Knowledge Navigated Quantum-inspired Tabu Search Algorithm for Reversible Circuit Synthesis

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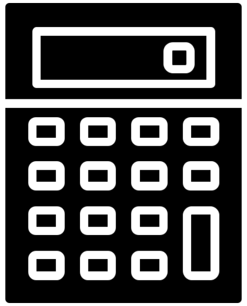
Outline

- Introduction
- Proposed Method
- Experiment Results
- Conclusion

Quantum computing

Shor's Algorithm

Prime factorization



Grover's Algorithm

Unstructured data searching

Quantum circuit synthesis

Quantum circuit

One-to-one and onto

input		output
a	**b**	**A**
0	0	0
0	1	1
1	0	1
1	1	0

Classical
Irreversible

input		output	
a	**b**	**A**	**B**
0	0	0	0
0	1	1	0
1	0	1	1
1	1	0	1

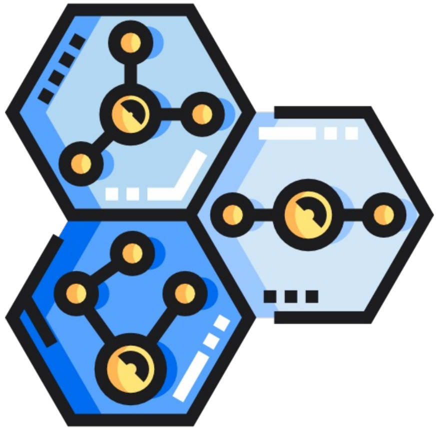
Quantum
Reversible

New synthesis methods

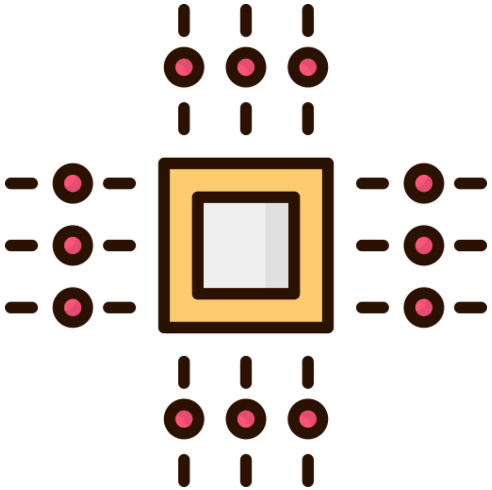
Achieve zero dissipation

Application

Nanotechnology



Optical computing



DNA computing

Reversible circuit

For n-bit circuit
there are $2^n!$ possible truth table

- 2-bits: $2^2! = 24$
- 3-bits: $2^3! = 40{,}320$
- 4-bits: $2^4! = 20{,}922{,}789{,}888{,}000$
- 5-bits: $2^5! = 263{,}130{,}836{,}933{,}693{,}530{,}167{,}218{,}012{,}160{,}000{,}000$

