Complex Roots

Root of $\mathbb{C}$ is $\mathbb{C}$

$\sqrt[n]{z_1} = z_2$ $\quad \forall z_1 \exists z_2$ where $z_1 = a + ib$, $z_2 = c + id$ — In cartesion and polar form

$\hookrightarrow = r \cdot e^{i\varphi}$, $\hookrightarrow = s \cdot e^{i\psi}$

$\Rightarrow z_1 = z_2^n$ ← cartesion

$\Rightarrow r \cdot e^{i\varphi} = s^n e^{in\psi}$ ← polar of n-th power (multiply two complex numbers $\Rightarrow$ multiply length, and angle e.g. $z_1 * z_2 = r \cdot s\, e^{i(\varphi+\psi)}$)

$n\psi = \varphi + 2\pi k$ → $\varphi = \varphi + 2\pi k \;\forall k \geq 0$

↳ Hence, it is normally not written. But for complex root it is very important.

$$\Rightarrow \underbrace{s e^{i\psi}}_{z_2} = \sqrt[n]{r}\, e^{\overbrace{\frac{i\varphi + 2\pi k}{n}}}, \quad k = 0 \ldots n-1$$

that is the reason why in your graph the points lie closer to the origin than the original point (points ≙ numbers in cartesion plane)

There exist n solutions obtained by inserting $k = 0 \ldots n-1$. We know that $\varphi = \varphi + 2\pi k \;\forall k \geq 0$. Since the range of the angle is $0 - 2\pi$ ($0° - 360°$) there is no need of writing the $+2\pi k$. However, since $n\psi = \varphi (+2\pi k)$ $\Rightarrow \psi = (\varphi + 2\pi k)/\psi$. again, since the range is $0 - 2\pi$, we say $k = 0 \ldots n-1$.