Applications of the First Law of Thermodynamics in Measuring Properties of a Non-Adiabatic Isochoric System

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CHE260 PRA0101 - 9am-12pm - First Law of Thermodynamics

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1 Abstract

This lab report investigates the First Law of Thermodynamics using an ideal gas to study system properties related to heat and work. The first law states that all energy is conserved and can only be transferred from one form to another. Mathematically, it is expressed as $Q + W = \Delta E$; where Q is the heat, W is the boundary work, and ΔE is the change in energy of the system. The system studied in this lab is an acrylic tank whose temperature and pressure can be controlled, monitored, and adjusted. The results of the experiment validate the principles of the First Law and demonstrate how a mathematical model can be used to model heat transfer and losses, calculate c_v for air, and measure the work done by a propeller.

2 Introduction

The study of thermodynamics is critical to understanding the physical world. Many fields of physics and engineering are concerned with the process of energy transfer from one form to another, and as a result the First Law (also called the principle of Conservation of Energy) is fundamental. Applications of the First Law include refrigeration, aircraft, heat engines, power plants, and any process that deals with losses due to non-conservative forces. The first law is written as:

$$W + Q = \Delta E = \Delta U \tag{1}$$

Where W is the work done, Q the heat transfer, ΔE the change in energy, and ΔU internal energy is the sum of all the microscopic energies of the system (molecular, chemical, vibrational, rotational) [1]. Note that equation (1) can contain mechanical energy terms; however, the system in this lab remains stationary.

The objective of this experiment is to perform an energy balance of a real system by accounting for heat losses through different materials namely, aluminum and acrylic. Moreover, the constant volume specific heat capacity of air will be calculated, since the system is assumed to be isochoric. Finally, the work done by the tank's propeller will be analyzed using fan and blade performance equations. This is a specific application of the First Law, once again highlighting its ubiquity in industrial.

3 Experimental Method

The apparatus is comprised of pipes and two large acrylic tanks, only the left one will be used. The tank can be filled with air, where its pressure and temperature can be manipulated. All of these properties can be measured as a function of time, taken at 0.1 s intervals. This data is recorded and plotted in real time through the LabView software. The full

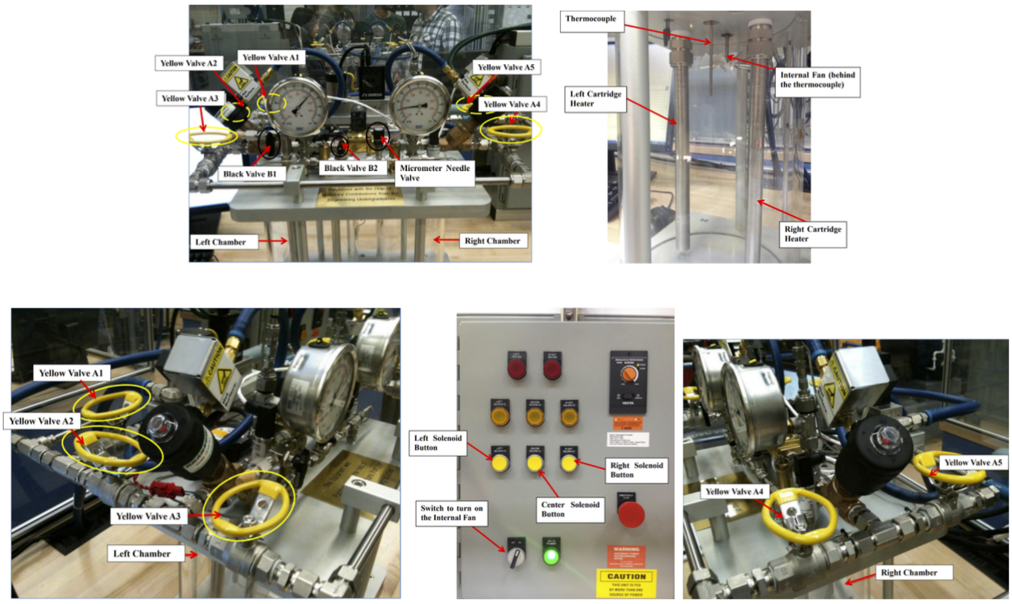

Figure 1: *From left to right, top to bottom: center of apparatus, left tank, left side view of apparatus showing the valves, control board, right side view of apparatus*

apparatus is labelled in Figure 1.