Large search space

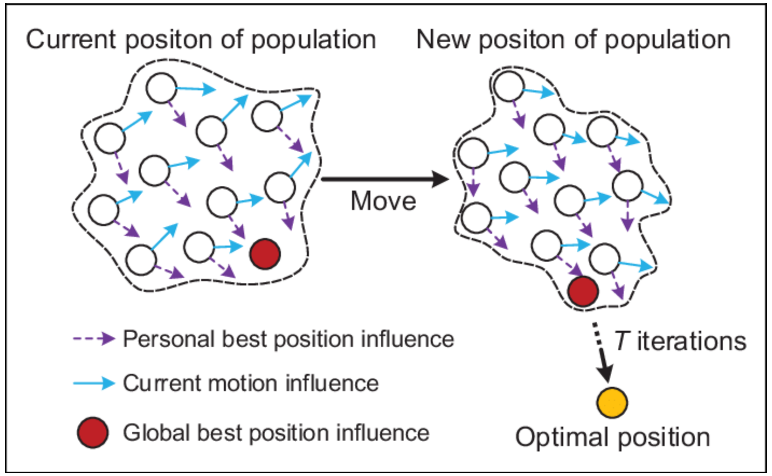
No dominate synthesis method

The motivation of quantum circuit synthesis

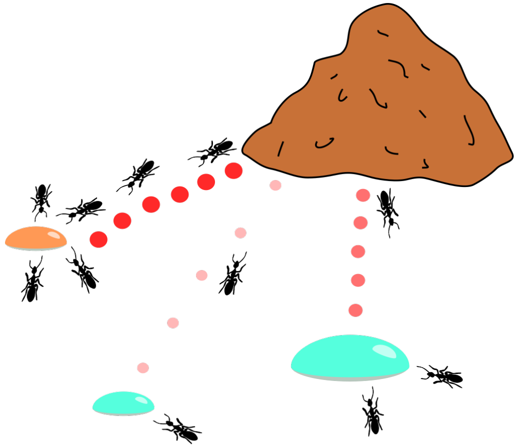
Metaheuristic algorithm

Genetic algorithms (GA)

