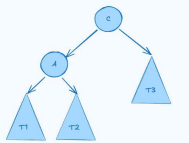
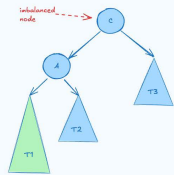


## STARTING TREE For LL and LR



Before an insertion, height(C(left)) and height(C(right)) differ by 1.

### Case 1: Left-Left Imbalance (Single Rotation)



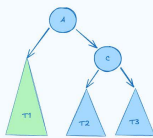
Then, there is an insertion somewhere in T1 that causes A's height to increase by 1. Now, height(C(left)) and height(C(right)) differ by 2.

#### Important Observations:

- All values in T1 are smaller than both A and C.
- All values in T2 are bigger than A but smaller than C.
- All values in T3 are bigger than C.

→ Since all values in T2 are bigger than A but smaller than C, that subtree could live to the right of A or the left of C.

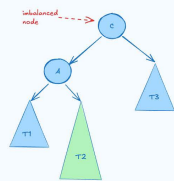
```
fun rotateLL(leftChild(C), parentOf(C)):  
  A = C.left  
  C.left = A.right  
  A.right = C  
  if C == root of tree:  
    root of tree = A  
  else:  
    if parentOf(C).left = C:  
      parentOf(C).left = A  
    else:  
      parentOf(C).right = A  
  updateHeight(C)  
  updateHeight(A)
```



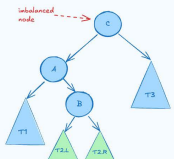
We can adjust for this imbalance with a SINGLE ROTATION by rotating the node of imbalance with its left child.

Notice after the re-balancing, A has the same height as C did in the original tree.

### Case 2: Left-Right Imbalance (Double Rotation)



An insertion in T2 causes C to become the node of imbalance.



B represents the root of T2. Inserting into T2 cannot be fixed by a single rotation (shown above). T2.L and T2.R are shown as half-way to the next level because only 1 (where the insertion happened) would extend down to the red --- level.

#### Observations:

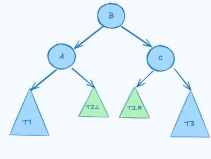
- All values in T2.L fall between A and B.
- So, T2.L could be the left child of B or the right child of A.
- All values in T2.R fall between B and C.
- So, T2.R could be the right child of B or the left child of C.

To fix:

- Step 1: Rotate A with its right child (B). (same rotation as RR imbalance)
- Step 2: Rotate C with its left child (now it is B after step 1).

```
fun doubleRotateLL(leftChild(C), parentOf(C)):  
  rotateLL(leftChild(C), parentOf(A))  
  rotateLL(leftChild(C), parentOf(C))
```

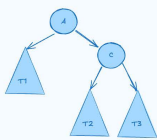
Step 1: Rotate A with Right Child B.



Step 2: Rotate C with Right Child B.

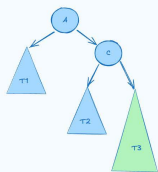
After this rotation, either T2.L or T2.R will be as deep as T1 and T3, but not both.

## STARTING TREE For RR and RL



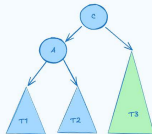
Before an insertion, height(A(left)) and height(A(right)) differ by 1.

### Case 4: Right Right Imbalance (Single Rotation)

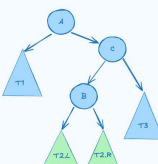


Insertion into T3 causes T to be 2 levels deeper than T1, making A the node of imbalance.

Remember: All values in T3 fall between A and C. So T2 could be connected to the left of C or the right of A.



### Case 3: Right Left Imbalance (Double Rotation)



As with Case 2, an insertion into T2 cannot be solved with a single rotation. So, we need to explicitly consider the root of T2, labeled as B here.

