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PHY293 Lab 1: Measuring the output resistance of a power source and the internal resistance of practical voltmeters and ammeters

I. Introduction

Question 1:

Explaining the difference in measurements of the non-ideal Voltmeter and Ammeter in Options 1 and 2

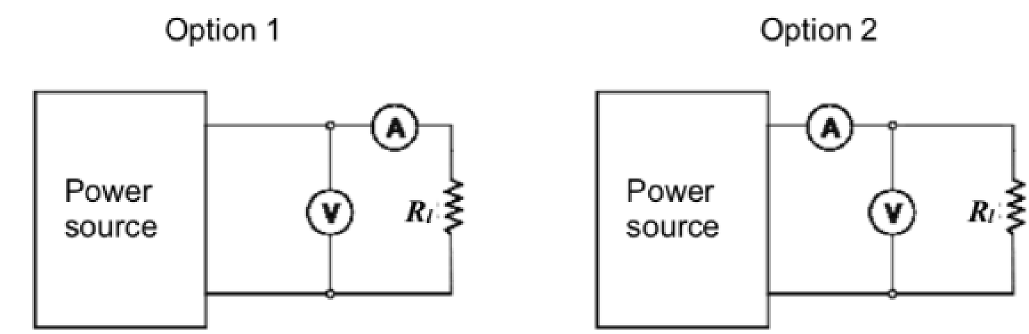

Fig. 3. Possible circuits for determining the output resistance of a power source.

Voltmeter Readings: It is expected that the voltmeter reading in 1 will be higher than in 2. Real ammeters have a non-zero internal resistance, so voltage division takes place in option 1. The voltage will be taken across both the real ammeter and the load resistance. Conversely, in option 2, the voltage will only be taken across the load resistance.

Ammeter Readings: It is expected that the current reading is more accurate in option 2, as it measures directly from the power source, whereas option 1 measures the current after the voltmeter in parallel. Practical voltmeters do not have infinite resistance, as such not all current enters through the ammeter. Thus, the measured current for option 1 is lower than that of option 2 because current division takes place before the ammeter takes the reading.

Question 2:

Deriving equations to calculate the internal resistance of the non ideal Voltmeter and Ammeter.

To find the internal resistance of the ammeter, set up the circuit as shown in option 1. The ammeter is in series with the resistor of the circuit, so its internal resistance is found through voltage division:

$$V_{measured} = IR_{ammeter} + IR_{load}$$

$$R_{ammeter} = (V_{measured} - IR_{load})/I$$

Where I is the measured current from the ammeter.

To find the internal resistance of the voltmeter, set up the circuit as shown in option 2. The current is divided between the voltmeter and resistor in parallel, so KCL is applied to find the internal resistance of the voltmeter:

$$I = (V_{measured})/(R_{voltmeter}) + (V_{measured})/(R_{load})$$

$$R_{voltmeter} = 1/(\frac{1}{V_{measured}} - \frac{1}{R_{load}})$$

II. The Experiment

Resistances (ohms)	**Reading (ohms)**	**Uncertainty (+-ohms)**	**Relative Uncertainty (%)**
100	99.78	0.005	0.0050
220	219.81	0.005	0.0023
470	463.7	0.005	0.0011
5 k	5.147 k	0.005 k	0.0971
2.7 k	2.708 k	0.005 k	0.1846
27 k	26.84 k	0.005 k	0.0186
100 k	101.68 k	0.005 k	0.0049

Table 1: Actual values of resistors and associated relative uncertainty

Question 3:

Four resistors, 470, 100k, 220, 100 Ω. with the lowest relative uncertainty [table 1] were chosen initially to minimize error propagation in proceeding sections of the experiment. Since there was sufficient time at the end of the experiment, the other three resistors were tested to record more data points to more accurately identify possible trends in record data.