Particle swarm optimization (PSO)

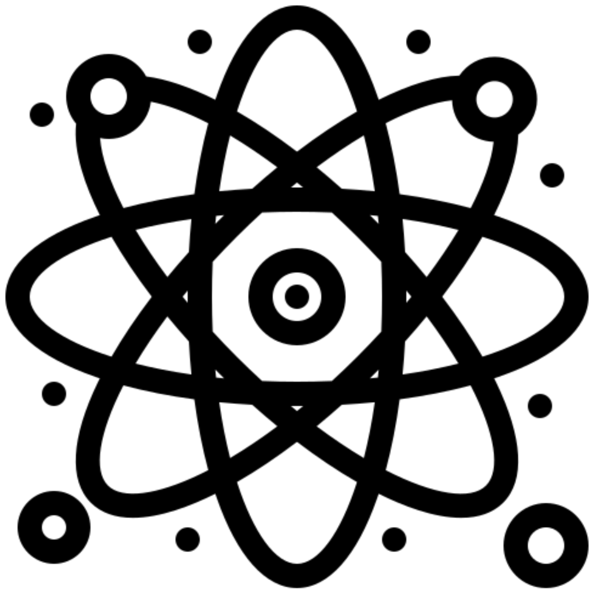


Ant colony optimization (ACO)



Metaheuristic algorithm

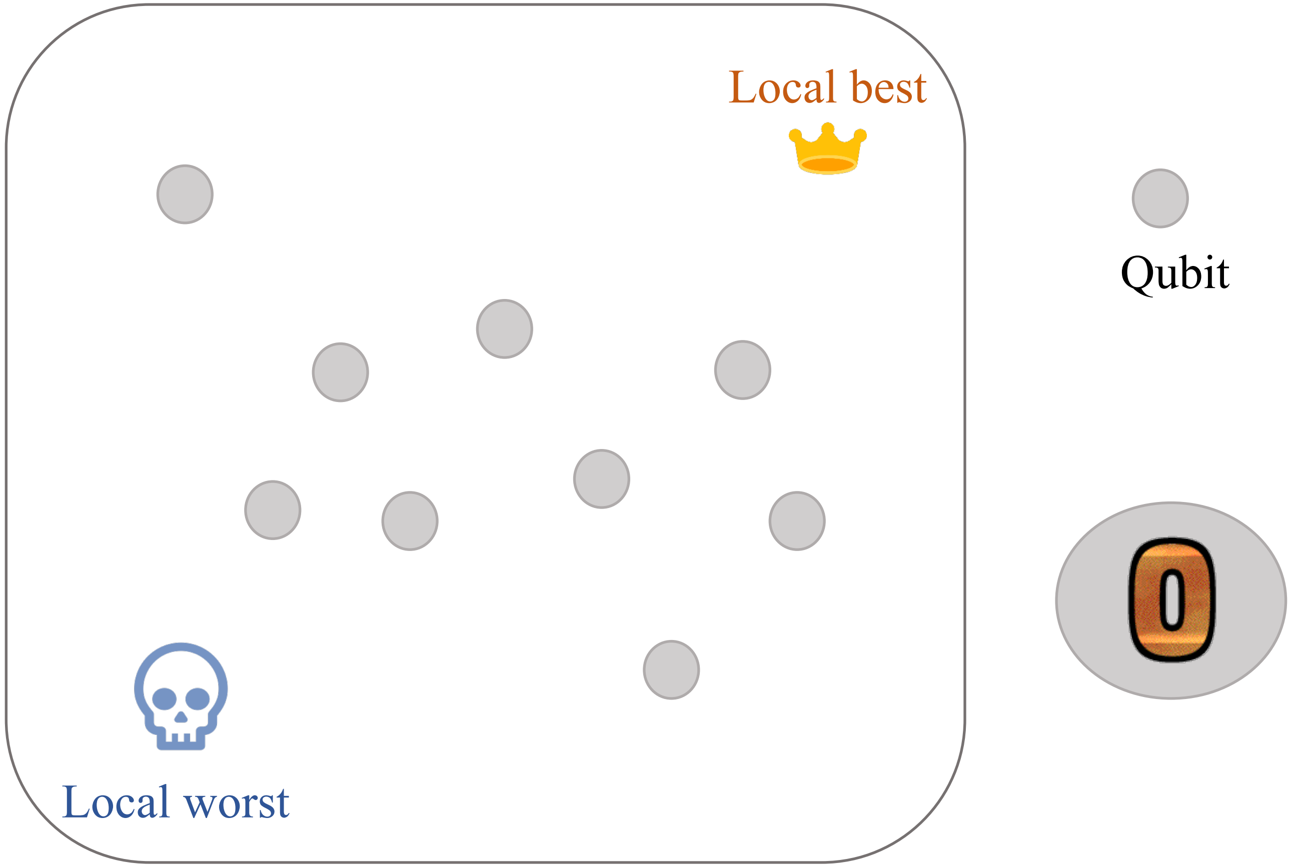
Quantum-inspired algorithms
(QA)



