EXPLORING PATH INDEPENDENCY OF THERMODYNAMIC STATE PROPERTIES AND APPLYING THE IDEAL GAS LAW

Abstract

This document presents our findings from the Ideal Gas Lab, which was divided into two parts. In part 1, two methods were used to expand air from one tank to another, demonstrating that despite the differing expansion processes, the final thermodynamic state properties—temperature and pressure—remained constant. This demonstrates the fact that state properties are independent of path functions during a process. In part 2, we determined the values of the initial mass of (9.05 ± 0.05g) and volume (7.0 ± 1.0L) of the left tank. Upon calculating the mass of air added into the left tank, the Ideal Gas Law was applied to find the value of initial mass and volume.

Introduction

The field of thermodynamics plays a pivotal role in our understanding of the physical world, serving as a foundation of science and engineering. By studying thermodynamics, we gain insights into how energy and matter interact within systems; this knowledge applies to multiple domains such as aerospace, transportation, and materials science.

The primary objective of this experiment is to elucidate the concept of path independence in thermodynamic state properties. By exploring two distinct scenarios of air expansion between tanks at varying initial pressures, we demonstrate that regardless of the specific process employed, the final state properties, namely temperature and pressure, remain consistent. This fundamental insight not only deepens our comprehension of thermodynamics but also has profound implications on the broader realm of engineering: it allows engineers to simplify and expedite complex calculations, enabling efficient design and optimization of various systems.

By engaging in this experiment, students are also able to acquire practical skills in determining unknowns of a system such as initial mass and volume using the Ideal Gas Law ($PV = mRT$ [Chandra, 2016]). In the following sections, we will delve into the experimental setup, procedures, results, and discussion.

Experimental Method

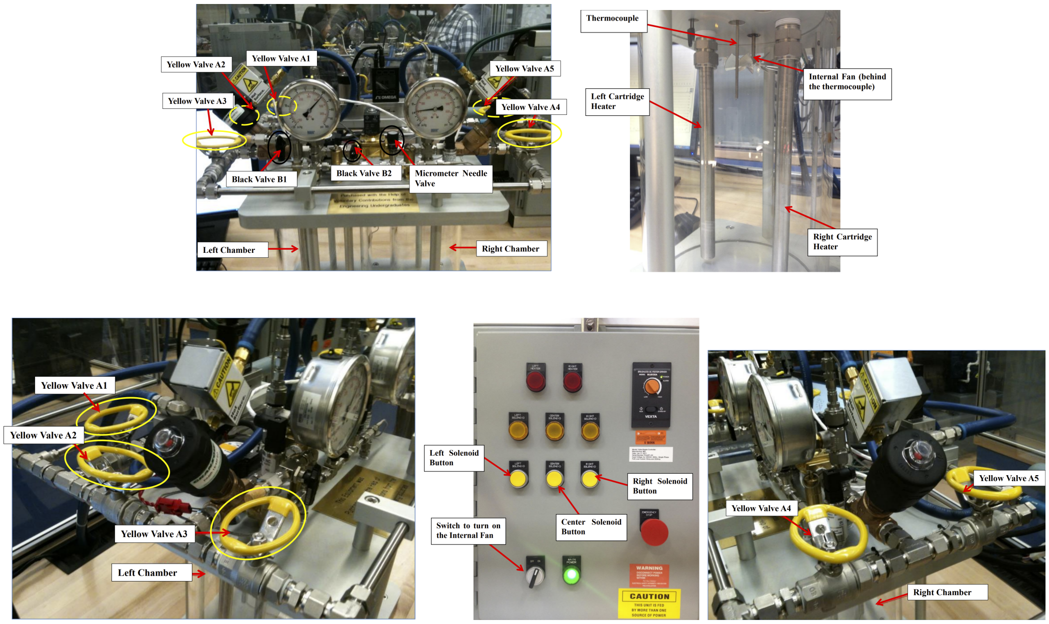

Figure 1: *From left to right, top to bottom: center view of apparatus, left tank, left side view of apparatus showing the manually controlled valves, apparatus control board, right side view of apparatus*

The apparatus for this experiment comprises a network of interconnected pipes and valves, linked to two transparent acrylic tanks. These tanks, while not perfect insulators, enable us to observe heat transfer phenomena between the system and its surroundings. Key components of this setup include solenoids, which can be precisely actuated to control the opening and closing of airways, and ball valves, which offer an additional layer of control by regulating the airflow. Crucially, the apparatus is equipped with a mass flow rate sensor, enabling the precise measurement of gas input into each tank. Further, the system is equipped with sensors that enable the continuous recording of temperature and pressure at 0.1 second intervals, providing adequate data for analysis.

