SFCalcSheet formula collection

Preface

This is an index of formulas used internally by SFCalcSheet, the science fiction calculation spreadsheet. Hopefully, and independent of whether you are a user of the spreadsheet, these formulas can help you with various worldbuilding and astronomy related tasks.

If you find a mistake in this document, don't hesitate and file an issue on GitHub. You can also contact the author on Reddit, Twitter or on his Discord server. Ideas, constructive feedback and praise are always appreciated.

Star formulas

Luminosity from absolute magnitude

$\sqrt[5]{100}^{M_{Sun}-M}$

L = luminosity (Suns), M = absolute magnitude

Absolute magnitude from luminosity

$M = M_{Sun} - 2.5 \cdot \log_{10}(L)$

M = absolute magnitude, L = luminosity (Suns)

Visible magnitude from absolute magnitude and distance

$m = M - 5 + 5 \cdot \log_{10}(d)$

m = visible magnitude, M = absolute magnitude, d = distance (pc)

Luminosity difference from absolute magnitudes

$\Delta L = \sqrt[5]{100}^{(M_{Big}-M_{Small})}$

L = luminosity, M = absolute magnitude

Luminosity from mass

$L = m^{3.5}$

L = luminosity (Suns), m = mass (Suns)

Radius from mass (stars less massive than the Sun)

$r = m^{0.8}$

r = radius (Suns), m = mass (Suns)

Radius from mass (stars as massive or more massive than the Sun)

$r = m^{0.57}$

r = radius (Suns), m = mass (Suns)

Surface temperature from mass

$T_s = 5778 \cdot m^{0.54}$

T_s = surface temperature (K), m = mass (Suns)

5778 is the Sun's surface temperature in Kelvin.

Inner rim of habitable zone

$r_{h_{inner}} = \sqrt{\frac{L}{1.1}}$

$r_{h_{inner}}$ = inner rim (AU), L = luminosity (Suns). This is a conservative formula resulting in a narrow HZ. Use the "habitable zone limits" formula for more flexibility.

Outer rim of habitable zone

$r_{h_{outer}} = \sqrt{\frac{L}{0.53}}$

$r_{h_{outer}}$ = outer rim (AU), L = luminosity (Suns). This is a conservative formula resulting in a narrow HZ. Use the "habitable zone limits" formula for more flexibility.

Main sequence life span from mass

$l = 10^{10} \cdot (\frac{1}{m})^{2.5}$

l = life span (years), m = mass (Suns)

Luminosity relative to ZAMS (zero age main sequence)

$L = 1 + 10^{-10} \cdot m^{2.5} \cdot t$

L = relative luminosity (ZAMS = 1), m = mass (Suns), t = age of star (years)

Orbital period of binary orbit from distance and mass

$T = 2\pi \cdot \sqrt{\frac{d^3}{G \cdot (m_1 + m_2)}}$

T = orbital period (s), d = separation of bodies' centers/sum of semi-major axes (m), G = gravitational constant, m_1 and m_2 = masses of orbiting bodies (kg)

Barycenter of binary orbit

$d_b = \frac{d}{1 + \frac{m_1}{m_2}}$

d_b = distance of barycenter to center of first body (m), d = distance between bodies' centers (m), m_1 and m_2 = masses of orbiting bodies (kg)

Stellar wind mass loss

$M_{Suns} = 10^{-12.76 + 1.3 \cdot log_{10} L_{Suns}} \cdot 5754400000000$

M_{Suns} = mass loss (Sun = 1); L_{Suns} = luminosity of star (Suns)

Stellar wind energy

$E_{Suns} = M \cdot (\frac{T}{5778})^4$

E_{Suns} = stellar wind energy (Sun = 1); M = stellar wind mass loss (Sun = 1); T = star surface temperature. 5778 is the Sun's surface temperature.

Stellar wind energy at distance

$E_d = \frac{E_{Suns}}{d^2}$

E_d = stellar wind energy at distance (Sun = 1); E_{Suns} = stellar wind energy; d = distance from star (AU)

Planet formulas

Radius from mass and density

$r = \sqrt[3]{\frac{3m}{4\pi \cdot \rho}}$

r = radius (m), m = mass (kg), ρ = density (kg/m³)

Gravitational acceleration from mass and radius

$g = \frac{G \cdot m}{r^2}$

g = gravitational acceleration (m/s²), G = gravitational constant, m = body mass (kg), r = body radius (m)

Escape velocity from mass and radius

$v_e = \sqrt{\frac{2m \cdot G}{r}}$

v_e = escape velocity (m/s), G = gravitational constant, m = mass (kg), r = radius (m)

Roche limit (rigid bodies) from densities

$d = r_p \cdot 1.26 \cdot \sqrt[3]{\frac{\rho_p}{\rho_s}}$

d = Roche limit (m), r_p = planet radius (m), ρ_p = density of planet (kg/m³), ρ_s = density of satellite (kg/m³)

Roche limit (fluid/loose bodies) from densities

$d = r_p \cdot 2.44 \cdot \sqrt[3]{\frac{\rho_p}{\rho_s}}$

d = Roche limit (m), r_p = planet radius (m), ρ_p = density of planet (kg/m³), ρ_s = density of satellite (kg/m³)

Orbital period from distance and mass

$T = \sqrt{\frac{4\pi^2 \cdot d^3}{G \cdot m}}$

T = orbital period (s), d = semi-major axis (m), G = gravitational constant, m = mass of orbited body (kg)

Hill sphere

$r_h = d \cdot \sqrt[3]{\frac{m}{3m_o}}$

r_h = radius of Hill sphere (m), d = distance to orbited body (m), m = mass of body (kg), m_o = mass of orbited body (kg)