Search ability

Preventing premature convergence

Quantum-inspired Tabu Search Algorithm, QTS



Approach the best solution and be far away from the worst solutions

Knowledge Navigated Quantum-inspired Tabu Search Algorithm, KNQTS

QTS + Knowledge Navigation / Quantum-Not gate = **KNQTS**

Knowledge Navigated Quantum-inspired Tabu Search Algorithm, KNQTS

QTS + Knowledge Navigation / Quantum-Not gate = **KNQTS**

Knowledge: the convergence level

Local best	1	3	2	0	1	3	2	1	0
Local worst	1	0	1	0	2	3	2	3	0

$\boldsymbol{Diff_i = 4}$

$Diff_i$: the difference in current iteration

$Diff_{i-1}$: the difference in last iteration

Knowledge Navigated Quantum-inspired Tabu Search Algorithm, KNQTS

QTS + Knowledge Navigation / Quantum-Not gate = **KNQTS**

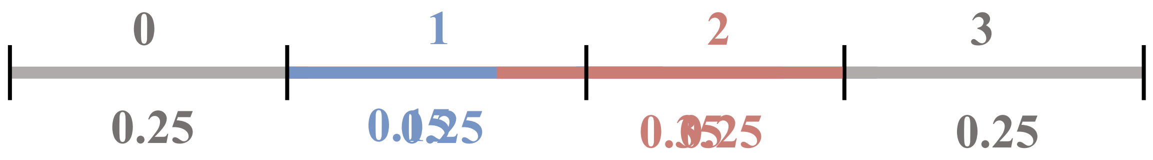
Knowledge: the convergence level

Compares the differences between the best and worst solutions

Use knowledge to adjust the convergence speed

$\boldsymbol{Diff_{i-1} = 2}$ $\boldsymbol{Diff_{i} = 4}$

larger → the convergence level is slow



the probability of a qudit

$Diff_i$: the difference in current iteration

$Diff_{i-1}$: the difference in last iteration

The algorithm accelerates its speed

Knowledge Navigated Quantum-inspired Tabu Search Algorithm, KNQTS

QTS + Knowledge Navigation / Quantum-Not gate = KNQTS

Knowledge: the convergence level

Compares the differences between the best and worst solutions

Use knowledge to adjust the convergence speed

$$\underbrace{Diff_{i-1} = 7 \qquad Diff_i = 4}$$

smaller → convergence level is fast

0	1	2	3
0.25			0.25

the probability of a qudit

The algorithm decelerates its speed

$Diff_i$: the difference in current iteration

$Diff_{i-1}$: the difference in last iteration

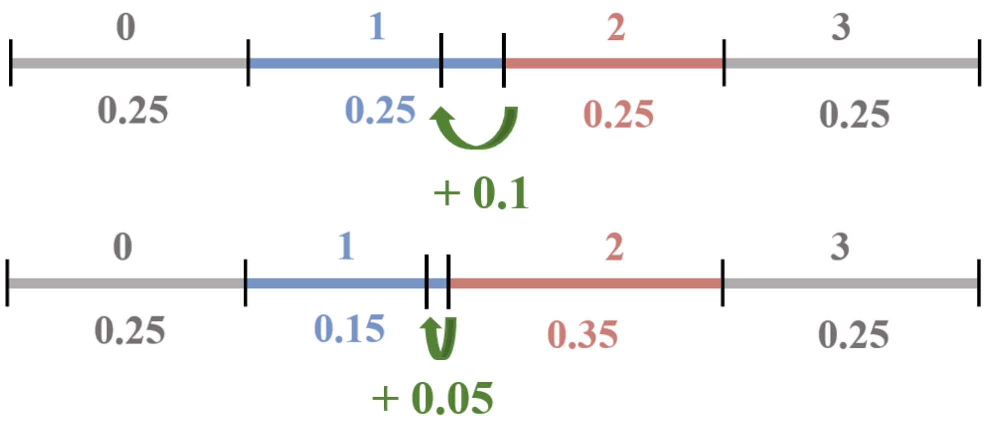
Knowledge Navigated Quantum-inspired Tabu Search Algorithm, KNQTS

QTS + Knowledge Navigation / Quantum-Not gate = KNQTS

Knowledge: the convergence level

Compares the differences between the best and worst solutions

Use knowledge to adjust the convergence speed



Not convergence

Convergence

The nonlinear adjustment of parameter

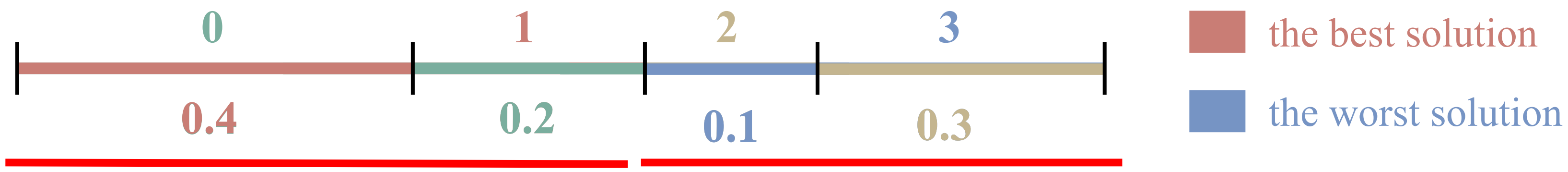
Navigate to a better solution

Knowledge Navigated Quantum-inspired Tabu Search Algorithm, KNQTS

QTS + Knowledge Navigation / Quantum-Not gate = **KNQTS**

The probability of the best solution is smaller than $\left(\frac{1}{\sqrt{4}}\right)^2$

