

Assignment 6

https://github.com/jchryssanthacopoulos/quantum_information/tree/main/assignment_6

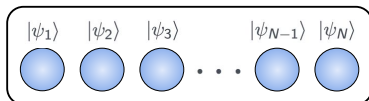
Quantum Information and Computing AA 2022–23

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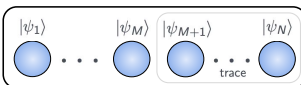


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- Quantum system composed of N subsystems, each with wavefunction $|\psi_i\rangle = \sum_{\alpha_i} c_i |\alpha_i\rangle$ in D -dimensional Hilbert space



- Overall wavefunction is $|\Psi\rangle = \sum_{\alpha_1 \dots \alpha_N} C_{\alpha_1 \dots \alpha_N} |\alpha_1 \dots \alpha_N\rangle$, where $C_{\alpha_1 \dots \alpha_N}$ are D^N complex coefficients
 - With normalization and phase constraints, there are $2D^N - 2$ real DOFs
- If state is **separable**, $|\Psi\rangle = |\psi_1\rangle \otimes \dots \otimes |\psi_N\rangle$, which has $(2D - 2)N$ real DOFs
- Density matrix** of a pure state is $\rho \equiv |\Psi\rangle \langle \Psi|$. To describe state of part of system, **reduced density matrix** is computed, tracing over rest of system. For example:

$$\rho_M = \text{Tr}_{M+1} \dots \text{Tr}_N |\Psi\rangle \langle \Psi| =$$


Separable versus Generic States



- **Separable.** $|\Psi\rangle$ needs only $\mathcal{O}(DN)$ parameters (linear in N), but it represents non-interacting systems and is less flexible (i.e., mean-field approximation)
- **Generic.** $|\Psi\rangle$ needs $\mathcal{O}(D^N)$ parameters (exponential in N), but it can capture arbitrary interactions

```
! generate separable state containing N * d parameters,  
! making sure to correctly normalize each wavefunction  
do ii = 1, N  
  norm = 0  
  do jj = 1, D  
    separable_state(ii, jj) = cmplx(rand(0) * 2 - 1, rand(0) * 2 - 1)  
    norm = norm + separable_state(ii, jj) * conjg(separable_state(ii, jj))  
  end do  
  separable_state(ii, :) = separable_state(ii, :) / sqrt(norm)  
end do  
  
if (debug_level .ge. DEBUG_LEVEL_1) then  
  print *, "Wavefunction factorization = "  
  call print_complex_matrix(separable_state)  
end if  
  
! populate tensor  
state = 1  
  
do ii = 1, dim  
  mat_indices = tensor2mat(ii, N, D)  
  
  if (debug_level .ge. DEBUG_LEVEL_2) then  
    print *, "Matrix indices for tensor index = ", ii, " are ", mat_indices  
  end if  
  
  do jj = 1, N  
    state(ii) = state(ii) * separable_state(jj, mat_indices(jj))  
  end do  
end do
```

Since $|\Psi\rangle$ is still D^N -dimensional,
function used to convert between
generic and separable indices

```
function prepare_generic_state(N, D) result(state)  
  implicit none  
  
  integer N, D, dim  
  real*8 norm  
  integer ii  
  
  complex*16, dimension(:), allocatable :: state  
  
  dim = D ** N  
  allocate(state(dim))  
  
  norm = 0  
  
  do ii = 1, dim  
    state(ii) = cmplx(rand(0) * 2 - 1, rand(0) * 2 - 1)  
    norm = norm + state(ii) * conjg(state(ii))  
  end do  
  
  state = state / sqrt(norm)  
  
end function
```

```
function tensor2mat(tt, N, D) result(mat_idx)  
  implicit none  
  
  integer tt, N, D  
  integer ii  
  integer mat_idx(N)  
  
  do ii = 1, N  
    mat_idx(ii) = modulo((tt - 1) / D ** (ii - 1), D) + 1  
  end do  
  
end function
```

Computing Density Matrices



```
compiled/density_matrix --N 6 --D 4 --M 3 --type generic --output_filename data/density_mat_N6_D4_M3.txt
```

