MACM 316 – Computing Assignment 5

- Read the *Guidelines for Assignments* first.
- Submit a one-page PDF report to Canvas and upload you Matlab scripts (as m-files). Do not use any other file formats.
- Keep in mind that Canvas discussions are open forums.
- You must acknowledge any collaborations/assistance from colleagues, TAs, instructors etc.

Perturbations of polynomial roots

You have seen in class that the roots of a polynomial

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0,$$

can be computed efficiently via the `roots` command in Matlab. You may wish to take a look at the in-class demo *RandPolyRoots.m* to remind yourself how to do this.

Suppose that $[a_n, a_{n-1}, \ldots, a_1, a_0]$ is a given set of coefficients of a polynomial $p(x)$. Your goal in this assignment is to investigate the behaviour of polynomial roots of when the largest coefficient a_n is perturbed. Specifically, you will investigate the behaviour of the roots of the polynomial

$$q(x) = re^{i\theta}a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0,$$

where $r > 0$ and $0 \leq \theta \leq 2\pi$ are parameters that you will vary.

For example, suppose that $n = 1$ and $[a_1, a_0] = [2, -1]$. Then the polynomial

$$p(x) = 2x - 1,$$

has its root at $x^* = 1/2$. The perturbed polynomial

$$q(x) = 2re^{i\theta}x - 1,$$

has its root at $x^* = \frac{1}{2r}e^{-i\theta}$. Varying θ from 0 to 2π, the perturbed roots describe a circle of radius $1/(2r)$. This is shown in the figure on the next page.

Your task is to investigate the behaviour for larger polynomial degrees n by producing plots similar to the one shown on the next page. I recommend you begin with a randomly-generated vector of coefficients $[a_n, a_{n-1}, \ldots, a_1, a_0]$. Compute and plot the roots of this polynomial, and then perturb r and θ. Use a fine grid of values for θ, equally-spaced between 0 and 2π, and for each fixed value of r plot the corresponding curves of polynomial roots.

Present your figures and explain your conclusions in a one-page report. Make sure to discuss the *qualitative* behaviour of the graphs (i.e. how do the roots behave as r and θ are varied), as well as the parameter values you used and your justification for them. The best figures will be showcased on Canvas, and bonus marks may be awarded for particularly good reports.

Hint: When you plot the perturbed roots, make sure you use appropriate plot markers. If R is a vector of perturbed roots, I suggest you use the command

```
plot(real(R),imag(R),'.','MarkerSize',ms);
```

Here `ms` is variable that controls the marker size. You should select it appropriately so you can visualize the roots.

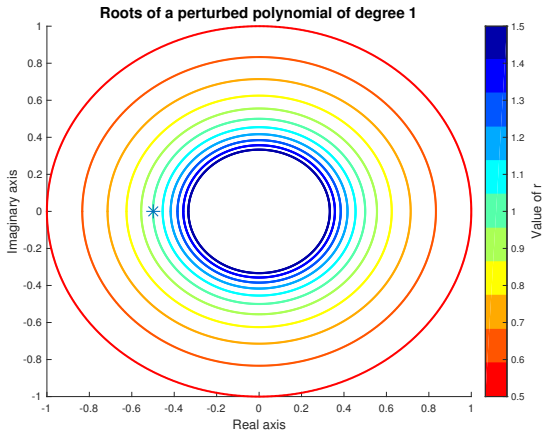

Figure 1: Roots of the perturbed polynomial $q(x) = 2re^{i\theta}x - 1$. Each colour represents a fixed value of r. The blue star is the original root $x^* = 1/2$.