Exchanges the probability



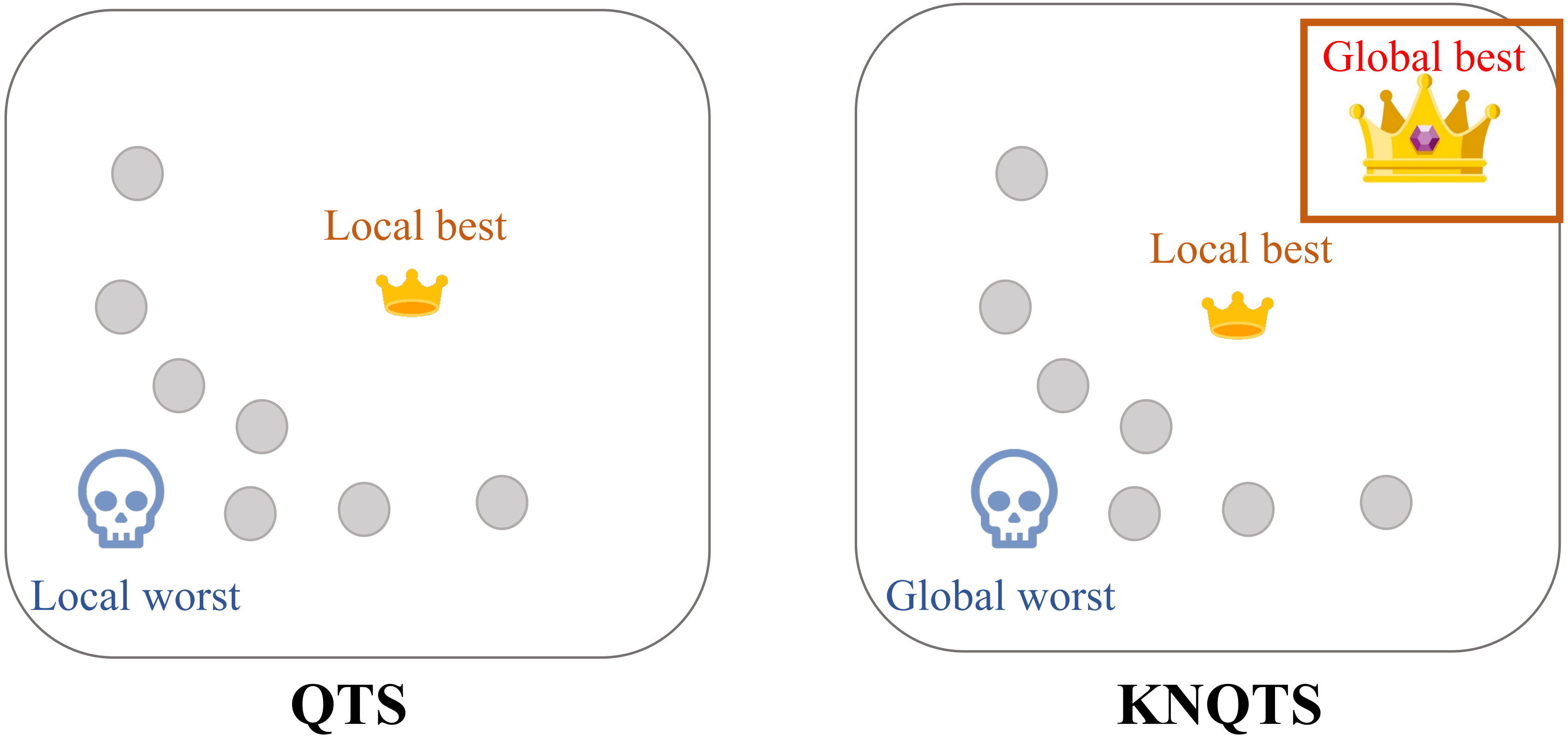
the probability of a qudit

Jump out of local optimum

Guide the algorithm to a more promising way

Knowledge Navigated Quantum-inspired Tabu Search Algorithm, KNQTS

QTS + [Knowledge Navigation / Quantum-Not gate] = KNQTS



Powerful search ability

Gate library

Multiple-Control Toffoli, MCT

Control line


