku3D Propositional Logic Representation Notes

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A 3D sudoku is an $N \times N \times N$ structure (with $N \in \mathbb{N}$), where every row, colum and layer needs to contain exactly one digit $d \in \{1, \ldots, N\}$. A single row, column or layer may not use the same digit more than once. This can be expressed as (TODO: reference to Weber):

$$\text{valid}(x_1, \ldots, x_N) := \bigwedge_{d=1}^{N} \bigvee_{i=1}^{N} \bigwedge_{1 \leq d \leq d' \leq N} \neg x_i^d \vee \neg x_i^{d'} \tag{1}$$

Then the constraints of the sudoku can be represented as:
TODO: resolve inconsistent representations of x_i and $x_{i,j,k}$.

$$\text{sudoku}(\{x_{i,j,k}\}) := \bigwedge_{k=1}^{N} (\bigwedge_{i=1}^{N} \text{valid}(x_{i,1,k}, \ldots, x_{i,N,k}) \wedge \bigwedge_{j=1}^{N} \text{valid}(x_{1,j,k}, \ldots, x_{N,j,k}))$$
$$\wedge \bigwedge_{j=1}^{N} (\bigwedge_{k=1}^{N} \text{valid}(x_{1,j,k}, \ldots, x_{N,j,k}) \wedge \bigwedge_{i=1}^{N} \text{valid}(x_{i,j,1}, \ldots, x_{i,j,N}))$$
$$\tag{2}$$