

- Maximal Contiguous Subsequence sum.

A. Big-Oh  $O(n^3)$

$$\sum_{i=0}^{n-1} \left( \sum_{j=i}^{n-1} \left( \sum_{k=j}^{n-1} (1) + 2 \right) \right)$$

$$\begin{aligned}\sum_{k=j}^{n-1} (1) &= j - i + 1 \\ &= j - (i - 1)\end{aligned}$$

$$\sum_{i=0}^{n-1} \left( \sum_{j=i}^{n-1} (j - (i - 1) + 2) \right)$$

$$\sum_{i=0}^{n-1} \left( \sum_{j=i}^{n-1} (j - (i - 3)) \right)$$

$$\begin{aligned}\sum_{j=i}^{n-1} (j - (i - 3)) &= \sum_{j=i}^{n-1} (j) - \sum_{j=i}^{n-1} (i - 3) \\ &= (\sum_{j=0}^{n-1} (j) - \sum_{j=0}^{i-1} (j)) - (i - 3) \sum_{j=i}^{n-1} (1) \\ &= \left( \frac{n(n+1)}{2} - \frac{i(i-1)}{2} \right) + (i-3)(i-n) \\ &= \left( \frac{i^2 - in - 3i}{2} - \frac{i^2 - i}{2} \right) + 3n + \frac{n(n-1)}{2} \\ &= \frac{i(i+1)}{2} - i(n+3) + 3n + \frac{n(n-1)}{2} \\ &= \frac{i^2}{2} + \frac{i}{2} - i(n+3) + 3n + \frac{n(n-1)}{2} \\ &= \frac{i^2}{2} - i(n + \frac{5}{2}) + \frac{n(n+5)}{2}.\end{aligned}$$

$$\sum_{i=0}^{n-1} \left( \frac{i^2}{2} - i(n + \frac{5}{2}) + \frac{n(n+5)}{2} \right)$$

$$= \left(\frac{1}{2}\right) \sum_{i=0}^{n-1} (i^2) - (n + \frac{5}{2}) \sum_{i=0}^{n-1} (i) + \left(\frac{1}{2}\right) (n(n+5)) \sum_{i=0}^{n-1} (1)$$

$$= \frac{1}{2} \left( \frac{n(n-1)(2n-1)}{6} \right) - \frac{(n + \frac{5}{2})(n)(n-1)}{2} + \frac{n^2(n+5)}{2}$$

$$= \frac{n(n-1)}{2} \left( \frac{2n-1}{6} - \frac{2n+5}{2} \right) + \frac{n^2(n+5)}{2}$$

$$C(n) = \frac{n(n^2 + 9n + 8)}{6} \approx \frac{n^3}{6} \in O(n^3)$$

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Because best case, average case, worst case  
are irrelevant b/c entire array must be traversed  
all runs

$$\Theta(n^3) = O(n^3)$$

B. Big-Oh  $O(n^2)$

$$\begin{aligned} \cdot \sum_{i=0}^{n-1} (\sum_{j=i}^{n-1} (2) + 1) &= \\ &= \sum_{i=0}^{n-1} (2(n-1-i+1) + 1) \\ &= \sum_{i=0}^{n-1} (2(n-i) + 1) \\ &= \sum_{i=0}^{n-1} (2n+1) - \sum_{i=0}^{n-1} (2i) \\ &= (2n+1) \sum_{i=0}^{n-1} (1) - 2 \sum_{i=0}^{n-1} (i) \\ &= n(2n+1) - 2 \frac{n(n-1)}{2} \\ C(n) &= n(n+2) \end{aligned}$$

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$$\begin{aligned} C(n) &= n(n+2) \\ &= \frac{n^2 + 2n}{n^2} \in O(n^2) \end{aligned}$$

C. Big-Oh  $O(n)$

$$\begin{aligned} \cdot \sum_{i=0}^{n-1} (2) &= \\ &= \sum_{i=0}^{n-1} (2) \\ &= 2 \sum_{i=0}^{n-1} (1) \end{aligned}$$

$$C(n) = 2n$$

$$C(n) = 2n \underset{n \in O(n)}{\approx}$$

Cubed		
N	T(n)	T(N)/N^3
125	881300	0.4512256
250	2051700	0.1313088
500	14755200	0.1180416
1000	115616700	0.1156167
2000	790822500	0.098852813

- Because the function is of  $O(n^3)$ ,  $T(n) = c(n^3)$ .
- Based on the data acquired, we use the empirical data point  $T(n) = 2051700$ ns