Procedure Expanding the Tank (Part 1): Measure the ambient air pressure using the manometer. On the control board, turn the fan switch to the "ON" state. Assure that the yellow and black valves are closed. Start data collection on the LabView interface, then press the left solenoid button and physically open yellow valve A2 by rotating. At this point, air can be expanded into the left tank, so gradually increase the mass flow rate from 0 g/min to 50 g/min by clicking the increase arrow in LabView. As soon as the pressure reaches $P = 40$ $psig$, close the solenoid and valve A2. Reset the mass flow rate to 0 g/min and wait for the pressure and temperature graphs to plateau. Record the data, and then reset.

Procedure Heating the Tank (Part 2): Start data collection in LabView and set the temperature to $T = 40°C$ in the box labelled "Target Temperature". Turn on the heaters by clicking the "Hold" button. Wait for the temperature to reach $T = 40°C$ and then wait for 5 minutes. Save the data. Now, follow the subsequent steps to cool the tank:

Cooling the Tank: Start collecting data again (to monitor the temperature). Open the left solenoid valve and pull on the black bar valve behind the left tank. There should be a loud hissing noise, which is normal. Once the temperature has cooled down to to $T = 30°C$, close the bar valve and the solenoid. Note that the process of cooling down can take 5-10 minutes of air circulation.

Repeat the above steps 3 more times given the following conditions:

(a) 40 $psig$ and $T = 40°C$. Cool the tank to $T = 30°C$ (This trial was described above)

(b) 70 $psig$ and $T = 40°C$. Cool the tank to $T = 30°C$

(c) 40 $psig$ and $T = 60°C$. Cool the tank to $T = 40°C$

(d) 70 $psig$ and $T = 60°C$. Cool the tank to $T = 40°C$

4 Results

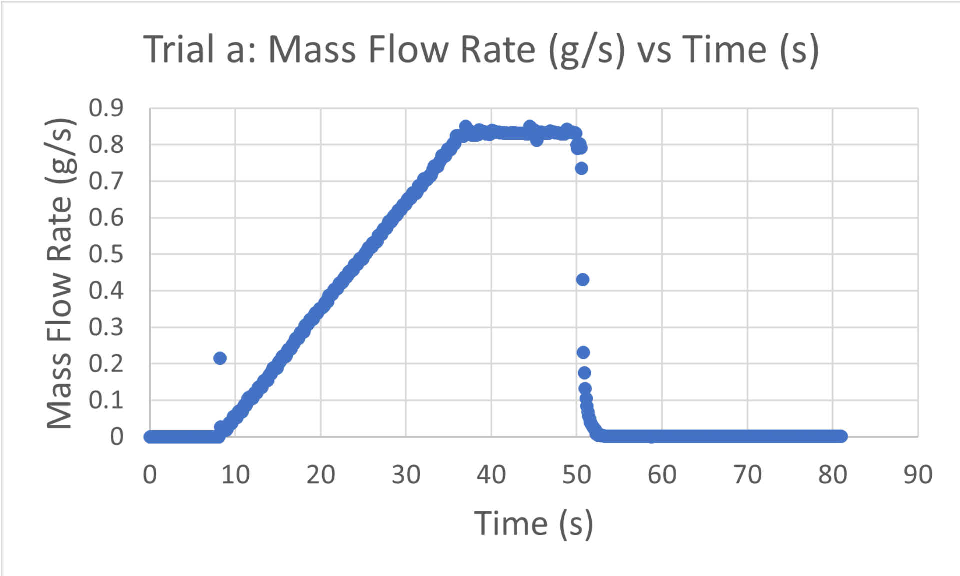

Figure 2: Graph of mass flow rate, in g/s, over time, in s, for Trial a.

Trial	m_{added} $[g]$	T_1 $[K]$	T_2 $[K]$	$P_{1\ abs}$ $[kPa]$	$P_{2\ abs}$ $[kPa]$
a	23.69	296.8	298.0	105.9	376.9
b	41.35	301.9	302.5	105.9	584.4
c	24.34	302.7	303.5	105.9	379.0
d	41.06	311.6	310.6	105.9	585.8
Δ $[\pm]$	Δm_{added} $[\pm g]$	ΔT_1 $[\pm K]$	ΔT_2 $[\pm K]$	$\Delta P_{1\ abs}$ $[\pm kPa]$	$\Delta P_{2\ abs}$ $[\pm kPa]$
a	0.50	0.1	0.2	2.07	2.76
b	0.50	0.2	0.1	2.07	3.45
c	0.50	0.1	0.1	2.76	3.45
d	0.50	0.1	0.1	3.45	4.83

Table 1: Mass added to the tank across each trial, with their associated uncertainties. Mass added for Trial a found by integrating the function numerically in Figure 2 (and similarly for Trials b-d with their respective mass flow rates). Temperature and pressure data also collected for state 1, before pressurization, and state 2, after pressurization.