Circuit Option 1:

	Resistance	Uncertainty	Voltage V	Uncertainty V	Current mA	Uncertainty mA	Resistance of Ammeter	Uncertainty
1	99.78	0.005	6.498	0.0005	63.69	0.005	**2.25**	**1.57E-05**
2	219.81	0.005	6.5	0.0005	29.281	0.0005	**2.18**	**0.000222**
3	463.7	0.005	6.501	0.0005	13.955	0.0005	**2.15**	**0.000465**
4	2708	5	6.501	0.0005	2.394	0.0005	**7.54**	**0.00271**
5	26840	5	6.501	0.0005	0.239	0.0005	**360.8**	**0.0272**
6	101680	5	6.501	0.0005	0.06	0.0005	**6670**	**0.108**
						Average:	2.19	0.028

Table 2: Resistance of the ammeter and its uncertainty for different values of load resistance
*Average resistance of the ammeter is calculated with resistors one to three.

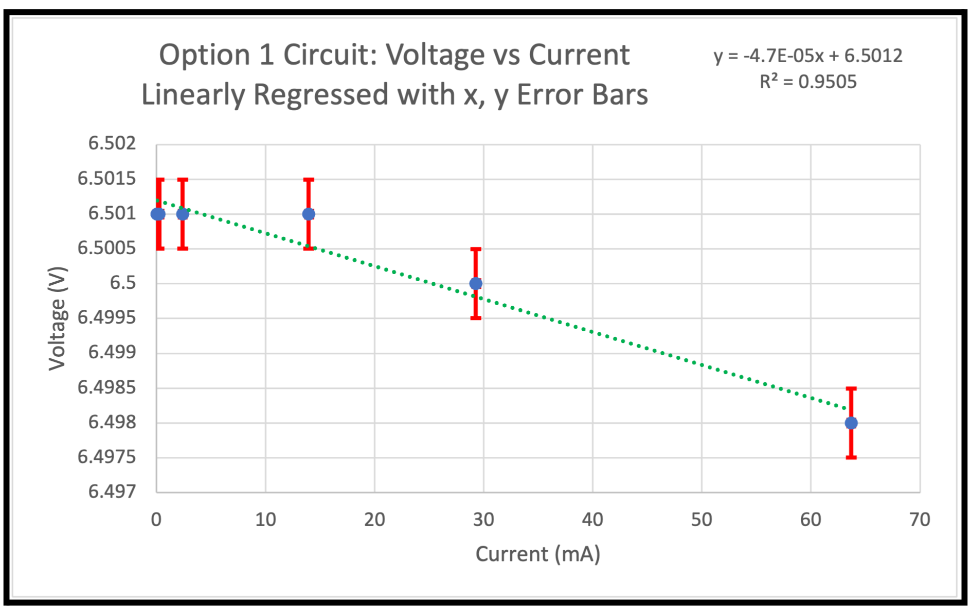

Figure 1: Circuit 1 V vs I linearly regressed. Note that the x error bars (+- 0.005 mA) are too small to be visible.