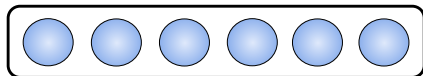
number of
subsystems

number of
subsystems in
right subsystem

number of
dimensions

type of state to prepare
(e.g., separable, generic, bell)

matrices saved to file



left subsystem

right subsystem

$$\rho = \begin{pmatrix} 5.64 \times 10^{-4} & \cdots & -2.27 \times 10^{-4} - 1.08 \times 10^{-4}i \\ \vdots & \ddots & \vdots \\ -2.27 \times 10^{-4} + 1.08 \times 10^{-4}i & \cdots & 1.11 \times 10^{-4} \end{pmatrix}$$

$$D^N = 4^6 = 4096 \text{ rows and columns}$$

Computing left reduced matrix has $\mathcal{O}(D^{2N-M})$ complexity

$$D^M = 4^3 = 64 \text{ rows and columns}$$

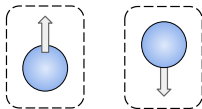
```
do ii = 1, dim
  do jj = 1, dim
    do kk = 1, D ** M
      idx1 = kk + (ii - 1) * D ** M
      idx2 = kk + (jj - 1) * D ** M
      rho_reduced_L(ii, jj) = rho_reduced_L(ii, jj) + rho(idx1, idx2)
      if (debug_level .ge. DEBUG_LEVEL_2) then
        print "('Added rho index ', (i4), (i4), ' to rho reduced L index ',
          idx1, idx2, ii, jj)
      end if
    end do
  end do
end do
```

$$\rho_L = \begin{pmatrix} 1.62 \times 10^{-2} & \cdots & -1.14 \times 10^{-3} - 3.08 \times 10^{-4}i \\ \vdots & \ddots & \vdots \\ -1.14 \times 10^{-3} + 3.08 \times 10^{-4}i & \cdots & 1.59 \times 10^{-2} \end{pmatrix}$$

$$\rho_R = \begin{pmatrix} 1.63 \times 10^{-2} & \cdots & -1.02 \times 10^{-3} - 1.36 \times 10^{-3}i \\ \vdots & \ddots & \vdots \\ -1.02 \times 10^{-3} + 1.36 \times 10^{-3}i & \cdots & 1.74 \times 10^{-2} \end{pmatrix}$$

- Density matrices are Hermitian with trace one, as expected
- $\rho_{L,R}$ of maximally entangled state exhibits total loss of coherence
- Von Neumann entropy $-\text{Tr}(\rho \log \rho)$ computed by diagonalizing ρ

Separable



$$|\Psi\rangle = [(-0.743 - 0.548i)|0\rangle + (0.380 - 0.061i)|1\rangle] \times [(0.058 - 0.499i)|0\rangle + (-0.804 + 0.318i)|1\rangle]$$

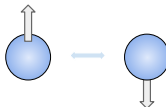
$$\rho = \begin{pmatrix} 0.21 & -0.06 - 0.06i & -0.18 + 0.33i & 0.15 - 0.04i \\ -0.06 + 0.06i & 0.04 & -0.05 - 0.15i & -0.03 + 0.06i \\ -0.18 - 0.33i & -0.05 + 0.15i & 0.64 & -0.19 - 0.19i \\ 0.15 + 0.04i & -0.03 - 0.06i & -0.19 + 0.19i & 0.11 \end{pmatrix}$$

$$\rho_L = \begin{pmatrix} 0.25 & -0.21 + 0.38i \\ -0.21 - 0.38i & 0.75 \end{pmatrix}$$

$$\rho_R = \begin{pmatrix} 0.85 & -0.25 - 0.25i \\ -0.25 + 0.25i & 0.15 \end{pmatrix}$$

$$S_{L,R} = 0$$

Generic (Partially Entangled)



$$(-0.57 - 0.42i)|00\rangle + (0.29 - 0.05i)|01\rangle + (0.04 - 0.32i)|10\rangle + (-0.52 + 0.20i)|11\rangle$$

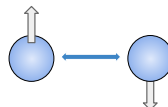
$$\begin{pmatrix} 0.50 & -0.15 - 0.15i & 0.11 - 0.20i & 0.21 + 0.33i \\ -0.15 + 0.15i & 0.09 & 0.03 + 0.09i & -0.16 - 0.04i \\ 0.11 + 0.20i & 0.03 - 0.09i & 0.10 & -0.08 + 0.16i \\ 0.21 - 0.33i & -0.16 + 0.04i & -0.08 - 0.16i & 0.31 \end{pmatrix}$$

$$\begin{pmatrix} 0.59 & -0.05 - 0.23i \\ -0.05 + 0.23i & 0.41 \end{pmatrix}$$

$$\begin{pmatrix} 0.60 & -0.23 + 0.01i \\ -0.23 - 0.01i & 0.40 \end{pmatrix}$$

$$0.56$$

Maximally Entangled



$$0.71|01\rangle - 0.71|10\rangle$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.50 & -0.50 & 0 \\ 0 & -0.50 & 0.50 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0.50 & 0 \\ 0 & 0.50 \end{pmatrix}$$

$$\begin{pmatrix} 0.50 & 0 \\ 0 & 0.50 \end{pmatrix}$$

$$\log 2 = 0.69$$

Entropy of Qubit Systems



- Entropy computed for different N and sizes of right bipartition M
- When $M = 0$, entropy is computed over entire system, so it is equal to zero since state is pure
- As M increases, entropy increases since more of system is traced over, but when $M > N/2$, entropy decreases as more of system is included

