MACM 316 – Computing Assignment 2

- Read the *Guidelines for Assignments* first.
- Submit a one-page PDF report to Canvas and upload you Matlab scripts (as m-files). Do not use any other file formats.
- Keep in mind that Canvas discussions are open forums.
- You must acknowledge any collaborations/assistance from colleagues, TAs, instructors etc.

Robustness of Gaussian Elimination

Note: Download the in-class demo *LSRandom.m* before completing this assignment.

As seen in class, Gaussian elimination with partial pivoting in floating point arithmetic may incur errors due to round-off. In this assignment you will investigate this error.

Let A be a random $n \times n$ matrix, $\underline{x} = (1, 1, \ldots, 1)^\top$ be an n-vector of ones and $\underline{b} = A\underline{x}$ be the right-hand side vector. As in class, let $\underline{z} = (z_j) \in \mathbb{R}^n$ be the result of solving the system $A\underline{x} = \underline{b}$ in finite precision using the backslash command. To measure the error between $\underline{x}$ and $\underline{z}$, we let

$$\delta = \max_{j=1,\ldots,n} |x_j - z_j|,$$

be the maximum componentwise difference between the two vectors. Since A is a random matrix, we need to run this calculation a number of times with different realizations of A in order to get a reasonable value for δ. Let M be the number of trials and suppose that for the k^{th} trial the error is $\delta^{(k)}$. We define the mean error as follows:

$$E_n = \frac{1}{M}\left(\delta^{(1)} + \delta^{(2)} + \ldots + \delta^{(M)}\right).$$

Your goal is to determine the approximate size of matrix $n = n^*$ at which the mean error is $E_n \approx 1$. In other words, the point at which round-off error in Gaussian elimination is of the same magnitude as the vector $\underline{x}$.

In practice, your computer will not have the memory or processing power to find n^* exactly. Instead, you should extrapolate your data. Find E_n for reasonable values of n, make a plot of $\log_{10}(E_n)$ versus $\log_{10}(n)$ and then perform a suitable extrapolation.

Your conclusions should be explained in a one-page report. Your report should include the following:

(a) A plot of $\log_{10}(E_n)$ versus $\log_{10}(n)$.

(b) Justification for the values of n and M you chose.

(c) Explanation of how you do the extrapolation.

(d) An estimation of the number n^*.

Finally, repeat the above experiment, but now with A replaced by $A^\top A$ (here $\top$ denotes the transpose of A). Compare with your previous results and discuss.