The Stability of Euclidean Wormholes

Exploring the Factorization Problem
in the AdS/CFT Correspondence

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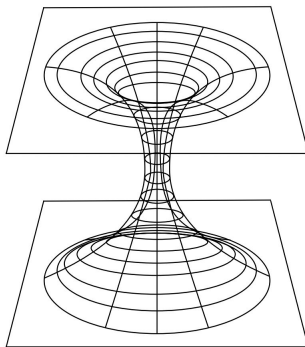25 October 2022

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Outline

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Wormholes are solutions to general relativity
connecting distant asymptotic regions of spacetime

Euclidean wormholes emerge as saddles of gravitational path integral, and can be used to approximate it
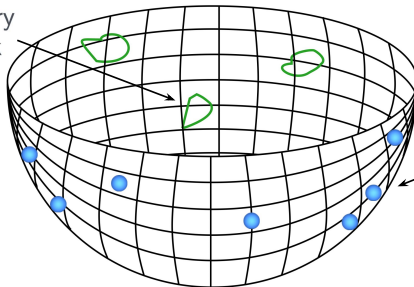
$$Z = \int \mathcal{D}g \; e^{-S[g]}$$

Outline

Concrete example of holography relating
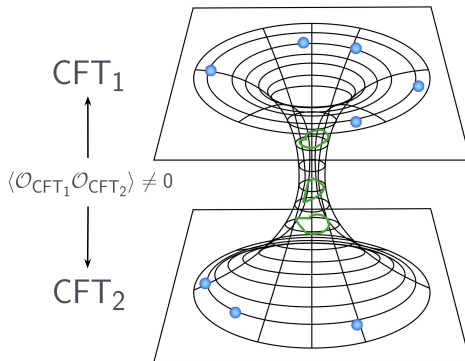partition functions of different theories

$$Z_{\text{CFT}} = Z_{\text{string}}$$

string theory
in the bulk

quantum field theory
on the boundary

Gravity is dual to two CFTs over different boundaries, but if Z_{string} has extra wormhole contribution, CFTs must be correlated

CFT$_1$

$\langle \mathcal{O}_{\text{CFT}_1} \mathcal{O}_{\text{CFT}_2} \rangle \neq 0$

CFT$_2$

On gravity side, this means path integral fails to factorize

$$\langle Z \rangle =$$

$$\langle Z^2 \rangle = \left(\quad \quad \right) +$$
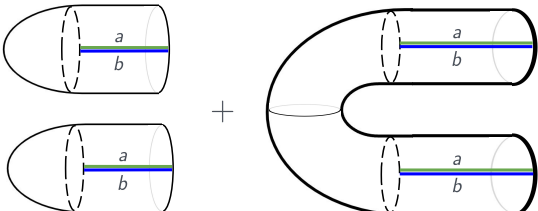
$$\neq \langle Z \rangle^2$$

Outline

Wormholes can help explain how information
can be recovered from black hole
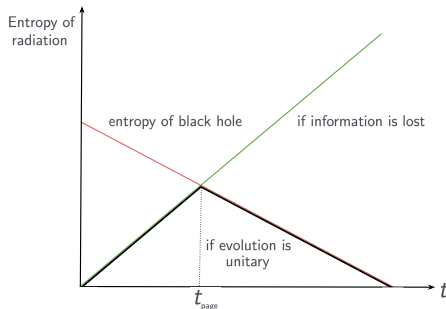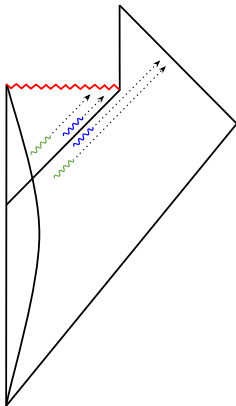
The entropy can be computed using

$$S = (1 - n\partial_n) \log \mathrm{Tr}(\rho^n)\Big|_{n=1} \quad \text{with } \rho_{ab} = \langle\Psi_a|\Psi_i\rangle\langle\Psi_i|\Psi_b\rangle$$

