## Assignment 8

https://github.com/jchryssanthacopoulos/quantum\_information/tree/main/assignment\_8

# Quantum Information and Computing AA 2022–23

James Chryssanthacopoulos 9 January 2023



### Real-Space Renormalization Group



- **Real-space renormalization group** algorithm can be used to compute ground-state energy  $E_0$  of quantum Ising model with traverse field in one dimension
- Following steps are repeated until  $|E_0^{(i+1)} E_0^i| < \epsilon$ , where i is iteration number:
  - **1** Starting with Hamiltonian of system with N sites,  $\hat{H}_N$ , construct Hamiltonian by replicating system:

$$\hat{H}_{2N} = \hat{H}_N \otimes \mathbb{1}_N + \mathbb{1}_N \otimes \hat{H}_N + \hat{H}_{int}$$

where  $\hat{H}_{\text{int}} = \hat{A}_N \otimes \hat{B}_N$  is interaction between left and right bipartitions, initialized to  $\hat{A}_N = \mathbb{1}_{N-1} \otimes \sigma^x$ ,  $\hat{B}_N = \sigma^x \otimes \mathbb{1}_{N-1}$ .  $\hat{H}_N$  is initialized to Hamiltonian of Ising model with N sites

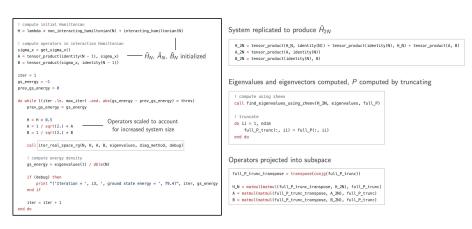
- 2 Diagonalize  $\hat{H}_{2N}$ , obtaining ground-state energy  $E_0^i$ . Construct projector onto  $2^N$  eigenvectors with lowest energy,  $P = \sum_{i=1}^{2^N} |E_i\rangle \langle E_i|$
- 3 Project operators into subspace spanned by chosen eigenvectors:

$$\hat{H}_N = P^{\dagger} \hat{H}_{2N} P, \quad \hat{A}_N = P^{\dagger} (\hat{A}_N \otimes \mathbb{1}_N) P, \quad \hat{B}_N = P^{\dagger} (\mathbb{1}_N \otimes \hat{B}_N) P$$

#### **Implementation**



Program computes ground-state energy given number of sites N, interaction strength  $\lambda$ , termination threshold  $\epsilon$ , and maximum number of iterations



#### Results



■ Energy density,  $e \equiv E_0/N$ , computed using RSRG compared to mean-field result:

$$\mathsf{e}_\mathsf{MF} = \begin{cases} -1 - \lambda^2/4, & |\lambda| \leq 2 \\ -|\lambda|, & |\lambda| > 2 \end{cases}$$

- Energy densities are very similar, but deviate the most when  $0 < |\lambda| < 2$ , when external field is present but not strong enough to coordinate all spins
- Experiments showed that energy density doesn't change with N, as expected
- $\blacksquare$  Number of iterations increases with N, except when field is weak