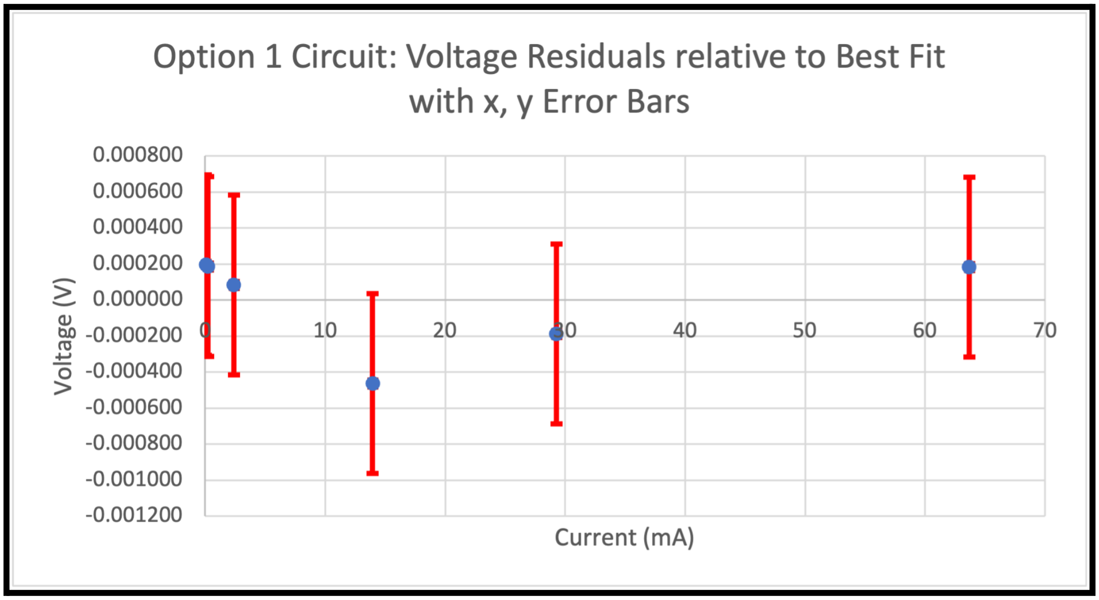

Figure 2: Circuit 1 residuals from relative to the model in Figure 1. All error bars fall within the range of residuals.

In Figure 1, we received a linear model of $V = -4.73 \times 10^{-5} I + 6.50\ [V]$. Using the "LINEST" built in function in excel, our slope had a standard error of $\pm 5.44 \times 10^{-6}$, y-intercept an error of $\pm 1.58 \times 10^{-4}$, and the R^2 of this fit was found to be 0.951. Using the "CHISQ.TEST" built in function in Excel, the reduced χ^2 was found to be an ideal value of 1.00., which indicates that the data has an excellent fit.

Circuit Option 2:

	Resistance	**Uncertainty**	**Voltage V**	**Uncertainty V**	**Current mA**	**Uncertainty mA**	**Resistance of Voltmeter**	**Uncertainty**
1	99.78	0.005	6.407	0.0005	64.12	0.005	**-70202.326**	-7.381
2	219.81	0.005	6.458	0.0005	29.373	0.0005	**-933369.045**	-75.185
3	463.7	0.005	6.481	0.0005	13.974	0.0005	**-2392325.824**	-192.142
4	2708	5	6.498	0.0005	2.398	0.0005	**-4173762.808**	-333.9301
5	26840	5	6.501	0.0005	0.241	0.0005	**-5358932.432**	-429.228
6	101680	5	6.502	0.0005	0.063	0.0005	**-6875242.928**	-550.600
						Average:	-4700065.998	200.239

Table 3: Resistance of the voltmeter and its uncertainty for different values of load resistance

*Average resistance of the voltmeter calculated with resistors three to six.