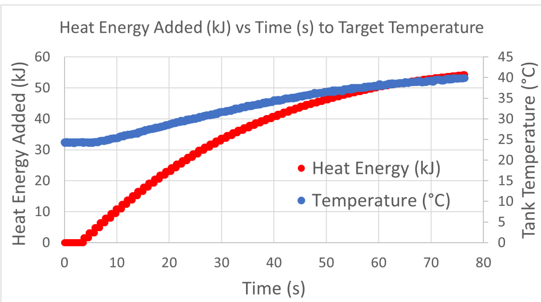

Figure 3: Heat energy added, in [kJ], over time, in [s], to reach the target temperature for Trial a.

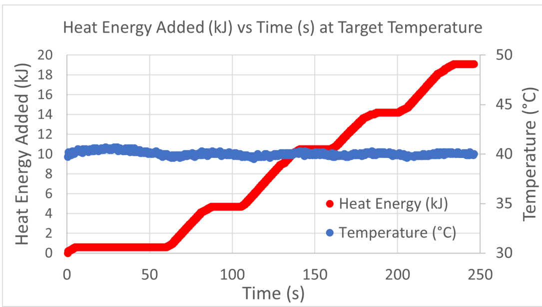

Figure 4: Heat energy added after reaching target temperature, in [kJ], over time, in [s], for Trial a.

Trial	$T_{Ambient}$ $[°C]$	T_{Trial} $[°C]$	Time Maintained $[s]$	Q to Maintain $[kJ]$
a	23.4	40.0	246.0	19.1
b	23.4	40.0	294.0	28.4
c	23.4	60.0	250.4	43.4
d	23.4	60.0	312.6	63.2
Δ $[\pm]$	$\Delta T_{Ambient}$ $[\pm°C]$	ΔT_{Trial} $[\pm°C]$	Δt $[\pm s]$	ΔQ $[\pm kJ]$
a	0.1	0.1	1.00	0.2
b	0.1	0.1	1.00	0.2
c	0.1	0.1	1.00	0.2
d	0.1	0.1	1.00	0.2

Table 2: Heat energy added to maintain the tank temperature, along with the time the tank was maintained at the trial temperature. The energy to maintain the temperature and the time maintained are deduced for Trial a from Figures 3 and 4 (and similarly for Trials b-d with their respected heat energies added) as the amount of heat energy added and amount of time after the target temperature is reached.

5 Discussion

5.1 Part 1

In order to calculate the mass of the left tank, the following formula is given:

$$m_{left\ tank} = m_{added\ to\ tank}[1 + \frac{1}{(P_2T_1)/(P_1T_2) - 1}] \tag{2}$$

where T_1, T_2 and P_1, P_2 are the temperatures and pressures before and after expanding gas into the left tank. The relevant measurements along with corresponding uncertainties are tabulated in Table 3. m_{added} is calculated through numerical integration by summing up discrete rectangles corresponding to the change in mass flow rate over a fixed time interval $\Delta\dot{m} \times \Delta t$.

Below is a sample calculation of m_{left} for trial a:

$$m_{left\ tank} = 23.69\ g \times [1 + \frac{1}{\frac{(376.9\ kPa \times 296.8\ K)}{(105.9\ kPa \times 298.0\ K)} - 1}] \rightarrow m_{left\ tank} = 33.00\ g$$

In order to propagate uncertainties, the partial derivatives of Equation 2 with respect to each variable $(m_{added}, T_1, T_2, P_1, P_2)$ are squared, summed and then square rooted as follows [2]:

$$\Delta m_{left} = \sqrt{(\frac{\partial m_l}{\partial T_1}\Delta T_1)^2 + (\frac{\partial m_l}{\partial T_2}\Delta T_2)^2 + (\frac{\partial m_l}{\partial P_1}\Delta P_1)^2 + (\frac{\partial m_l}{\partial P_2}\Delta P_2)^2 + (\frac{\partial m_l}{\partial m_{added}}\Delta m_{added})^2}$$

Trial	m_{left} $[g]$	Δm_{left} $[\pm g]$
a	33.00	0.9
b	50.52	0.8
c	33.82	1.0
d	50.09	0.8

Table 3: Calculated total mass in the tank after pressurization, with values from Table 1.