In Lab 1, this apparatus facilitates the manipulation of gas flow between the two tanks, each initially possessing distinct properties. The rate at which these tanks achieve thermal equilibrium can be finely adjusted, either through a micrometer or solenoid. In the second experiment (Lab 2), the mass flow rate sensor plays a pivotal role in determining the mass added to a tank. Coupled with the continuous recording of temperature and pressure, this data enables calculations of the tank's volume using the ideal gas law.

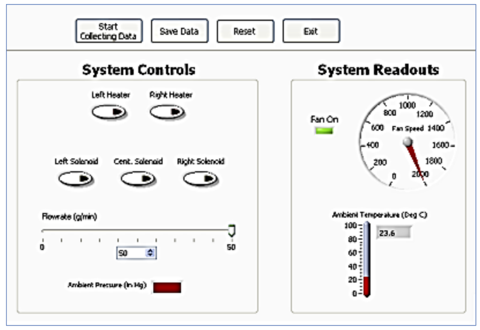

Figure 2: LABView Program Interface

The LABView program interface (Figure 2), which runs on a computer attached to the apparatus, is also vital to the experiment, as it contains several buttons which manipulate the apparatus' solenoids (Left/Center/Right Solenoid buttons), control the flowrate (Flowrate slider), and collect data via computer-generated graphs (Start Collecting Data/Save Data buttons).

Procedure

Part 1: *Path Independency of State Properties and Determining Volume Ratio of Tanks*

1. Measure the current air pressure in the laboratory by observing the manometer.
2. Start recording data by clicking the 'Start Collecting Data' button in the LabVIEW software.
3. Increase the pressure in the left tank to 40 psi:
   a. Open Valve A2, activate Left Solenoid in LabView.
   b. Within LabVIEW, gradually increase the air flow rate to 50 grams per minute.
   c. When the pressure in the left chamber reaches 40 psi, close the left solenoid in LabView.
   d. Close Valve A2 to stop the airflow, set air flow rate to zero in LabView.
4. Decrease the pressure in the right tank to -6 psi:
   a. Open Valves A1 and A5 to enable air flow; activate the Right Solenoid in LabView.
   b. Within LabVIEW, gradually increase the air flow rate to 50 grams per minute.
   c. When the pressure in the right chamber reaches -6 psi, close right solenoid in LabView.
   d. Close Valves A1 and A5 to stop the airflow, set the air flow rate in LabView to zero.

Method 1: Using the Center Solenoid Valve

5. Use the LabVIEW program to open the center solenoid valve by pressing the ‘Cent. Solenoid’ button once, and keep the valve open until the temperature difference between the two tanks becomes negligible.
    a. In LabVIEW, stop recording data, and save it as 'Lab 2 – Part 1a.'
    b. Reset LabVIEW, and complete parts 1 - 4 again before proceeding to the next step.

Method 2: Using the Micrometer Needle Valve

6. Open the small ball valve 'B2' and adjust micrometer valve by turning it counterclockwise four times. Keep both valves open until pressure difference between two tanks becomes negligible. This should take approximately 10 minutes.

a. In LabVIEW, stop recording data, and save it as 'Lab 2 – Part 1b.'
b. Reset LabVIEW software and close both small ball valve and micrometer valve, empty tanks.

Part 2: <u>*Determine the Initial Mass and Volume of the Left Tank*</u>

1. Measure air pressure in the lab with the manometer.
2. Click the 'Start Collecting Data' button in the LABView interface.
3. Increase pressure in left tank to 40 psi until both pressure and temperature become stable:
    a. Open Valve A2 to let air in, activate the Left Solenoid using LabView.
    b. Within LabVIEW, gradually increase the air flow rate to 50 grams per minute.
    c. When pressure in the left chamber reaches 40 psi, close the Left Solenoid in LabView.
    d. Wait until pressure and temperature values have stabilized on the LabVIEW graphs.
4. In LabVIEW, click 'Stop Recording,' save data, and use Valve B1 to empty the tank completely.

