MACM 316 – Computing Assignment 8

- Read the *Guidelines for Assignments* first.
- Submit a one-page PDF report to Canvas and upload you Matlab scripts (as m-files). Do not use any other file formats.
- Keep in mind that Canvas discussions are open forums.
- You must acknowledge any collaborations/assistance from colleagues, TAs, instructors etc.

Predicting the population of BC

The logistic equation is a simple initial value problem describing the behaviour of a population $P(t)$ over time given an initial population α. It is defined by

$$\frac{dP}{dt} = bP(t) - k(P(t))^2, \qquad a \le t \le b, \qquad P(a) = \alpha, \tag{1}$$

where b and k are parameters. Your objective in this assignment is to use this model to predict the future population of British Columbia.

1) Go to the BC Stats website[1] and download the total year-by-year population data for BC from 1986 (the earliest year in the database) to 2015 (the last year in the database). Make sure you select the correct options. If you've done this correctly, you should have the data 3,003,621 for 1986 and 4,683,139 for 2015. Import this data into Matlab and store it as a 30×2 array with the year in the first column and population in the second column.

2) You first need to estimate the parameters b and k in the logistic model. You will do this by fitting a curve of the form $bx - kx^2$ to some appropriate y-values. First, pick an initial year N between 1986 and 2015 (you will change N later, you may wish to start with $N = 1986$ and go from there). Define an array `P` of population values for the years $N, N+1, \ldots, 2015$. To perform the fit you need to find approximate values for $\frac{dP}{dt}$. For this, you should use the forwards difference formula $\frac{dP}{dt} \approx P(t+1) - P(t)$. Construct an array `dP` of forwards differences based on the entries of `P`. You now have the x-values and y-values to perform your fit. To estimate the parameters b and k, use the `fit` command in Matlab:

```
parfit = fit(P(1:end-1), dP, fittype('b*x-k*x^2'), 'start', [0,0] )
```

The estimated values of b and k can now be obtained as follows:

```
b_est = b.parfit;  k_est = k.parfit;
```

3) You now have the parameters b and k for the logistic equation. We next want to solve the IVP (1) to predict the future population. You should do this using Matlab's `ode45` command. If you do not know how to use this command, look at the Matlab documentation[2] or at the demo *ode45demo.m*. You should use the parameters $[a, b] = [N, 2040]$ and α be the population at year N. This means you will be predicting the population from 2015 to 2040.

[1] `http://www.bcstats.gov.bc.ca/statisticsbysubject/demography/populationestimates.aspx`
[2] `https://www.mathworks.com/help/matlab/ref/ode45.html`

Compute the solution using `ode45` and plot a graph of population versus year. This graph should contain (i) the exact data values of the population from year N to 2015, and (ii) the population output from `ode45` from year N to 2040.

Does your prediction look reasonable? Discuss.

4) Now change the parameter N and repeat the above procedure for a range of different values of N. How does the choice of N affect your prediction? Which value (or values) of N would you recommend using for the prediction? Using that value(s), give an estimate for BC's population in 2040. Comment on the reliability of your estimate.