Tidal force

$F_t = G \cdot m \cdot \frac{2r}{d^3}$

F_t = tidal force (m/s²), G = gravitational constant, m = mass of causing body (kg), r = radius of affected body (m), d = distance between bodies (m)

Habitable zone limits

$d = \sqrt{\frac{L}{S_e}}$

d = distance of HZ limit to the star (AU), L = luminosity (Suns), S_e = normalized stellar flux (runaway greenhouse = 1.41; moist greenhouse = 1.107; 1st condensation = 0.53; maximum greenhouse = 0.356)

Solar irradiance

$R = L \cdot G_{SC} \cdot (\frac{1}{d})^2$

R = irradiance (W/m²), L = luminosity of star (Suns), G_{SC} = solar constant (1,361 W/m²), d = distance of planet to star (AU)

Effective temperature

$T_e = \sqrt[4]{\frac{(1-A) \cdot L}{16\pi \cdot d^2 \cdot \sigma}}$

T_e = effective temperature (K), A = bond albedo of planet, L = luminosity of star (W), d = distance to star (m), σ = Stefan-Boltzmann constant

Surface temperature

$T_s = T_e \cdot \sqrt[4]{\frac{1}{1-\frac{\epsilon}{2}}}$

T_s = surface temperature (K), T_e = effective temperature (K), ε = atmospheric absorption (0-1)

Land area

$A_l = 4\pi \cdot r^2 \cdot A_{l_p}$

A_l = land area (km²), r = planet radius (km), A_{l_p} = land percentage (0-1)

Satellite formulas

Visual diameter

$d_v = 2 \cdot \arcsin(\frac{r}{d})$

d_v = visual diameter (rad), r = radius of satellite (km), d = distance to satellite (km)

Visual area from visual diameter

$A_v = \pi \cdot (\frac{d_v}{2})^2$

A_v = visual area (rad²), d_v = visual diameter (rad)

Illuminance

$E_v = \frac{A_v \cdot (\frac{A_b}{0.12})}{(\frac{\pi}{48900})}$

E_v = illuminance (full moons; $\approx$ 0.25 lux), A_v = visual area (rad²), A_b = bond albedo (0-1)

48900 is a shorthand for 1 divided by half the Moon's angular diameter (in radians) squared. 0.12 is the Moon's bond albedo.

Compact object formulas

Schwarzschild radius

$r_s = \frac{2G \cdot m}{c^2}$

r_s = Schwarzschild radius (m), G = gravitational constant, m = mass (kg), c = speed of light (m/s)

Black hole lifetime

$t_l = m^3 \cdot \frac{5120 \cdot \pi \cdot G^2}{\hbar \cdot c^4}$

t_l = lifetime (s), m = mass (kg), G = gravitational constant, $\hbar$ = Dirac constant, c = speed of light (m/s)

Disaster formulas

Impactor mass

$m = \frac{4}{3} \cdot \pi \cdot \rho \cdot r^3$

m = impactor mass (kg), ρ = impactor density (kg/m³), r = impactor radius (m)

Impactor kinetic energy

$K = \frac{1}{2} \cdot m \cdot v^2$

K = impactor kinetic energy (J), m = impactor mass (kg), v = impactor velocity (m/s)

Impact crater approximate size

$d_c = (\frac{K}{(4.184 \cdot 10^{12})})^{0.3} \cdot 46$

d_c = approximate crater diameter (m), K = impactor kinetic energy (J)

$4.184 \cdot 10^{12}$ is the explosion energy of 1 kt TNT in joules. 46 is the diameter of the crater left by the explosion of 1 kt TNT (in meters).

Impact crater size lower bound

$d_{crlower} = \frac{d_c}{3.48}$

$d_{crlower}$ = crater diameter lower bound, d_c = approximate crater diameter

Impact crater size upper bound

$d_{crupper} = d_c \cdot 3.48$

$d_{crupper}$ = crater diameter upper bound, d_c = approximate crater diameter

Energy of matter/antimatter annihilation

$E = mc^2$

E = released energy (J), m = total reaction mass (kg), c = speed of light (m/s)

Bomb blast maximum fireball radius

$r_{max} = E^{0.4} \cdot 39$

r_{max} = maximum fireball radius (m), E = bomb yield (kt TNT)

This is an intermediate formula between the approximate fireball radii of aerial and ground-based detonations. Use a multiplicator of 33.5 for aerial and 44 for ground-based detonations.

Bomb blast fireball duration

$t = E^{0.45} \cdot 0.2$

t = fireball duration (s), E = bomb yield (kt TNT)

Bomb blast crater diameter

$d = E^{0.3} \cdot 46$

d = crater diameter (m), E = bomb yield (kt TNT)

Bomb blast shockwave radius

$r = E^{\frac{1}{5}} \cdot \Delta t^{\frac{2}{5}} \cdot \rho^{-\frac{1}{5}}$

r = shockwave radius (m), E = explosion energy (J), Δt = time since detonation (s), ρ = mass density of medium (kg/m³)

Earthquake energy

$E = 10^{4.4+1.5M}$

E = released energy (J), M = earthquake magnitude

Travel formulas

Time dilation (outside observer) / relativistic mass

$\Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

t = time, v = velocity (m/s), c = speed of light (m/s)

Uniform acceleration time

$t_a = \frac{v_f - v_i}{a}$

t_a = acceleration time (s), v_f = final velocity (m/s), v_i = initial velocity (m/s), a = acceleration (m/s²)

Uniform acceleration distance

$d = \frac{(v_i+v_f) \cdot t_a}{2}$

d = distance (m), v_i = initial velocity (m/s), v_f = final velocity (m/s), t_a = acceleration time (s)

3D distance

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

d = distance, $x_1 y_1 z_1$ = start point, $x_2 y_2 z_2$ = endpoint