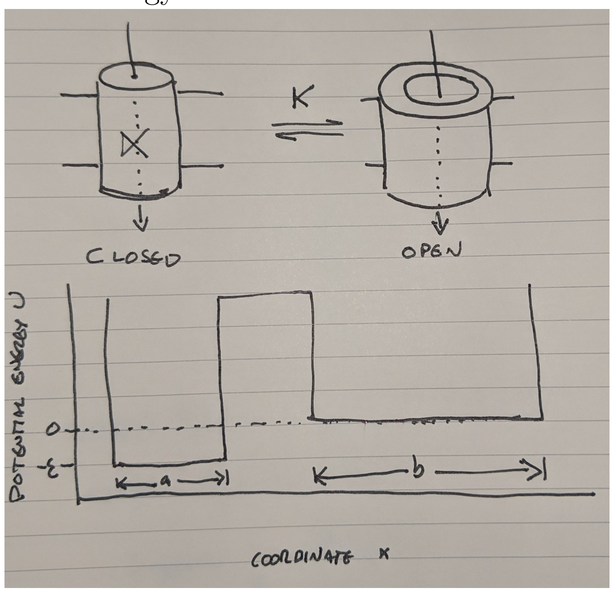
**Statistical Mechanics Exam Question**

Suppose a temperature-sensitive ion channel can populate one of two conductance states: *closed*, where it does not conduct current, and *open*, where it allows current to pass through.

As a simple way of getting an idea of how temperature can modulate the ion conductance of this channel, let's model the two states of this channel according to the one-dimensional potential energy landscape where the closed state has width $a$ (in some arbitrary distance units, such as Å) and potential energy $-\epsilon$ while the open state has width $b$ (in the same distance units) and potential energy 0.




*(a)* What is the enthalpic contribution to the free energy difference $\Delta H \equiv H_{\text{open}} - H_{\text{closed}}$ between the two states (written in terms of $\epsilon$, $a$, and $b$)?
*(b)* What is the entropic contribution $T\Delta S = TS_{\text{open}} - TS_{\text{closed}}$ (written in terms of $\epsilon$, $a$, and $b$)?
*(c)* What is the temperature-dependent free energy difference $\Delta G(T) = G_{\text{open}}(T) - G_{\text{closed}}(T)$ between the two states (written in terms of $\epsilon$, $a$, and $b$)?
*(d)* What is the equilibrium constant $K(T) = \frac{p_{\text{open}}(T)}{p_{\text{closed}}(T)}$ between the two states?
*(e)* At what temperature $T_0$ is $K(T_0) = 1$, such that $p_{\text{open}}(T_0) = p_{\text{closed}}(T_0)$?

**Statistical Mechanics Exam Answer**

*(a)* $\Delta H = H_{\text{open}} - H_{\text{closed}} = 0 - (-\epsilon) = \epsilon$
*(b)* $T\Delta S = TS_{\text{open}} - TS_{\text{closed}} = Tk_B \ln b - Tk_B \ln a$ which could also be written $k_BT\ln(b/a)$.
*(c)* $\Delta G(T) = \Delta H - T\Delta S = \epsilon - k_BT\ln(b/a)$
*(d)* $K(T) = \frac{p_{\text{open}}(T)}{p_{\text{closed}}(T)} = e^{-\beta\Delta G(T)} = (b/a)e^{-\frac{\epsilon}{k_BT}}$
*(e)*

$$K(T_0) = 1 \tag{1}$$

implies that

$$(b/a)e^{-\frac{\epsilon}{k_BT_0}} = 1 \tag{2}$$

which we can solve for $T_0$ as

$$\frac{\epsilon}{k_B\ln(b/a)} = T_0 \tag{3}$$