Lab 1 Discussion:

For methods A and B, 3 graphs with respect to time were recorded. Note that error bars on the graphs have been omitted, as each reading has a constant instrumental error.

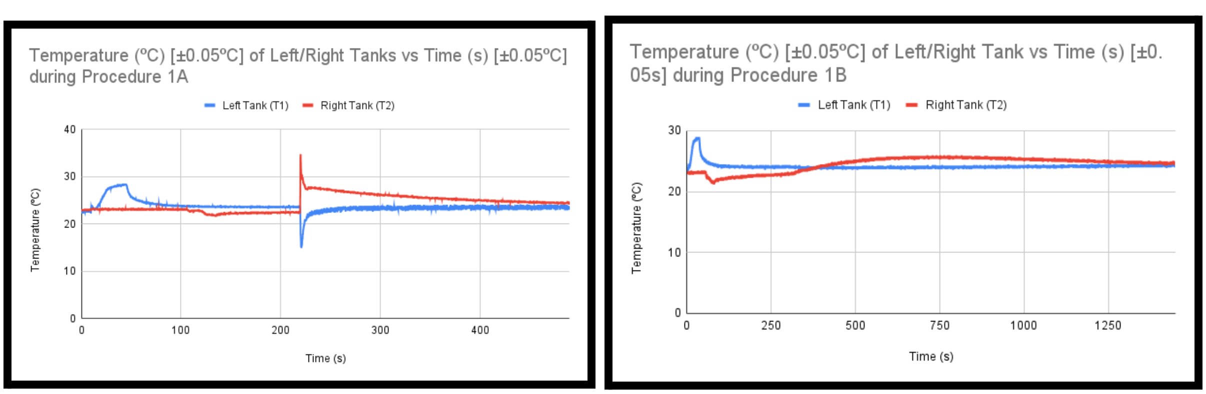

Figure 3: *Temperature vs Time (A, B)*

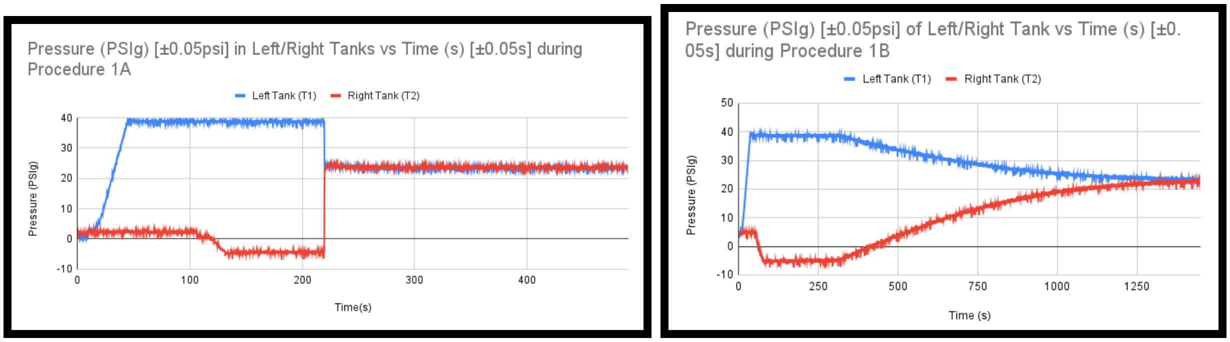

Figure 4: *Pressure vs Time (A, B)*

Discussion Question 1: **Using the ideal gas law and conservation of mass determine the volume ratio between the two tanks.**