1-control

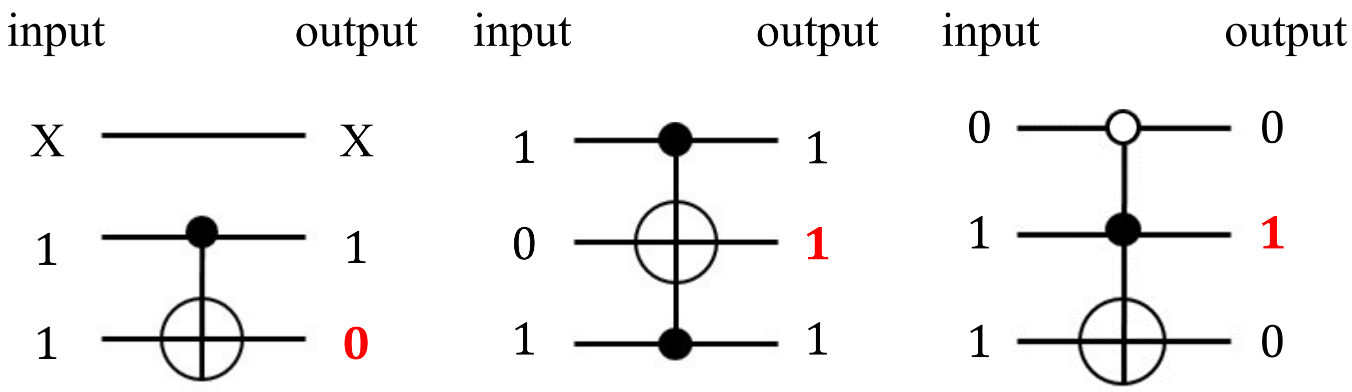

0-control

$\leq$ **(n-1)**

Target line

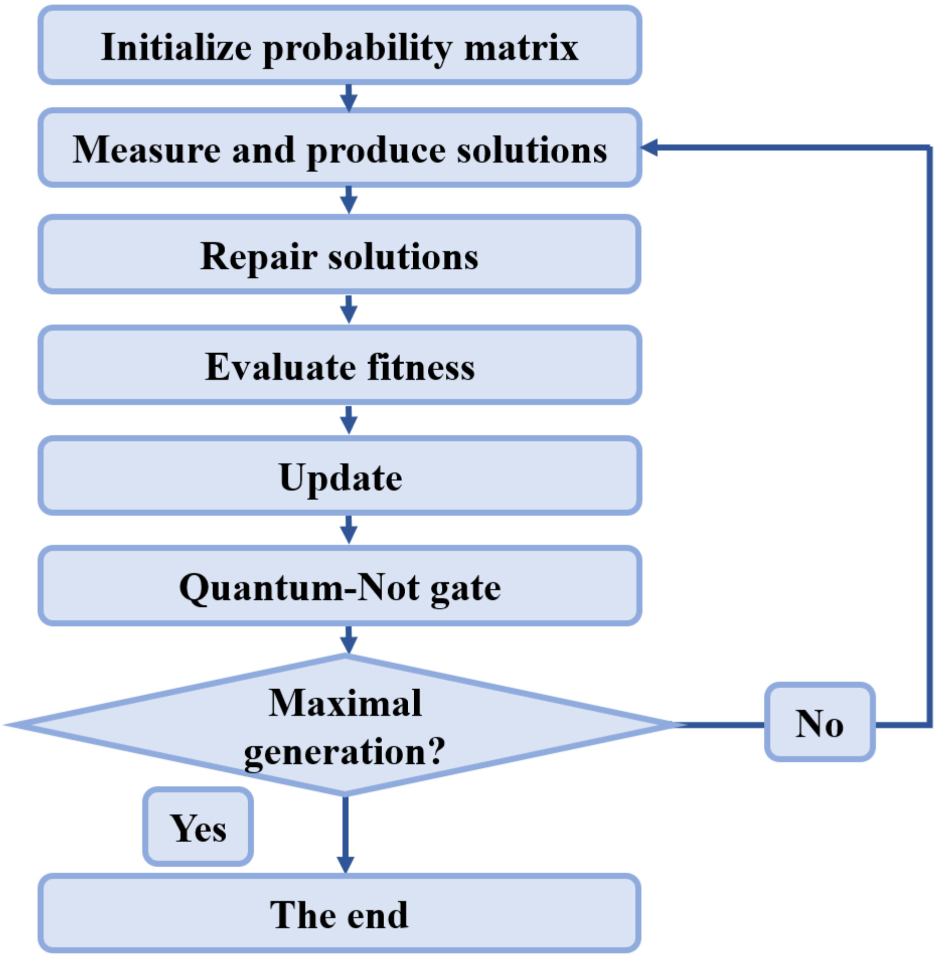
= 1

Circuit



X: don't care

KNQTS flowchart



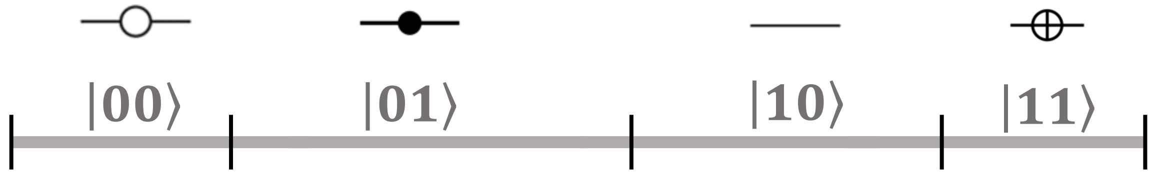
Encode scheme and initialization

$$\begin{bmatrix} q_{00} & \cdots & q_{0m} \\ \vdots & \ddots & \vdots \\ q_{n0} & \cdots & q_{nm} \end{bmatrix}$$