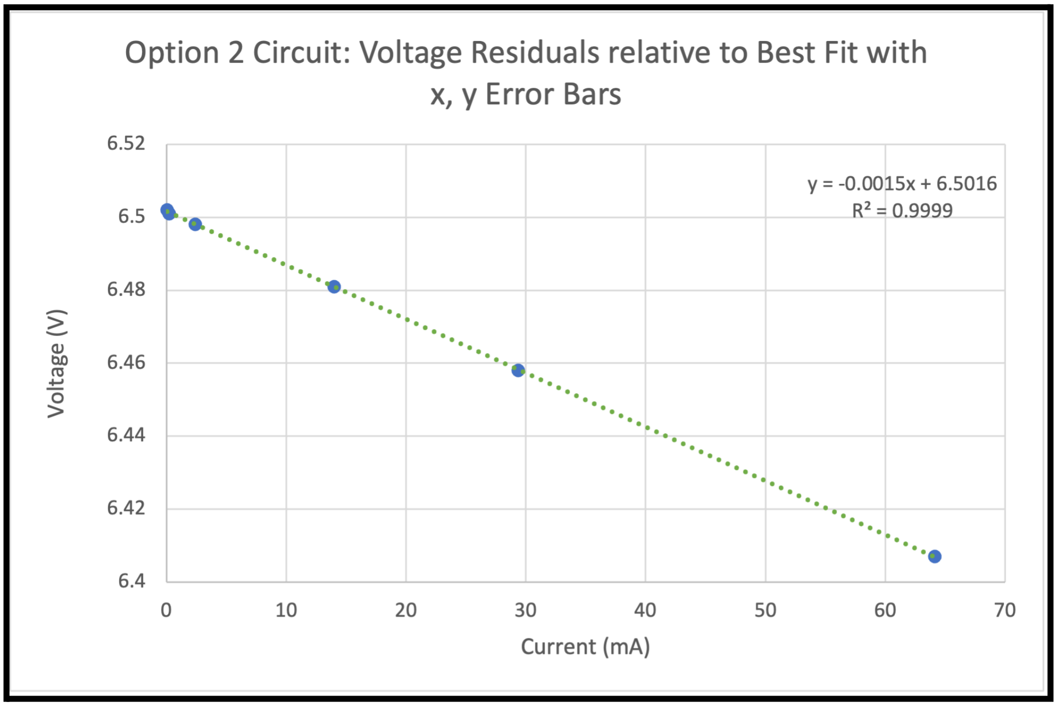

Figure 3: Circuit 3 V vs I linearly regressed. Note that both x and y error bars (± 0.005 mA and ± 0.005 V, respectively), are too small to be seen in the graph

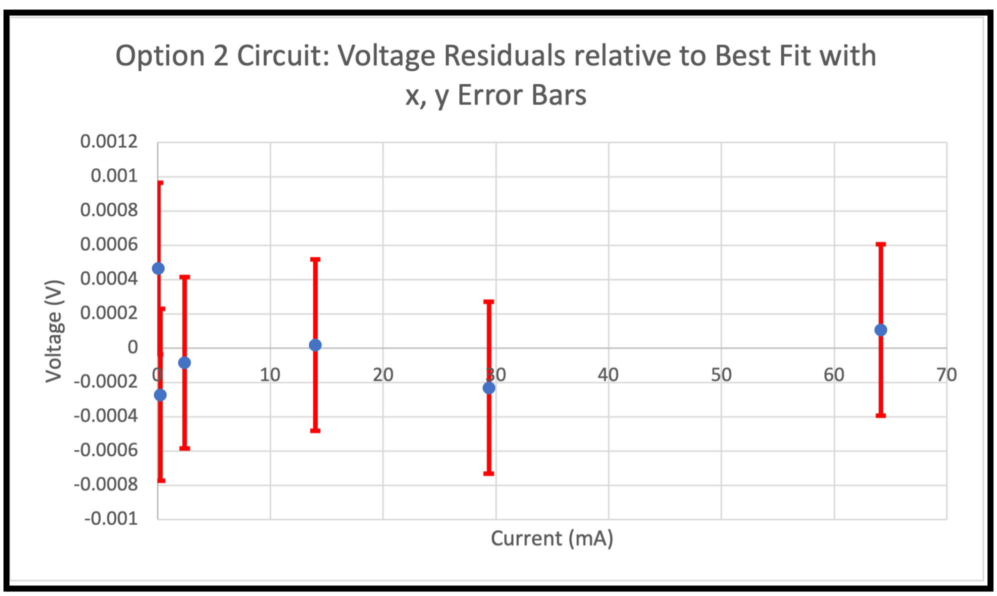

Figure 4: Circuit 2 residuals from relative to the model in Figure 3. All error bars fall within the range of residuals.

In Figure 2, we received a linear model of $V = -0.00148\,I + 6.51\,[V]$. Using the "LINEST" built in function in Excel, our slope had a standard error of $\pm 5.36 \times 10^{-6}$, the y-intercept an error of $\pm 1.57 \times 10^{-4}$, and the R^2 of this fit was found to be 0.999. Using the "CHISQ.TEST" built in function in Excel, the reduced χ^2 was found to be an ideal value of 1.00, indicating that the data had an excellent fit.