	Procedure A			**Procedure B**		
	Left Tank Initial	**Right Tank Initial**	**Final State**	**Left Tank Initial**	**Right Tank Initial**	**Final State**
P (psig)	38.7 ± 0.05	-4.1 ± 0.05	23.3 ± 0.05	39.2 ± 0.05	-5 ± 0.05	23.0 ± 0.05
P_{abs} (kPa)	367.6 ± 0.8	72.5 ± 0.8	261.5 ± 0.8	371.1 ± 0.8	66.3 ± 0.8	259.4 ± 0.8
T (C)	23.6 ± 0.05	22.5 ± 0.05	23.3 ± 0.05	23.5 ± 0.05	23 ± 0.5	24.1 ± 0.5
T (K)	296.8 ± 0.5	295.7 ± 0.5	296.5 ± 0.5	296.2 ± 0.5	296.2 ± 0.5	297.3 ± 0.5

Figure 5: Final and Initial data with conversions and instrumental errors for procedures A and B

We make two assumptions that will allow us to calculate the ratio:

1. Mass is conserved between the two tanks ($m_1 + m_2 = m_f$)
2. The volume of the piping connecting the two tanks is negligible ($V_1 + V_2 = V_f$)

From the conservation of mass, the final mass is equal to the sum of the two masses in tanks T_1 and T_2.

Moreover, the volume of the final state is equal to the combined volumes of tank 1 and 2.

From the ideal gas law we have $PV = mR_{air}T \rightarrow m = \frac{PV}{R_{air}T}$.

Substituting the above equation into Assumption 1: $\frac{P_1V_1}{R_{air}T_1} + \frac{P_2V_2}{R_{air}T_2} = \frac{P_fV_f}{R_{air}T_f}$

In order to find a ratio between V_1 and V_2, V_f is replaced with the sum of the individual volumes taken according to Assumption 2: $\frac{P_1 V_1}{R_{air} T_1} + \frac{P_2 V_2}{R_{air} T_2} = \frac{P_f (V_1 + V_2)}{R_{air} T_f} \rightarrow \frac{V_1}{V_2} = \frac{V_{left}}{V_{right}} = (\frac{P_f}{T_f} - \frac{P_2}{T_2})/(\frac{P_1}{T_1} - \frac{P_f}{T_f})$

Taking the data from Figure 5, the two ratios are received, one for each procedure:

Procedure A: $\frac{V_1}{V_2} = 1.78 \pm 0.02$ Procedure B: $\frac{V_1}{V_2} = 1.72 \pm 0.02$

Discussion Question 2: **Explain the heat transfer between the tanks and the surroundings for both of the expansion cases. What is the net heat transfer?**

From Figure 5, the initial states of tanks 1 and 2 (temperature and pressure) for both methods were relatively similar, with slight variations due to error in human judgment. Tank 1 had higher internal energy at a pressure (367.7, 371.1 kPa for Procedures A, B respectively), while Tank 2 was on average at a lower pressure (72.6, 66.3 kPa for Procedures A, B respectively). In both cases, Tank 1 transferred heat to Tank 2 until both of the systems reached thermal equilibrium. The system can be assumed to be closed and isochoric, as there were no leakages from the system nor were there any deformations to the apparatus. Thus, no boundary work was done on the system:

$$Q + W = \Delta E \rightarrow Q = \Delta E$$

Since the apparatus itself had no noticeable kinetic or potential energy from our frame of reference:

$$\Delta E = \Delta PE + \Delta KE + \Delta U \rightarrow \Delta E = \Delta U$$

Combining the two equations, we have $Q = \Delta U$: the heat transfer between the two tanks is equal to the difference in their internal energies. However, since the system was not adiabatic and heat transfer is governed by a difference in temperature, heat losses were inevitable. Since the temperature of the gasses was rarely at the ambient temperature of around 23.7 ºC, heat could have either entered or exited the system. Thus, heat transfer is split into the transfer between the surroundings and the system and the transfer between the two tanks themselves:

$$Q_{tanks} + Q_{surroundings} = \Delta U$$

Discussion Question 3: How does this experiment show that the state properties are path independent? Use a figure to assist your explanation.

The state properties considered in this experiment were pressure and temperature. In order to show path independence, we observe the final states of both processes A and B. It can be seen from Figure 6 that the pressure and temperature in both methods end at the same state regardless of the rate at which the two gasses mix.

