The Stability of Euclidean Wormholes

Exploring the Factorization Problem in the AdS/CFT Correspondence

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Outline

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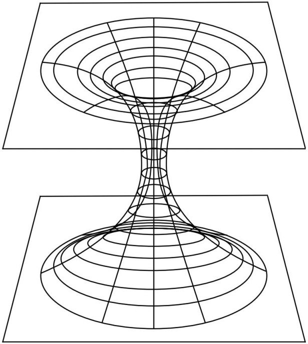
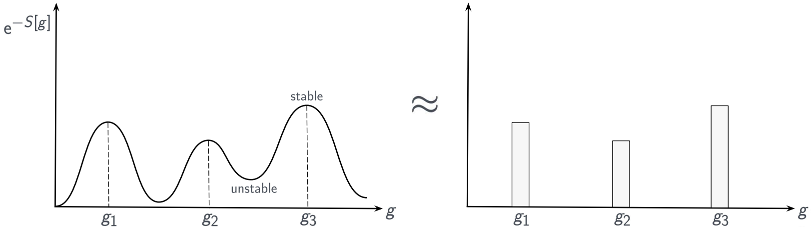
Wormholes are solutions to general relativity connecting distant asymptotic regions of spacetime



Euclidean wormholes emerge as saddles of gravitational path integral, and can be used to approximate it

$$Z = \int \mathcal{D}g \, e^{-S[g]}$$



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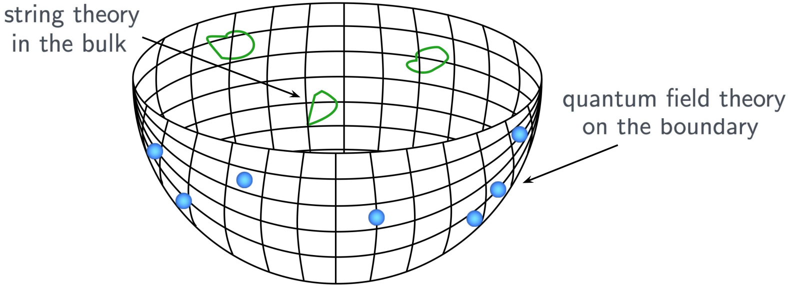
Concrete example of holography relating partition functions of different theories

$$Z_{\text{CFT}} = Z_{\text{string}}$$



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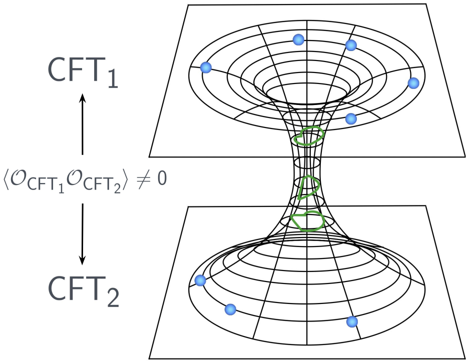
Gravity is dual to two CFTs over different boundaries, but if Z_{string} has extra wormhole contribution, CFTs must be correlated



Factorization Problem

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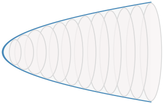
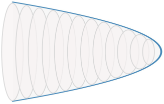
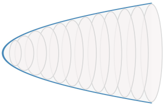
On gravity side, this means path integral fails to factorize

$$\langle Z \rangle =$$



$$\langle Z^2 \rangle = \Bigg($$





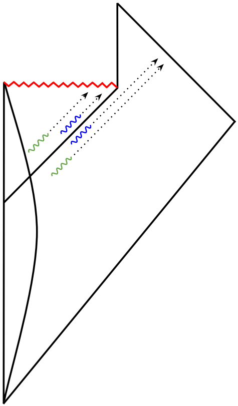
$$\Bigg) +$$

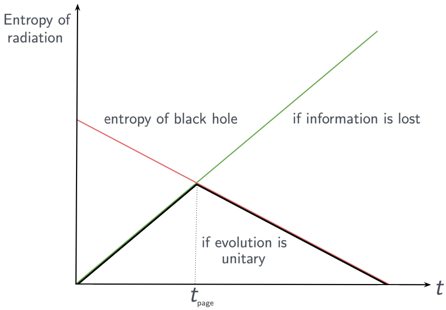
$$\neq \langle Z \rangle^2$$

Outline

Entropy of Hawking Radiation

Wormholes can help explain how information can be recovered from black hole



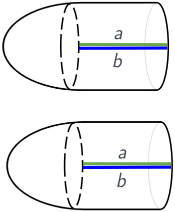


The Replica Trick

The entropy can be computed using

$$S = (1 - n\partial_n) \log \mathrm{Tr}(\rho^n)\Big|_{n=1} \quad \text{with } \rho_{ab} = \langle \Psi_a | \Psi_i \rangle \langle \Psi_i | \Psi_b \rangle$$

$\mathrm{Tr}(\rho^2) \approx$



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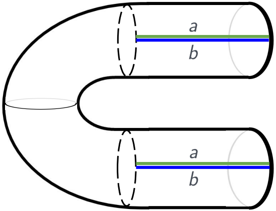