III. Analysis

Derive a relationship among m1, Rv, and R1, to find the output resistance with its uncertainty. Show the steps of your error propagation calculation for the uncertainty of R1.

$$R_1 = \frac{m_1 R_v}{m_1 + R_v}, \text{ where } m_1 = \frac{v}{I}$$

$$R_1 = \frac{-4.7\times10^{-2} \times -4700065}{-4.7\times10^{-2} - 4700065} = -4.7 \times 10^{-2}\,\Omega$$

Using data from the experiment and the derived formula for R_1, $R_1 = -4.7 \times 10^{-5}\,\Omega$. The error propagation was done using the following formula, where $u(m_1)$ is the standard error on the slope from figure 1 found from the "LINEST" Excel function. The value of m_1 was found by multiplying the slope of the graph in figure 1 by 10^3 to convert KΩ to Ω.

$$u(Z_1) = \sqrt{\left(\frac{u(m_1)}{m_1}\right)^2 + \left(\frac{u(R_V)}{R_V}\right)^2} \cdot (m_1 R_v)$$

$$u(Z_2) = \sqrt{u(m_1)^2 + u(R_v)^2}$$

$$Z = \sqrt{\left(\frac{u(Z_1)}{m_1 R_V}\right)^2 + \left(\frac{u(Z_2)}{m_1 + R_V}\right)^2} \cdot \frac{m_1 R_V}{m_1 + R_V}$$

Using Excel, the value of Z was found to be $6.1 \times 10^{-6}\,\Omega$

Derive a relationship among m2, RA and R2, to find the output resistance with its uncertainty. Show the steps of your error propagation calculation for the uncertainty of R2.

$$R_2 = -(R_A + m_2), \text{ where } m_2 = \frac{v}{I}$$

$$R_2 = -(2.19 - 1.5) = -0.69\,\Omega$$

Using data from the experiment and the derived formula for R_2, $R_2 = -0.69\,\Omega$. The error propagation was done using the following formula, where $u(m_2)$ is the standard error on the slope from figure 3 found from the "LINEST" Excel function. The value of m_2 was found by multiplying the slope of the graph in figure 3 by 10^3 to convert KΩ to Ω.

$$z = \sqrt{\left(u(m_2)\right)^2 + (u(R_A))^2}$$
$$z = 0.0050\,\Omega$$

$$R_1 = -0.047 \pm 6.1 \times 10^{-6}\,\Omega$$
$$R_2 = -0.69 \pm 0.005\,\Omega$$

IV. Conclusion

It is expected that R_1 and R_2 are equivalent, but instead they disagree by a factor of 10. This may be due to the inaccurate collection of voltmeter and ammeter internal resistances, as they began to vary when different load resistances were attached, despite expecting to remain constant. Moreover, the jumper cables used had some internal resistance as well that was not accounted for. While the values would have been small, it would have had a larger impact when using the lower resistances. Note that the uncertainties in both cases are negligibly small compared to the value itself. We round the uncertainty up to the nearest degree of precision.

$$R_1 = -0.047\,\Omega$$
$$R_2 = -0.69 \pm 0.01\,\Omega$$

V. Appendix:

Error propagation calculation for Circuit Option 1, uncertainty:

$$z = \sqrt{\left(\left(\frac{\sqrt{u(V)^2 - \left(\sqrt{\left(\frac{u(I)}{I} \right)^2 + \left(\frac{u(R)}{R} \right)^2} \cdot RI \right)^2}}{V - IR} \right)^2 + \left(\frac{u(I)}{I} \right)^2 \right)} \cdot \frac{(V - IR)}{I}$$

Error propagation calculation for Circuit Option 2, uncertainty:

$$z = \sqrt{\left(\frac{u(V)}{V} \right)^2 + \left(\sqrt{u(I)^2 - \sqrt{\left(\frac{u(V)}{V} \right)^2 + \left(\frac{u(R)}{R} \right)^2} \cdot \frac{V}{R}} \right)^2} \cdot \left(\frac{V}{I - \frac{V}{R}} \right)$$