$$\mathrm{Tr}(\rho^2) \approx \qquad + \qquad + \cdots$$

Outline

Low-energy model very similar to string theory that captures important properties while remaining analytically tractable [**?**]

$$S = -\int_{\mathcal{M}} d^4x \sqrt{g} \left(R + \frac{6}{L^2} - \sum_{i=1}^{3} F_{\mu\nu}^i F_i^{\mu\nu} \right) - 2\int_{\partial\mathcal{M}} d^3x \sqrt{h}K + \mathcal{S}_{\mathcal{B}}$$

with ansatzes

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{f(r)} + g(r)\,\mathrm{d}\Omega^2 \qquad A^i = L\frac{\sigma_i}{2}\Phi(r)$$

Since calculations are elaborate, different programs were written to produce and analyze results

There are two wormhole solutions with different throats r_0
if source of Maxwell fields Φ_0 is large enough
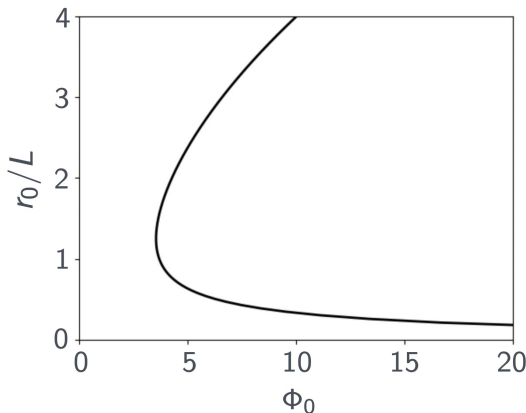
Large wormhole has lower action than disconnected solution in certain regime, suggesting it may dominate path integral
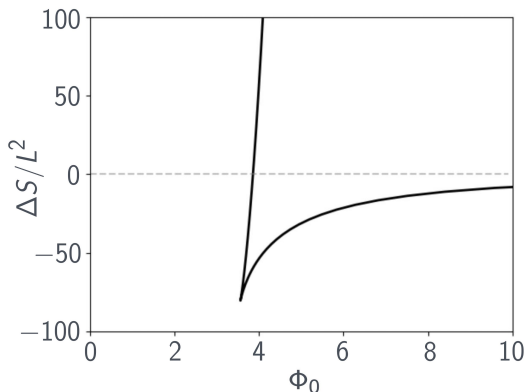
Wormhole can dominate in path integral if it is local minimum, meaning it is perturbatively stable

$$S^{(2)} = \frac{\pi^2}{4} \int dr \sqrt{\frac{\overline{g}}{\overline{f}}} q \left[\underbrace{-\sqrt{\frac{\overline{f}}{\overline{g}}} \left(\sqrt{\overline{f}\,\overline{g}} K q' \right)' + V q}_{\lambda q} \right]$$

$\lambda > 0$ $\lambda < 0$
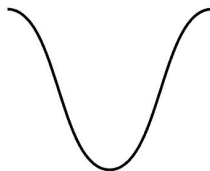
stable **unstable**

Discretizing eigenvalue problem, negative modes can be searched for by varying r_0, demonstrating large wormhole is free of them

$r_0/r_0^{\mathbf{min}}$	**Eigenvalues**
0.8	One negative eigenvalue, $\lambda = -118$
1.0	No negative eigenvalues, $\lambda_{\mathsf{min}} = 1.92 \times 10^{-7}$
1.1	No negative eigenvalues, $\lambda_{\mathsf{min}} = 1.41 \times 10^{-6}$

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Conclusion

- Wormholes introduce factorization problem, which suggests extra correlations in AdS/CFT but only serious if wormholes free of negative modes

- In low-energy model inspired by string theory, large wormhole is perturbatively stable and dominates over disconnected solution, analogous to Page transition of black hole entropy

- Other kinds of perturbations, non-perturbative channels, and string theory models need to be considered

- Factorization problem is still open and likely object of future research

References

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A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, and A. Tajdini, "The Entropy of Hawking Radiation," Reviews of Modern Physics, vol. 93, no. 3, jul 2021. [Online]. Available: https://doi.org/10.1103

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