# SFCalcSheet formula collection

This is an index of most formulas used internally by SFCalcSheet. If you find a mistake here, don't hesitate and file an issue.

## Star formulas

## Luminosity from absolute magnitude

$$L = \sqrt[5]{100}^{M_{Sun}-M}$$

L = Luminosity (Suns), M = Absolute magnitude

## **Absolute magnitude from luminosity**

$$M = M_{Sun} - 2.5 \cdot \log_{10}(L)$$

M = Absolute magnitude, L = Luminosity (Suns)

## Visible magnitude from absolute magnitude and distance

$$m = M - 5 + 5 \cdot \log_{10}(d)$$

m = Visible magnitude, M = Absolute magnitude, d = Distance (pc)

## Luminosity difference from absolute magnitudes

$$\Delta L = \sqrt[5]{100}^{(M_{big} - M_{small})}$$

L = Luminosity, M = Absolute magnitude

## **Luminosity from mass**

$$L = m^{3.5}$$

L = Luminosity (Suns), m = Mass (Suns)

# Radius from mass (stars less massive than the Sun)

$$r = m^{0.8}$$

r = Radius (Suns), m = Mass (Suns)

# Radius from mass (stars as massive or more massive than the Sun)

$$r = m^{0.57}$$

r = Radius (Suns), m = Mass (Suns)

#### Inner rim of habitable zone

$$r_{h_{inner}} = \sqrt{\frac{L}{1.1}}$$

r(h(inner)) = Inner rim (AU), L = Luminosity (Suns). This is a conservative formula resulting in a narrow HZ. Use the "habitable zone limits" formula for more flexibility.

#### Outer rim of habitable zone

$$r_{h_{outer}} = \sqrt{\frac{L}{0.53}}$$

r(h(outer)) = Outer rim (AU), L = Luminosity (Suns). This is a conservative formula resulting in a narrow HZ. Use the "habitable zone limits" formula for more flexibility.

## Main sequence life span from mass

$$l = 10^{10} \cdot (\frac{1}{m})^{2.5}$$

I = Life span (years), m = Mass (Suns)

## Luminosity relative to ZAMS (zero age main sequence)

$$L = 1 + 10^{-10} \cdot m^{2.5} \cdot t$$

L = Relative luminosity (ZAMS = 1), m = Mass (Suns), t = Age of star (years)

### Orbital period of binary orbit from distance and mass

$$T = 2\pi \cdot \sqrt{\frac{d^3}{G \cdot (m_1 + m_2)}}$$

T = Orbital period (s), d = Separation of bodies' centers/sum of semi-major axes (m), G = Gravitational constant, m(1) and m(2) = Masses of orbiting bodies (kg)

## Barycenter of binary orbit

$$d_b = \frac{d}{1 + \frac{m_1}{m_2}}$$

d(b) = Distance of barycenter to center of first body (m), d = Distance between bodies' centers (m), m(1) and m(2) = Masses of orbiting bodies (kg)

## **Planet formulas**

#### Radius from mass and density

$$r=\sqrt[3]{\frac{3m}{4\pi\cdot\rho}}$$

 $r = Radius (m), m = Mass (kg), \rho = Density (kg/m<sup>3</sup>)$ 

### Gravitational acceleration from mass and radius

$$g = \frac{G \cdot m}{r^2}$$

g = Gravitational acceleration ( $m/s^2$ ), G = Gravitational constant, m = Body mass (kg), r = Body radius (m)

## Escape velocity from mass and radius

$$v_e = \sqrt{\frac{2m \cdot G}{r}}$$

v(e) = Escape velocity (m/s), G = Gravitational constant, m = Mass (kg), r = Radius (m)

## Roche limit (rigid bodies) from densities

$$d = r_p \cdot 1.26 \cdot \sqrt[3]{\frac{\rho_p}{\rho_s}}$$

d = Roche limit (m), r(p) = Planet radius (m),  $\rho(p)$  = Density of planet (kg/m³),  $\rho(s)$  = Density of satellite (kg/m³)

## Roche limit (fluid/loose bodies) from densities

$$d = r_p \cdot 2.44 \cdot \sqrt[3]{\frac{\rho_p}{\rho_s}}$$

d = Roche limit (m), r(p) = Planet radius (m),  $\rho(p)$  = Density of planet (kg/m³),  $\rho(s)$  = Density of satellite (kg/m³)

## Orbital period from distance and mass

$$T = \sqrt{\frac{4\pi^2 \cdot d^3}{G \cdot m}}$$

T = Orbital period (s), d = Semi-major axis (m), G = Gravitational constant, m = Mass of orbited body (kg)

#### Hill sphere

$$r_h = d \cdot \sqrt[3]{\frac{m}{3m_o}}$$

r(h) = Radius of Hill sphere (m), d = Distance to orbited body (m), m = Mass of body (kg), m(o) = Mass of orbited body (kg)

#### **Tidal force**

$$F_t = G \cdot m \cdot \frac{2r}{d^3}$$

F(t) = Tidal force (m/s<sup>2</sup>), G = Gravitational constant, m = Mass of causing body (kg), r = Radius of affected body (m), d = Distance between bodies (m)

#### Habitable zone limits

$$d = \sqrt{\frac{L}{S_e}}$$

d = Distance of HZ limit to the star (AU), L = Luminosity (Suns), S(e) = Normalized stellar flux (runaway greenhouse = 1.41; moist greenhouse = 1.107; 1st condensation = 0.53; maximum greenhouse = 0.356)

#### Solar irradiance

$$R = L \cdot G_{SC} \cdot (\frac{1}{d})^2$$

R = Irradiance (W/m<sup>2</sup>), L = Luminosity of star (Suns), G(SC) = Solar constant (1,361 W/m<sup>2</sup>), d = Distance of planet to star (AU)