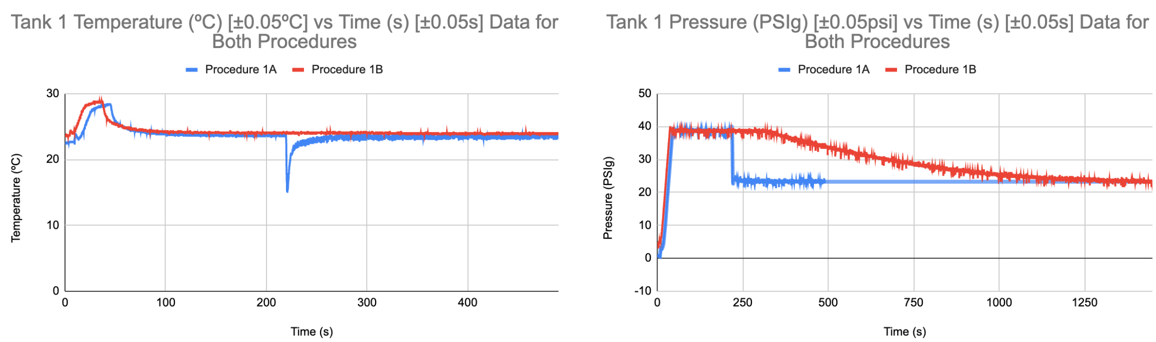

Figure 6: *Path functions of temperature and pressure for method A and B. Given the same initial state and environment, the final state will be the same.*

Lab 2 Discussion:

Before discussion of the lab, these following equations are defined:

$$m = \int mass\ flow\ rate\ dt$$

Equation 1

$$V = \frac{mRT}{P}$$

Equation 2

Calculate the Initial Mass and Volume of the Left Tank:

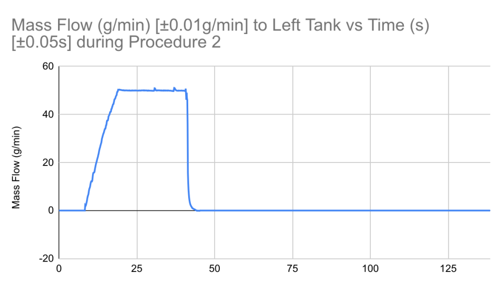

Figure 7: *Graph depicting the Mass Flow (g/min) to Left Tank vs Time (s) during Second Procedure showing a spike in mass flow at t=8.5s continuing to a maintained mass flow, f=51.02g/min. At t=41.3s mass flow drops to 0.00 g/min.*

As in Equation 1, the mass added to the system is the integral of the mass flow rate function, the flow rate in units of g/min are divided by 60 to be converted to g/s. This calculation was done using Python and the csv data collected by the LabView interface.

The following results were shown:

$$R_L = 23.61g \pm 0.01g \qquad R_R = 23.47g \pm 0.01g$$

R_R, R_L represent the right and left riemann sums. The mass added to the system used in further calculations will be referred to as $m_{add} = \frac{R_L + R_R}{2} = 23.54g \pm 0.01g$. To find the volume of the system, the initial and final conditions are analyzed. The initial condition is described by $P_i V = m_i R T_i$ where i represents the initial values; these initial values correspond to the air present in the tank before it is pressurized—the final condition can similarly be shown as $P_f V = m_f R T_f$ where f represents the final values. Furthermore, m_f can be represented as the sum of the mass added to the system and the mass initially present in the system: $m_f = m_i + m_{add}$ as per the conservation of mass. This can be used to simplify $V = \frac{m_i R T_i}{P_i} = \frac{(m_i + m_{add}) R T_f}{P_f}$ to $m_i = \frac{m_{add} T_f P_i}{T_i P_f - T_f P_i}$ where

$$P_i = P_{ambient} + P_{sig} = 105.6 \pm 0.1kPa \qquad T_i = 296.85 \pm 0.05K \qquad P_{ambient} = 100.8 \pm 0.1kPa$$

