EN.600.461. - Computer Vision
Homework #1

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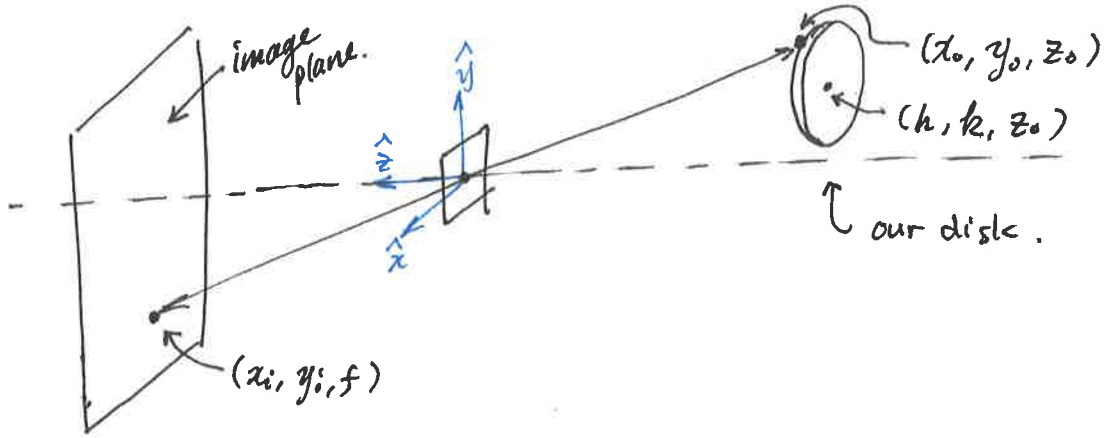
Section: 461

1) a. Let the center of the circle be denoted (h, k) and radius r.



And other notations as depicted above.
Then, by similar triangles, we have $\frac{x_i}{f} = \frac{x_0}{z_0}$ and $\frac{y_i}{f} = \frac{y_0}{z_0}$.
Our disk can be expressed as $(x_0-h)^2+(y_0-k)^2 = r^2$.
Substitute x_0 with $\frac{z_0}{f}x_i$ and y_0 with $\frac{z_0}{f}y_i$.

$$\left(\frac{z_0}{f}x_i - h\right)^2 + \left(\frac{z_0}{f}y_i - k\right)^2 = r^2$$

$$\frac{z_0^2}{f^2}x_i^2 - 2\frac{z_0}{f}h x_i + h^2 + \frac{z_0^2}{f^2}y_i^2 - 2\frac{z_0}{f}k y_i + k^2 = r^2$$

Divide both sides by $\frac{z_0^2}{f^2}$.

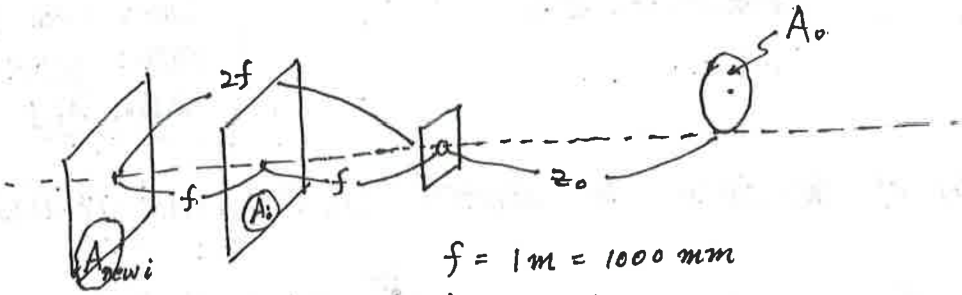
$$x_i^2 - 2\frac{f}{z_0}h x_i + \frac{f^2}{z_0^2}h^2 + y_i^2 - 2\frac{f}{z_0}k y_i + \frac{f^2}{z_0^2}k^2 = \frac{f^2r^2}{z_0^2}$$

$$\left(x_i - \frac{f}{z_0}h\right)^2 + \left(y_i - \frac{f}{z_0}k\right)^2 = \left(\frac{fr}{z_0}\right)^2.$$

Thus, the image is a circle with center $\left(\frac{f}{z_0}h, \frac{f}{z_0}k, f\right)$ and radius $\frac{fr}{z_0}$.

$\blacksquare$

b.



$f = 1m = 1000\ mm$

$A_i = 1\ mm^2$

The original magnification $m_o = \frac{f}{z_o}$

$$\frac{A_i}{A_o} = m_o^2 = \left(\frac{f}{z_o}\right)^2 = \frac{f^2}{z_o^2} = \frac{1}{A_o}$$

Thus, $A_o = z_o^2 f^{-2}$ — (1)

The changed magnification $m_c = \frac{2f}{z_o}$

$$\frac{A_{new\,i}}{A_o} = m_c^2 = \left(\frac{2f}{z_o}\right)^2 = \frac{4f^2}{z_o^2}$$

Thus, $A_0 = A_{new\,i} \cdot z_0^2 \cdot f^{-2} \cdot \frac{1}{4}$ — (2)

By setting (1) = (2), we get

$$\cancel{z_o^2 f^{-2}} = A_{new_i} \cdot \cancel{z_o^2 f^{-2}} \cdot \frac{1}{4}$$

$\therefore A_{new_i} = 4\ mm^2$

c. Let us assume perspective projection and a viewpoint external to the sphere. Then, the boundary that is formed by the viewpoint and the circle on the sphere will be a cone.
The image of the sphere will then be determined by intersecting this "cone" with the image plane, which produces a conic section. Thus, we can get a circle, ellipse, parabola, or hyperbola depending on the circumstances.