5.2 Part 2

5.2.1 Determining heat loss from the tank

In order to calculate the heat loss, 3 assumptions must be made to simplify Equation 1.

1. Assuming the acrylic cylinder is isochoric (V = constant), there is no boundary work, so the work done by/on the system is 0 [1].
2. The system itself is not moving, meaning there is no bulk movement, so $\Delta PE + \Delta KE = 0$.
3. At the "hold temperature", the temperature remains constant, so $\Delta U = 0$.

From the above assumptions, Equation 1 reduces to the following:

$$Q + W = \Delta U \rightarrow \Sigma\dot{Q} = 0 \quad \rightarrow \quad \dot{Q}_{TotalAdded} - \dot{Q}_{Loss} = 0 \quad \rightarrow \quad \dot{Q}_{TotalAdded} = \dot{Q}_{Loss}$$

The rate of heat energy added is given as the final energy minus the initial energy, over the time period. The initial energy is recorded at the first instant the tank reaches the target temperature, and the final energy is taken at the last instant of recorded data.

$$\dot{Q}_{Loss} = \frac{Q_{final} - Q_{initial}}{t_{final} - t_{initial}} = \frac{\Delta Q}{\Delta t} \tag{3}$$

Trial	$\dot{Q}_{loss}$ $[W]$	$\Delta\dot{Q}_{loss}$ $[\pm W]$
a	77.6	0.9
b	96.6	0.8
c	173.3	1.1
d	202.2	0.9

Table 4: The rate of energy loss, using Equation 3 with values from Table 2, with uncertainties for Trials a-d.

The results of Table 4 show that for trials at the same temperature (i.e. a & b and c & d) the rate of heat loss is slightly higher for higher pressures. For trials at the same pressure but different temperatures (a & c and b & d), the rate of heat loss is much higher for higher temperatures.

5.2.2 Calculating heat loss through the top and bottom plates of the tank

The rate of heat loss through the cylindrical walls of the tank is given by

$$\dot{Q}_{Acrylic} = 2k\pi l \frac{\Delta T}{\ln(\frac{r_2}{r_1})} \tag{4}$$

where k is the conductive heat transfer coefficient, l is the length of the cylinder, ΔT is the difference in temperature between the inside and outside of the walls, and r_1 and r_2 are the inner and outer radii of the cylinder, respectively.

	Imperial	Metric
r_2	4 ± 0.02"	0.1016 ± 0.0005 m
r_1	3.625 ± 0.06"	0.09208 ± 0.00152 m
l	11.25 ± 0.001"	0.28575 ± 0.00003 m
k	N/A	0.185 ± 0.15 $\frac{W}{mK}$

Table 5: Constants in imperial and metric units for Equation 4 [3].

Assuming heat was lost only through the acrylic walls of the cylinder and the top and bottom plates gives the following energy balance:

$$\dot{Q}_{Loss} = \dot{Q}_{Acrylic} + \dot{Q}_{Plates} \rightarrow \dot{Q}_{Plates} = \dot{Q}_{Loss} - \dot{Q}_{Acrylic} \tag{5}$$

A sample calculation is given for Trial a:

$$\dot{Q}_{Plates} = 77.6\ W - 2 \times 0.185 \times \pi \times 0.28575 \frac{40.0 - 23.4}{\ln{(0.1016/0.09208)}} = 21.5\ W$$

Where the uncertainty is propagated through the following calculation:

$$\Delta Q_p = \sqrt{\left(\frac{\partial Q_p}{\partial k}\Delta k\right)^2 + \left(\frac{\partial Q_p}{\partial l}\Delta l\right)^2 + \left(\frac{\partial Q_p}{\partial T_2}\Delta T_2\right)^2 + \left(\frac{\partial Q_p}{\partial T_1}\Delta T_1\right)^2 + \left(\frac{\partial Q_p}{\partial R}\Delta R\right)^2 + \left(\frac{\partial Q_p}{\partial Q}\Delta Q\right)^2}$$

Trial	$\dot{Q}_{Plates}$ $[W]$	$\Delta\dot{Q}_{Plates}$ $[\pm W]$
a	21.5	1.1
b	40.4	0.9
c	49.5	1.2
d	78.3	1.0

Table 6: Calculated heat loss through the top and bottom plates of the tank, using data from Tables 2 and 5.