#### Effective temperature

$$T_e = \sqrt[4]{\frac{(1-A)\cdot L}{16\pi \cdot d^2 \cdot \sigma}}$$

T(e) = Effective temperature (K), A = Bond albedo of planet, L = Luminosity of star (W), d = Distance to star (m),  $\sigma$  = Stefan-Boltzmann constant

#### Surface temperature

$$T_s = T_e \cdot \sqrt[4]{\frac{1}{1 - \frac{\epsilon}{2}}}$$

 $T(s) = Surface temperature (K), T(e) = Effective temperature (K), <math>\varepsilon = Atmospheric absorption (0-1)$ 

#### Land area

$$A_l = 4\pi \cdot r^2 \cdot A_{lp}$$

A(I) = Land area (km<sup>2</sup>), r = Planet radius (km), <math>A(I(p)) = Land percentage (0-1)

## Satellite formulas

### Visual diameter

$$d_v = 2 \cdot \arcsin(\frac{r}{d})$$

d(v) = Visual diameter (rad), r = Radius of satellite (km), d = Distance to satellite (km)

#### Visual area from visual diameter

$$A_v = \pi \cdot (\frac{d_v}{2})^2$$

A(v) = Visual area (rad<sup>2</sup>), d(v) = Visual diameter (rad)

#### Illuminance

$$E_v = \frac{A_v \cdot \left(\frac{A_b}{0.12}\right)}{\left(\frac{\pi}{48900}\right)}$$

 $E(v) = Illuminance (full moons; \approx 0.25 lux), A(v) = Visual area (rad²), A(b) = Bond albedo (0-1)$ 

48900 is a shorthand for 1 divided by half the Moon's angular diameter (in radians) squared. 0.12 is the Moon's bond albedo.

# **Compact object formulas**

#### Schwarzschild radius

$$r_s = \frac{2G \cdot m}{c^2}$$

r(s) = Schwarzschild radius (m), G = Gravitational constant, m = Mass (kg), c = Speed of light (m/s)

## **Black hole lifetime**

$$t_l = m^3 \cdot \frac{5120 \cdot \pi \cdot G^2}{\hbar \cdot c^4}$$

t(l) = Lifetime (s), m = Mass (kg), G = Gravitational constant,  $\hbar$  = Dirac constant, c = Speed of light (m/s)

## **Disaster formulas**

#### Impactor mass

$$m = \frac{4}{3} \cdot \pi \cdot \rho \cdot r^3$$

 $m = Impactor mass (kg), \rho = Impactor density (kg/m<sup>3</sup>), r = Impactor radius (m)$ 

## Impactor kinetic energy

$$K = \frac{1}{2} \cdot m \cdot v^2$$

K = Impactor kinetic energy (J), m = Impactor mass (kg), v = Impactor velocity (m/s)

## Impact crater approximate size

$$d_c = \left(\frac{K}{(4.184 \cdot 10^{12})}\right)^{0.3} \cdot 46$$

d(c) = Approximate crater diameter (m), K = Impactor kinetic energy (J)

 $4.184 \times 10^{12}$  is the explosion energy of 1 kt TNT in joules. 46 is the diameter of the crater left by the explosion of 1 kt TNT (in meters).

## Impact crater size lower bound

$$d_{crlower} = \frac{d_c}{3.48}$$

d(crlower) = Crater diameter lower bound, d(c) = approximate crater diameter

## Impact crater size upper bound

$$d_{crupper} = d_c \cdot 3.48$$

d(crupper) = Crater diameter upper bound, d(c) = approximate crater diameter

#### Energy of matter/antimatter annihilation

$$E = mc^2$$

E = Released energy (J), m = Total reaction mass (kg), c = Speed of light (m/s)

### Bomb blast maximum fireball radius

$$r_{max} = E^{0.4} \cdot 39$$

r(max) = Maximum fireball radius (m), E = Bomb yield (kt TNT)

This is an intermediate formula between the approximate fireball radii of aerial and ground-based detonations. Use a multiplicator of 33.5 for aerial and 44 for ground-based detonations.

#### Bomb blast fireball duration

$$t=E^{0.45}\cdot 0.2$$

t = Fireball duration (s), E = Bomb yield (kt TNT)

#### Bomb blast crater diameter

$$d = E^{0.3} \cdot 46$$

d = Crater diameter (m), E = Bomb yield (kt TNT)

#### Bomb blast shockwave radius

$$r = E^{\frac{1}{5}} \cdot \Delta t^{\frac{2}{5}} \cdot \rho^{-\frac{1}{5}}$$

r = Shockwave radius (m), E = Explosion energy (J),  $\Delta t$  = Time since detonation (s),  $\rho$  = Mass density of medium (kg/m³)

## Earthquake energy

$$E = 10^{4.4+1.5M}$$

E = Released energy (J), M = Earthquake magnitude

## **Travel formulas**

## Time dilation (outside observer) / relativistic mass

$$\Delta_t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

t = Time, v = Velocity (m/s), c = Speed of light (m/s)

#### **Uniform acceleration time**

$$t_a = \frac{v_f - v_i}{a}$$

t(a) = Acceleration time (s), v(f) = Final velocity (m/s), v(i) = Initial velocity (m/s), a = Acceleration (m/s<sup>2</sup>)

## **Uniform acceleration distance**

$$d = \frac{(v_i + v_f) \cdot t_a}{2}$$

d = Distance (m), v(i) = Initial velocity (m/s), v(f) = Final velocity (m/s), t(a) = Acceleration time (s)