$$P_f = 40psi + P_i = 381.4 \pm 0.1kPa \qquad T_f = 297.85 \pm 0.05K$$

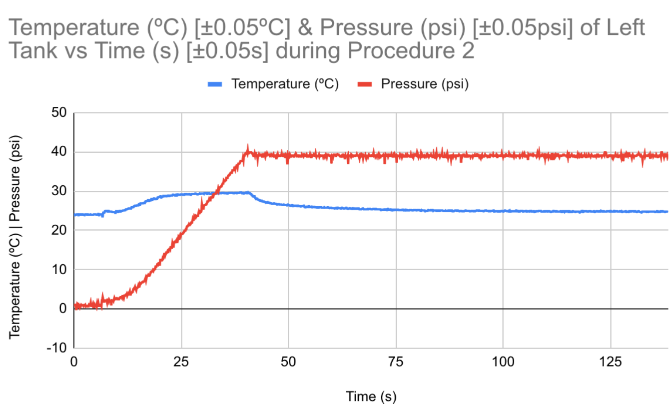

Figure 8: *The temperature and pressure shown on the same axis vs time as experienced by the left tank. The increase in pressure is also attributed to the mass flow increasing in Figure 7, alongside the increase in pressure. The stabilization of the pressure is also attributed to the prevention of mass flow, the temperature is then seen to approach equilibrium as the system stabilizes.*

T_f is calculated by collecting the last temperature result from the left tank sensor as shown in Figure 8. P_i is the sum between the ambient pressure and the pressure already present in the tank at t=0s, shown in Figure 8. P_f is the sum of P_i and the idealistic 40 psi added to the left tank.

Following these variables the initial mass of the left tank can be calculated as $m_i = 9.05g \pm 0.05g$.

To find the volume of the tank reference must be made to Equation 2, which rearranges the ideal gas equation to isolate for volume:

$V = \frac{m_f R T_f}{P_f}$, where $m_f = m_i + m_{add}$

The equation can then be solved with the constant $R = 287.0 J/kgK$ (Chandra, 2016) and mass represented as $m_f = m_i + m_{add} = 23.54g + 9.05g = 0.03259kg \pm 0.000005kg$. This results in the volume of the left tank being $V = 0.007m^3 \pm 0.001m^3 = 7.0L \pm 1.0L$.

What are the sources of error?

a) Ideal Gas Assumption: One prominent source of error is the false assumption that air is an ideal gas. Since air is not an ideal gas, our assumptions towards negligible volume, perfect elastic collisions, etc… do not reflect the properties of air.

b) Non-Constant Ambient Pressure and Temperature: Pressure and temperature readings were taken only once at the beginning of the lab; however, these conditions vary. This is a source of error which has not been considered in our calculations.

c) Error in Human Judgement: Since the mass flow rate was controlled by students through the LabView interface, and was thus dependent on human reaction time, it was impossible to consistently stop the pressure at exactly 40.0 psi.

d) Instrumental Error: The instrumentation errors have all been accounted for as uncertainties, but the mass flow rate sensor in particular would always create a spike of pressure when first opened from 0 g/min, which created inaccuracies in Riemann Sum results. Further, data collected was constantly noisy as shown in all graphs throughout this report.

e) Diabatic System + Open System: Since these equations were used with an idealistic interpretation of the ideal gas law, no measures were taken to account for pressure leaking out of the instruments, or heat transfer between the system and the surroundings.

Does the compressibility of air affect your results?

The compressibility of air creates uneven distribution of air density and pressure when the air enters the tank. This affects the readings of the pressure gauges within the system and causes inaccuracies with calculating the mass flow rate. In reference to Figure 8, there could be some attribution to the noise in the pressure data due to the compressibility of air creating irregularities that the sensor doesn't pick up instantaneously.

Conclusion

In conclusion, the exploration of path independence of thermodynamic state properties through this experiment yielded valuable insights into the fundamental principles governing the behavior of gasses. By examining two distinct scenarios of air expansion, we demonstrated that the final state properties, specifically temperature and pressure, remain invariant regardless of the expansion process. This was evinced by the final volume ratio of the two tanks after the two different processes, which were 1.78 ± 0.02 and 1.71 ± 0.02.

Further, we were able to use the Ideal Gas Law to determine the values of the initial mass and volume of the left tank (9.05 ± 0.05g and 7.0L $\pm$ 1.0L respectively)—these values were unknown to us beforehand, thereby demonstrating the utility of this formula in enabling students to obtain important properties of a system.

References

Chandra, S. (2016). Energy, Entropy and Engines. Wiley.