From Table 6, it can be concluded that a significant amount of heat is lost through the top and bottom plates. For instance, the heat lost in Trial a through the plates represents around 28% of the total heat lost throughout the constant temperature process.

Sources of Error:

(a) Equation 5 assumes that energy is only lost through the acrylic and aluminum plates. However, in the left tank are located a thermo-couple and a thermal fan that could absorb some of the heat energy as well.

(b) Equation 4 assumes ΔT is held constant; however, this is not the case, since the ambient temperature increases with time. In fact, the surroundings get noticeably warmer in the lab as time goes on. Thus, the difference in temperature is a function of time, and should decrease with time.

5.2.3 Estimation of constant volume specific heat of air

By definition, the constant volume specific heat c_v is

$$c_v \equiv \left(\frac{\partial u}{\partial T}\right)_v \tag{6}$$

Where u is the specific internal energy and T is the temperature. Manipulating Equation 6 in order to equate c_v in terms of measured values such as the rate of temperature rise, $\frac{dT}{dt}$, gives the following:

$$c_v \equiv \left(\frac{\partial u}{\partial T}\right)_v = \frac{\frac{1}{m}\frac{dU}{dt}}{\frac{dT}{dt}}$$

The time derivative of temperature is approximated as follows: $\frac{dT}{dt} = \frac{T_x - T_{ambient}}{\Delta t}$

The internal energy was not directly measured in this case; however, in doing an energy balance, it can be seen that it equates to the rate of energy into minus rate of energy out of the system:

$$\frac{dU}{dt} = \dot{Q}_{in} - \dot{Q}_{out}$$

The rate of energy into the system can be measured as the energy required to heat the system to the holding temperature, over the time required to do so: $\dot{Q}_{in} = \frac{\Delta Q_{heating}}{t_f - t_i}$. However, methods of calculating $\dot{Q}_{out}$ during the heating period are unknown. Thus, $\dot{Q}_{out}$

is taken to be equal to the heat lost $\dot{Q}_{lost}$ (from Table 4) during the constant temperature process. This would result in an overestimate of the true value of $\dot{Q}_{out}$ because the rate of energy loss is higher when the system is at a higher temperature i.e. when $\dot{Q}_{loss}$ is calculated.

Combining the above simplifications:

$$c_v = \frac{\frac{1}{m} \times (\frac{\Delta Q_{heating}}{\Delta t} - \dot{Q}_{avg\ loss})}{\frac{T_x - T_{ambient}}{\Delta t}} \tag{7}$$

From this relationship, four values of c_v are calculated for Trials a-d.

Trial	m_{left} $[g]$	$\frac{\Delta Q_{heating}}{\Delta t}[W]$	$\dot{Q}_{avg\ loss}[W]$	δT [K]	$c_v[J/K * kg]$
a	33.00	707.0	77.6	15.7	93290.8
b	50.52	515.6	96.6	15.7	40673.4
c	33.82	1094.8	173.3	35.7	74040.4
d	50.09	940.0	202.2	35.7	36478.7
Uncertainties	Δm_{left} $[\pm g]$	$\Delta \dot{Q}_{in}[\pm W]$	$\Delta \dot{Q}_{avg\ loss}[\pm W]$	$\Delta T[\pm K]$	Δc_v
a	0.9	2.8	0.9	0.1	2652
b	0.8	2.7	0.8	0.1	748
c	1.0	2.4	1.1	0.1	2210
d	0.8	2.5	0.9	0.1	607

Table 7: Table of values for Equation 7 needed to calculate c_v. Note that uncertainties were propagated using the same partial derivative formula as prior.

The literature value of c_v at a temperature of 300 K is 0.716 $kJ/kgK = 716$ J/kgK [1]. The values computed in Table 7 are off by two orders of magnitude. It is also seen that the uncertainties Δc_v do not bring the calculated values of c_v to within the literature value. As such, it is deduced that there is a large systematic error with equation (7).

Sources of Error:

(a) The literature equation is in terms of derivatives which correspond to instantaneous rates of change; yet, in the above calculations, the average slope over the process is taken. This would be valid if the slopes of internal energy and temperature were linear with time, but they were not.

(b) The ambient temperature changes with time. At the beginning of each trial, the LabView equipment measures the ambient temperature, which is not a constant since the heat loss is to the surroundings.