Probability matrix

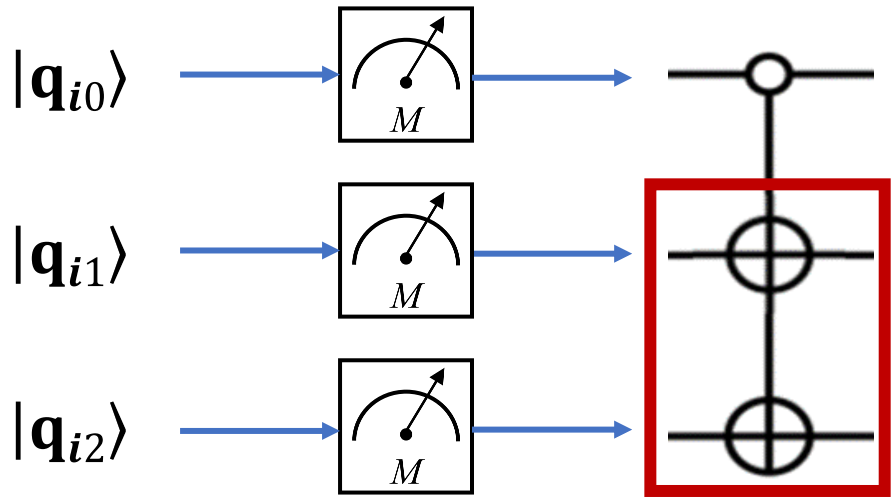
qudit: $|q_{ij}\rangle$

Initialization :



The KNQTS algorithm will produce circuit by measuring qudits

Repair mechanism

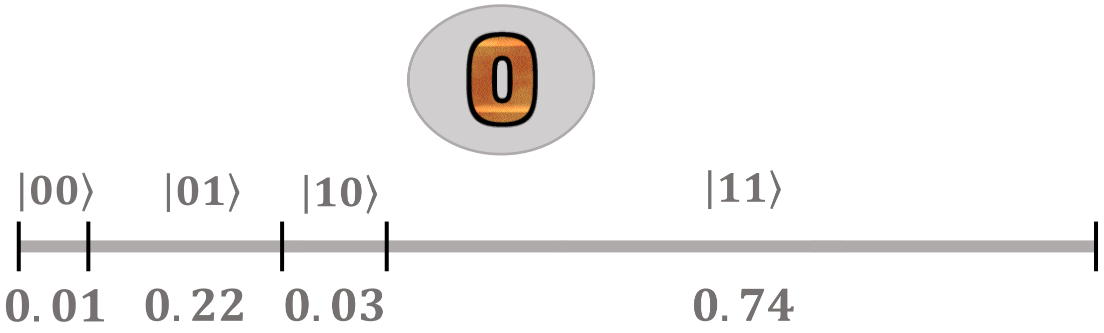


More than one target line

Infeasible circuit

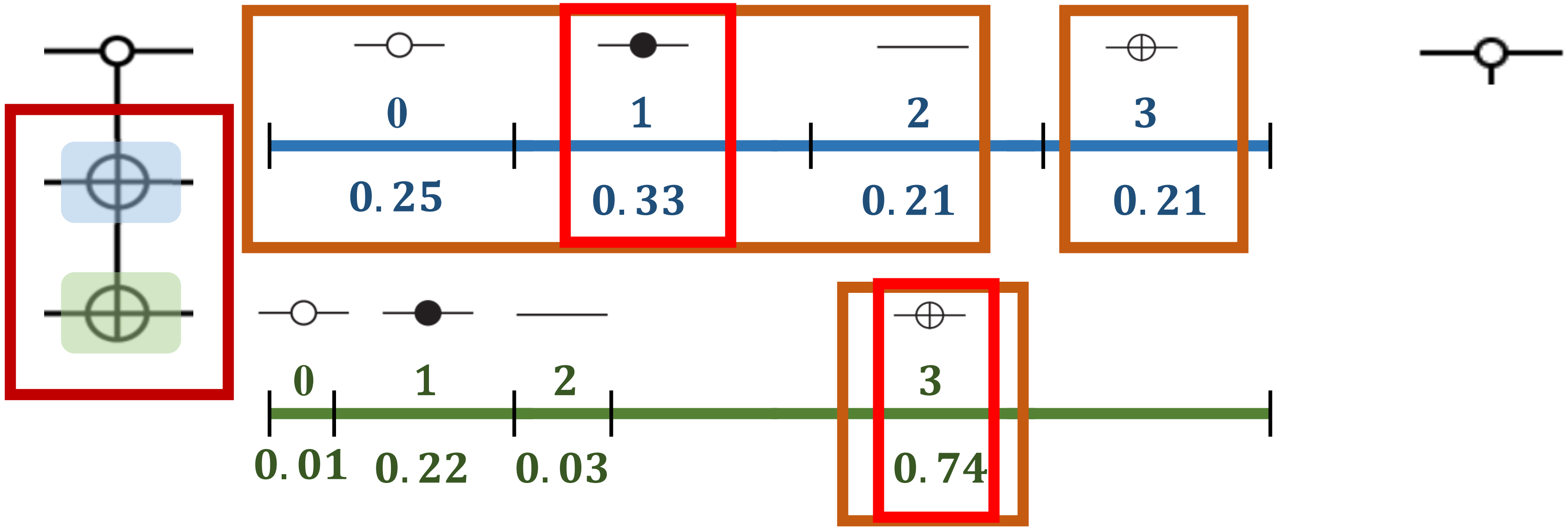
~~Traditional method: randomly repaired~~