$+ \cdots$

Outline

Einstein-Maxwell Theory in 4D AdS

Low-energy model very similar to string theory that captures important properties while remaining analytically tractable

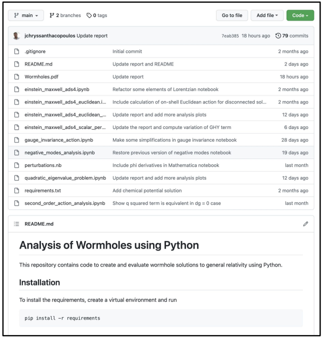
$$S = -\int_{\mathcal{M}} d^4x\sqrt{g}\left(R + \frac{6}{L^2} - \sum_{i=1}^{3} F^i_{\mu\nu}F_i^{\mu\nu}\right) - 2\int_{\partial\mathcal{M}} d^3x\sqrt{h}K + \mathcal{S}_{\mathcal{B}}$$

with ansatzes

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{f(r)} + g(r)\,\mathrm{d}\Omega^2 \qquad A^i = L\frac{\sigma_i}{2}\Phi(r)$$

Code-Driven Analysis

Since calculations are elaborate, different programs were written to produce and analyze results



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jchryssanthacopoulos Update report — 7eab385 18 hours ago — 79 commits

.gitignore	Initial commit	2 months ago
README.md	Update report and README	2 days ago
Wormholes.pdf	Update report	18 hours ago
einstein_maxwell_ads4.ipynb	Refactor some elements of Lorentzian notebook	2 months ago
einstein_maxwell_ads4_euclidean.i...	Include calculation of on-shell Euclidean action for disconnected sol...	2 months ago
einstein_maxwell_ads4_euclidean_...	Update report and add more analysis plots	12 days ago
einstein_maxwell_ads4_scalar_per...	Update the report and compute variation of GHY term	6 days ago
gauge_invariance_action.ipynb	Make some simplifications in gauge invariance notebook	28 days ago
negative_modes_analysis.ipynb	Restore previous version of negative modes notebook	19 days ago
perturbations.nb	Include phi derivatives in Mathematica notebook	last month
quadratic_eigenvalue_problem.ipynb	Update report and add more analysis plots	12 days ago
requirements.txt	Add chemical potential solution	2 months ago
second_order_action_analysis.ipynb	Show q squared term is equivalent in dg = 0 case	last month

README.md

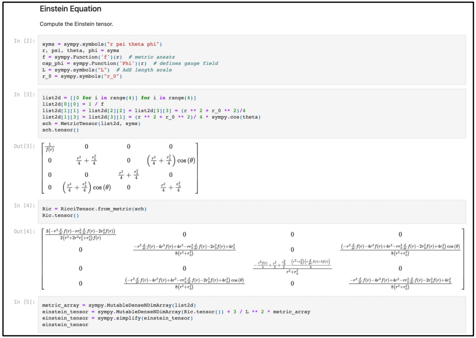
Analysis of Wormholes using Python

This repository contains code to create and evaluate wormhole solutions to general relativity using Python.

Installation

To install the requirements, create a virtual environment and run

```
pip install -r requirements
```



Einstein Equation

Compute the Einstein tensor.

In [2]:

```
syms = sympy.symbols("r psi theta phi")
r, psi, theta, phi = syms
f = sympy.Function('f')(r)  # metric ansatz
cap_phi = sympy.Function('Phi')(r)  # defines gauge field
L = sympy.symbols("L")  # AdS length scale
r_0 = sympy.symbols("r_0")
```

In [3]:

```
list2d = [[0 for i in range(4)] for i in range(4)]
list2d[0][0] = 1 / f
list2d[1][1] = list2d[2][2] = list2d[3][3] = (r ** 2 + r_0 ** 2)/4
list2d[1][3] = list2d[3][1] = (r ** 2 + r_0 ** 2)/ 4 * sympy.cos(theta)
sch = MetricTensor(list2d, syms)
sch.tensor()
```

Out[3]:

$$\begin{bmatrix} \frac{1}{f(r)} & 0 & 0 & 0 \\ 0 & \frac{r^2}{4}+\frac{r_0^2}{4} & 0 & \left(\frac{r^2}{4}+\frac{r_0^2}{4}\right)\cos(\theta) \\ 0 & 0 & \frac{r^2}{4}+\frac{r_0^2}{4} & 0 \\ 0 & \left(\frac{r^2}{4}+\frac{r_0^2}{4}\right)\cos(\theta) & 0 & \frac{r^2}{4}+\frac{r_0^2}{4} \end{bmatrix}$$

In [4]:

```
Ric = RicciTensor.from_metric(sch)
Ric.tensor()
```

Out[4]:

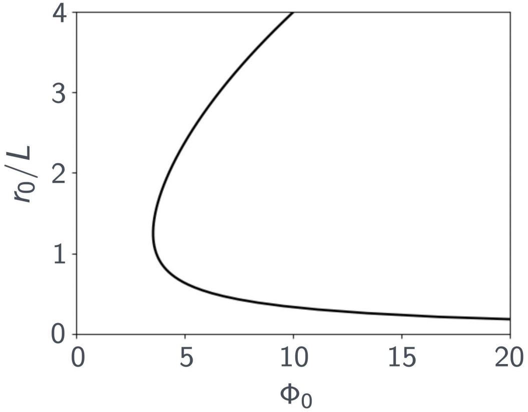
$$\begin{bmatrix} \frac{3\left(-r^3\frac{d}{dr}f(r)-rr_0^2\frac{d}{dr}f(r)-2r_0^2f(r)\right)}{2\left(r^4+2r^2r_0^2+r_0^4\right)f(r)} & 0 & 0 & 0 \\ 0 & \frac{-r^3\frac{d}{dr}f(r)-4r^2f(r)+4r^2-rr_0^2\frac{d}{dr}f(r)-2r_0^2f(r)+4r_0^2}{8\left(r^2+r_0^2\right)} & 0 & \frac{\left(-r^3\frac{d}{dr}f(r)-4r^2f(r)+4r^2-rr_0^2\frac{d}{dr}f(r)-2r_0^2f(r)+4r_0^2\right)\cos(\theta)}{8\left(r^2+r_0^2\right)} \\ 0 & 0 & \frac{-\frac{r^2f(r)}{4}+\frac{r^2}{2}+\frac{r_0^2}{2}-\frac{\left(r^2+r_0^2\right)\left(r\frac{d}{dr}f(r)+2f(r)\right)}{8}}{r^2+r_0^2} & 0 \\ 0 & \frac{\left(-r^3\frac{d}{dr}f(r)-4r^2f(r)+4r^2-rr_0^2\frac{d}{dr}f(r)-2r_0^2f(r)+4r_0^2\right)\cos(\theta)}{8\left(r^2+r_0^2\right)} & 0 & \frac{-r^3\frac{d}{dr}f(r)-4r^2f(r)+4r^2-rr_0^2\frac{d}{dr}f(r)-2r_0^2f(r)+4r_0^2}{8\left(r^2+r_0^2\right)} \end{bmatrix}$$

In [5]:

```
metric_array = sympy.MutableDenseNDimArray(list2d)
einstein_tensor = sympy.MutableDenseNDimArray(Ric.tensor()) + 3 / L ** 2 * metric_array
einstein_tensor = sympy.simplify(einstein_tensor)
einstein_tensor
```

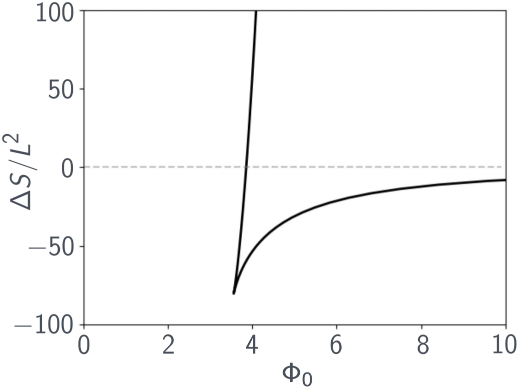
Small and Large Wormhole Solutions

There are two wormhole solutions with different throats r_0
if source of Maxwell fields Φ_0 is large enough



When Does Wormhole Dominate?

Large wormhole has lower action than disconnected solution in certain regime, suggesting it may dominate path integral

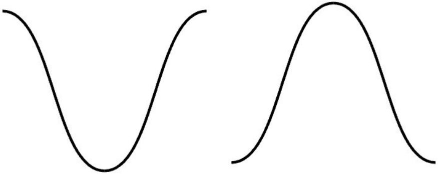


Wormhole Stability

Wormhole can dominate in path integral if it is local minimum, meaning it is perturbatively stable

$$S^{(2)} = \frac{\pi^2}{4}\int dr\sqrt{\frac{\bar{g}}{\bar{f}}}\,q\left[\underbrace{-\sqrt{\frac{\bar{f}}{\bar{g}}}\left(\sqrt{\bar{f}\,\bar{g}}\,Kq'\right)' + Vq}_{\lambda q}\right]$$

$\lambda > 0$ $\lambda < 0$



stable **unstable**

Negative Modes

Discretizing eigenvalue problem, negative modes can be searched for by varying r_0, demonstrating large wormhole is free of them

$r_0/r_0^{\mathbf{min}}$	**Eigenvalues**
0.8	One negative eigenvalue, $\lambda = -118$
1.0	No negative eigenvalues, $\lambda_{\text{min}} = 1.92 \times 10^{-7}$
1.1	No negative eigenvalues, $\lambda_{\text{min}} = 1.41 \times 10^{-6}$

Outline

Conclusion

- Wormholes introduce factorization problem, which suggests extra correlations in AdS/CFT but only serious if wormholes free of negative modes
- In low-energy model inspired by string theory, large wormhole is perturbatively stable and dominates over disconnected solution, analogous to Page transition of black hole entropy
- Other kinds of perturbations, non-perturbative channels, and string theory models need to be considered
- Factorization problem is still open and likely object of future research

References

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