(c) The rate of heat loss when increasing the temperature of the tank was assumed to be equal to the heat loss during the holding temperature.

5.3 Part 3

The power P of the propeller is given by:

$$P_2 = P_1 \frac{\rho_2}{\rho_1} \left(\frac{n_2}{n_1} \right)^3 \left(\frac{D_2}{D_1} \right)^5 \tag{8}$$

where ρ is the air density, n is the shaft pump speed, D is the propeller diameter. Variables denoted with a subscript 1 are given by the manufacturer as $P_1 = 0.7457$ W, $n_1 = 4200$ rpm, and $D_1 = 2.5$ in, at 273.15 K and 100 kPa [3]. Variables denoted with a subscript 2 are recorded in the lab as $n_2 = 2000$ rpm and $D_2 = D_1 = 2.5$ in.

Note that by the ideal gas law, $P = \rho RT$, the quantity $\frac{\rho_2}{\rho_1}$ can be expressed as

$$\frac{\rho_2}{\rho_1} = \frac{P(273.15\ K)}{T(100\ kPa)}$$

where P and T are the tank pressure and tank temperature, respectively, during the constant temperature portion of experiment (Tables 1 and 2) . Since power is equal to work done over time, the work can be expressed as $W = P\Delta t$, where Δt is the time during the constant temperature process.

The rate of temperature rise is derived from Equation 1, where assuming an adiabatic process ($Q = 0$) and dividing each term by the change in time gives:

$$\frac{\Delta T}{\Delta t} = \frac{P}{mc_v} \tag{9}$$

where $\frac{\Delta T}{\Delta t}$ is the expected rate of temperature rise, P is the propeller power, m is the mass of air in the tank, and c_v is given for air as $c_v = 717\ \frac{\text{J}}{\text{kgK}}$ [1]. Computing Equation 9 with the specific propeller power for each trial leads to the values compiled in Table 8.

Trial	$P_2\ [W]$	$W[J]$	$\frac{\Delta T}{\Delta t}[\frac{°C}{s}]$
a	0.265	65.2	0.0112
b	0.410	121	0.0113
c	0.250	62.6	0.0103
d	0.387	121	0.0108
Uncertainties	$\Delta P_2\ [\pm W]$	$\Delta W[\pm J]$	$\Delta\frac{\Delta T}{\Delta t}[\pm\frac{°C}{s}]$
a	0.002	0.6	0.0003
b	0.002	1	0.0002
c	0.002	0.6	0.0003
d	0.003	1	0.0002

Table 8: Tabulated results for the power of the propeller, the work done by the propeller on the system, and the rate of temperature change caused by the propeller.

Significance of the propeller work: The work done on the system by the propeller is insignificant compared to the heat added. This is concluded because the propeller work is less than the uncertainty in the measurement of the heat added. As an example, it is seen that the propeller work in Trial a makes up just 0.34% of the heat added, whereas the uncertainty of the heat added for Trial a is over 1% of the heat added.

Source of Error: Equation 8 assumes that the propeller is only supplying power through shaft work, but it is also possible that as the propeller may heat up as it runs for extended periods of time, and thus transfers heat through convection to the tank. The formulae above assume air is an ideal gas; however, this is not the case, as it experiences inter-molecular forces, and its constituents are not mono-atomic.

6 Conclusion

The First Law of Thermodynamics, given by $W + Q = \Delta E$, is a powerful equation in calculating the changes in the energy of a control mass system. Its application in Part 2 of this experiment show that the rate of heat loss for systems at the same temperature is slightly greater when the pressure is greater, and that the rate of heat loss for systems at the same pressure is much greater when the temperature is greater.

Further calculations show that a moderate amount of the heat lost in the constant temperature processes came from heat lost through the top and bottom plates of the cylinder.

Attempts to find the constant volume specific heat of air yielded values incorrect by two orders of magnitude, likely due to large systematic error in using the first law to derive Equation 7. In assessing the effects of the tank fan propeller's effect on the experiment, it was found that the work done by the propeller was insignificant when compared to the uncertainty of the heat added.

References

[1] S. Chandra. *Energy, Entropy, and Engines: An Introduction to Thermodynamics*. John Wiley & Sons, Ltd, United Kingdom, 2016.

[2] PHY293. *Uncertainty Propagation Formulae*. Year Accessed 2023

[3] CHE260. *Lab 2 – First Law of Thermodynamics*. Year Accessed 2023.