

The Stability of Euclidean Wormholes

Exploring the Factorization Problem
in the AdS/CFT Correspondence

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25 October 2022

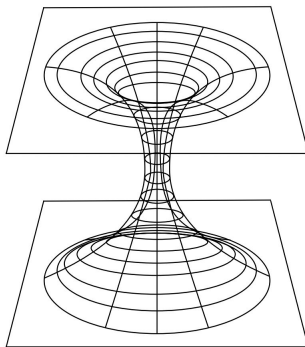


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- 1 Introduction to Wormholes
- 2 The AdS/CFT Correspondence and the Factorization Problem
- 3 Wormholes and the Black Hole Information Paradox
- 4 Einstein-Maxwell Theory in 4D AdS
- 5 Conclusion

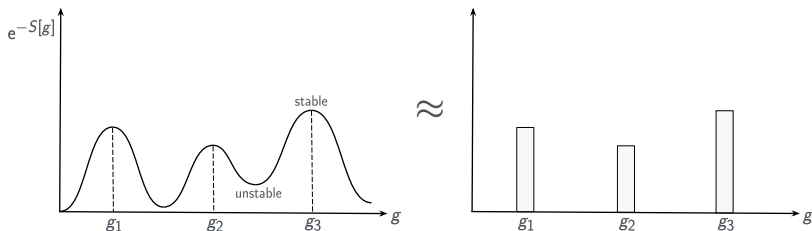
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Wormholes are solutions to general relativity
connecting distant asymptotic regions of spacetime



Euclidean wormholes emerge as saddles of gravitational path integral, and can be used to approximate it

$$Z = \int \mathcal{D}g e^{-S[g]}$$

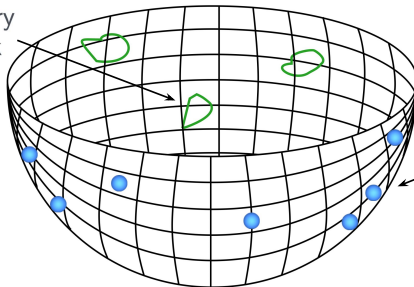


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Concrete example of holography relating
partition functions of different theories

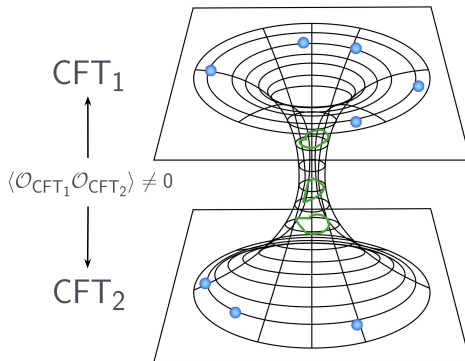
$$Z_{\text{CFT}} = Z_{\text{string}}$$

string theory
in the bulk



quantum field theory
on the boundary

Gravity is dual to two CFTs over different boundaries, but if Z_{string} has extra wormhole contribution, CFTs must be correlated



On gravity side, this means path integral fails to factorize

$$\langle Z \rangle = \text{[Diagram of a cone with many small circles inside, representing a path integral on a cone geometry.]}$$

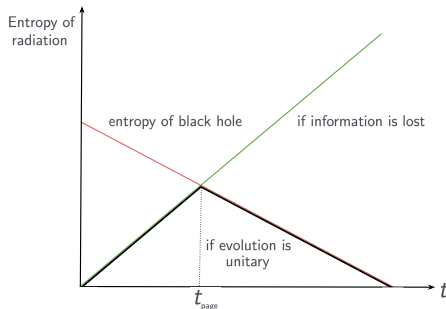
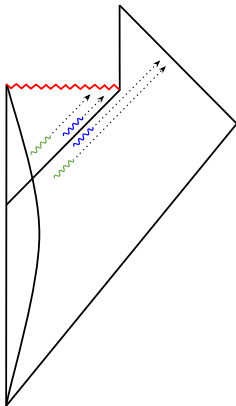
$$\langle Z^2 \rangle = \left(\text{[Diagram of two cones joined at their tips, representing a path integral on a genus-2 surface.]} \right) + \text{[Diagram of a hyperboloid of two sheets, representing a path integral on a genus-2 surface with a different topology.]}$$
$$\neq \langle Z \rangle^2$$

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Entropy of Hawking Radiation

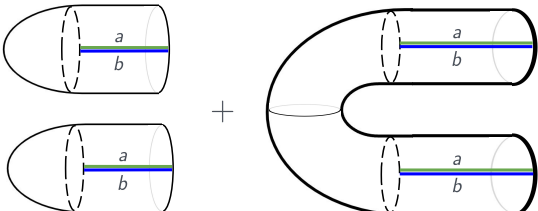


Wormholes can help explain how information
can be recovered from black hole



The entropy can be computed using

$$S = (1 - n \partial_n) \log \text{Tr}(\rho^n) \Big|_{n=1} \quad \text{with } \rho_{ab} = \langle \Psi_a | \Psi_i \rangle \langle \Psi_i | \Psi_b \rangle$$

$$\text{Tr}(\rho^2) \approx$$


$$+ \dots$$

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Low-energy model very similar to string theory that captures important properties while remaining analytically tractable [?]

$$S = - \int_{\mathcal{M}} d^4x \sqrt{g} \left(R + \frac{6}{L^2} - \sum_{i=1}^3 F_{\mu\nu}^i F_i^{\mu\nu} \right) - 2 \int_{\partial\mathcal{M}} d^3x \sqrt{h} K + S_B$$

with ansatzes

$$ds^2 = \frac{dr^2}{f(r)} + g(r) d\Omega^2 \quad A^i = L \frac{\sigma_i}{2} \Phi(r)$$

Since calculations are elaborate, different programs were written to produce and analyze results

main 2 branches 0 logs

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- gitignore Initial commit 2 months ago
- README.md Update report and README 2 days ago
- Wormholes.pdf Update report 16 hours ago
- elstein_reserve_ad4.ipynb Refactor some elements of Lorentzian notebook 2 months ago
- elstein_reserve_ad4_euclidean... Include calculation of on-shell Euclidean action for disconnected sol... 2 months ago
- elstein_reserve_ad4_euclidean... Update report and add more analysis plots 12 days ago
- elstein_reserve_ad4_scalar_part... Update the report and compute variation of GHY term 6 days ago
- georgi_invariance_action.ipynb Make some simplifications in gauge invariance notebook 28 days ago
- negative_modes_analysis.ipynb Restore previous version of negative modes notebook 19 days ago
- perturbations.nb Include phi derivatives in Mathematica notebook last month
- quadratic_ghenvalue_problem.ipynb Update report and add more analysis plots 12 days ago
- requirements.txt Add chemical potential solution 2 months ago
- second_order_action_analysis.ipynb Show q squared term is equivalent in d=0 case last month

README.md

Analysis of Wormholes using Python

This repository contains code to create and evaluate wormhole solutions to general relativity using Python.

Installation

To install the requirements, create a virtual environment and run

```
pip install -r requirements
```

Einstein Equation

Compute the Einstein tensor.

```
In [2]: sym = sympy.Symbol('r psi theta phi')
r, psi, theta, phi = sym
f = sympy.Function('f')(r) # metric Ansatz
exp_phi = sympy.Function('Psi')(r) # define gauge field
L = sympy.Symbol('L') # Add length scale
r_0 = sympy.Symbol('r_0')
```

```
In [3]: llistd = [[0 for i in range(4)] for i in range(4)]
llistd[0][0] = 1 / f
llistd[2][2] = llistd[3][3] = (r**2 + r_0**2) / 4
llistd[1][1] = llistd[1][3] = llistd[3][1] = (r**2 + r_0**2) / 4 + sympy.cos(theta)
sch = metricTensor(llistd, sym)
sch.tensor()
```

$$\begin{bmatrix} \frac{1}{f(r)} & 0 & 0 & 0 \\ 0 & \frac{r^2}{4} + \frac{r_0^2}{4} & 0 & \left(\frac{r^2}{4} + \frac{r_0^2}{4}\right) \cos(\theta) \\ 0 & 0 & \frac{r^2}{4} + \frac{r_0^2}{4} & 0 \\ 0 & \left(\frac{r^2}{4} + \frac{r_0^2}{4}\right) \cos(\theta) & 0 & \frac{r^2}{4} + \frac{r_0^2}{4} \end{bmatrix}$$

```
In [4]: Ric = RicciTensor.from_metric(sch)
Ric.tensor()
```

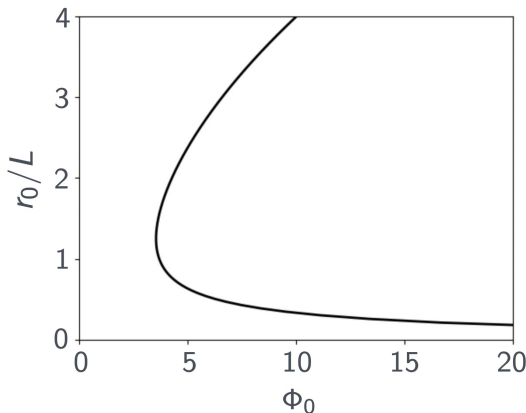
$$\begin{bmatrix} \frac{8(-r^2 f(r) - r_0^2 f(r) - 2r f'(r) - 2r_0 f'(r))}{3(r^2 + r_0^2 + 1)(r)} & 0 & 0 & 0 \\ 0 & \frac{-r^2 f(r) - 4r^2 f'(r) + 4r_0^2 f'(r) - 2r f'(r) + 4r_0 f'(r)}{3(r^2 + r_0^2 + 1)} & \frac{-r^2 f(r) - 4r^2 f'(r) + 4r_0^2 f'(r) - 2r f'(r) - 2r_0 f'(r) + 4r_0 f'(r)}{3(r^2 + r_0^2 + 1)} & \frac{(-r^2 f(r) - 4r^2 f'(r) + 4r_0^2 f'(r) - 2r f'(r) - 2r_0 f'(r) + 4r_0 f'(r)) \cos(\theta)}{3(r^2 + r_0^2 + 1)} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{(-r^2 f(r) - 4r^2 f'(r) + 4r_0^2 f'(r) - 2r f'(r) + 4r_0 f'(r)) \cos(\theta)}{3(r^2 + r_0^2 + 1)} & 0 & \frac{-r^2 f(r) - 4r^2 f'(r) + 4r_0^2 f'(r) - 2r f'(r) - 2r_0 f'(r) + 4r_0 f'(r)}{3(r^2 + r_0^2 + 1)} \end{bmatrix}$$

```
In [5]: metric_array = sympy.MutableDenseNDimArray(llistd)
einstein_tensor = sympy.MutableDenseNDimArray(Ric.tensor()) + 3 / L**2 * metric_array
einstein_tensor = sympy.simplify(einstein_tensor)
einstein_tensor
```

Small and Large Wormhole Solutions



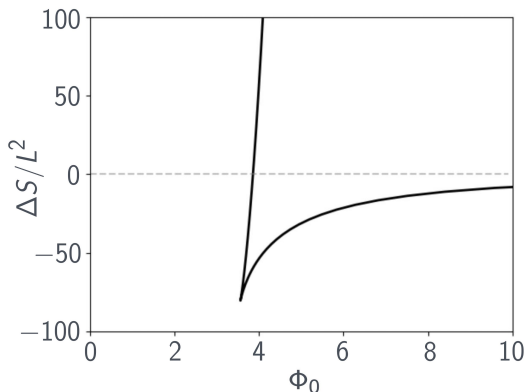
There are two wormhole solutions with different throats r_0
if source of Maxwell fields Φ_0 is large enough



When Does Wormhole Dominate?



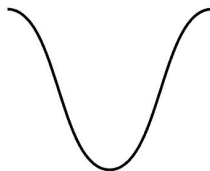
Large wormhole has lower action than disconnected solution in certain regime, suggesting it may dominate path integral



Wormhole can dominate in path integral if it is local minimum, meaning it is perturbatively stable

$$S^{(2)} = \frac{\pi^2}{4} \int dr \sqrt{\frac{\bar{g}}{\bar{f}}} q \left[\underbrace{-\sqrt{\frac{\bar{f}}{\bar{g}}} \left(\sqrt{\bar{f} \bar{g}} K q' \right)'}_{\lambda q} + V q \right]$$

$$\lambda > 0$$



stable

$$\lambda < 0$$






unstable

Discretizing eigenvalue problem, negative modes can be searched for by varying r_0 , demonstrating large wormhole is free of them

r_0/r_0^{\min}	Eigenvalues
0.8	One negative eigenvalue, $\lambda = -118$
1.0	No negative eigenvalues, $\lambda_{\min} = 1.92 \times 10^{-7}$
1.1	No negative eigenvalues, $\lambda_{\min} = 1.41 \times 10^{-6}$

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- Wormholes introduce factorization problem, which suggests extra correlations in AdS/CFT but only serious if wormholes free of negative modes
- In low-energy model inspired by string theory, large wormhole is perturbatively stable and dominates over disconnected solution, analogous to Page transition of black hole entropy
- Other kinds of perturbations, non-perturbative channels, and string theory models need to be considered
- Factorization problem is still open and likely object of future research

-  D. Marolf and J. E. Santos, “AdS Euclidean Wormholes,” Classical and Quantum Gravity, vol. 38, no. 22, p. 224002, oct 2021. [Online]. Available: <https://doi.org/10.1088>
-  A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, and A. Tajdini, “The Entropy of Hawking Radiation,” Reviews of Modern Physics, vol. 93, no. 3, jul 2021. [Online]. Available: <https://doi.org/10.1103>
-  J. Chryssathacopoulos. (2022) Analysis of Wormholes using Python. [Online]. Available: <https://github.com/jchryssanthacopoulos/wormholes>