KNQTS: repair the circuit according to the probability matrix



Repair mechanism

Replace the symbol with a more potent one



Find a better solution in a more reasonable way

Parameters setting

initial delta	μ	population (circuit)	generation
0.002	1.001	100	5000

μ = the parameter to adjust the delta

Compare with traditional QTS-based algorithm

Function	Gate Count		
	QTS	GNQTS	**KNQTS**
[1,0,3,2,5,7,4,6]	3	3	**3**
[7,0,1,2,3,4,5,6]	3	3	**3**
[0,1,2,3,4,6,5,7]	3	3	**3**
[0,1,2,4,3,5,6,7]	5	5	**5**
[0,1,14,15,4,5,10,11,7,9,6,8,12,13,2,3]	7	5	**5**
[1,2,3,4,5,6,7,0]	3	3	**3**
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,0]	4	4	**4**
[0,7,6,9,4,11,10,13,8,15,14,1,12,3,2,5]	4	4	**4**
[3,6,2,5,7,1,0,4]	6	6	**6**
[1,2,7,5,6,3,0,4]	5	5	**5**
[4,3,0,2,7,5,6,1]	5	5	**5**
[7,5,4,2,0,1,6,3]	5	4	**4**
[6,3,14,13,2,11,7,10,0,5,8,1,12,15,9,4]	10	9	**8**
[0,9,10,5,4,15,14,8,11,2,6,3,12,7,1,13]	10	7	**7**
[6,4,11,0,9,8,12,2,15,5,3,7,10,13,14,1]	10	10	**10**
[13,1,14,0,9,2,15,6,12,8,11,3,4,5,7,10]	9	9	**9**
Average gate count	5.75	5.3125	**5.25**
Achievement Ratio	-	79.75%	**85.38%**

Success rate:
We did 50 experiments with 16 functions using different random numbers of seeds
50 x 16 = 800
The success rate is the sum of the "number of times the correct circuit was found" and divided by 800.

KNQTS

Lower number of logical gates

Higher success rate

QTS : Quantum-inspired Tabu Search Algorithm
GNQTS :QTS with Quantum Not gate
KNQTS : :QTS with Knowledge Navigation strategy

Compare with other traditional synthesis methods

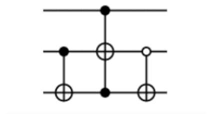
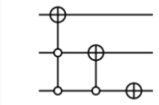
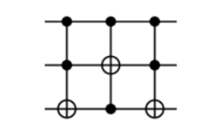
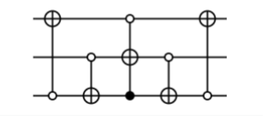
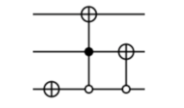
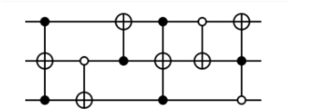
Function	Gate Count											
	PPRM [6]	MOSAIC [7]	SA-QM and ACO [19]	QTS [11]	ACO-based [9]	AGA [20]	EQIEA [21]	GA [22]	PSO [10]	ROCBDD [14]	KNQTS	Compared with the best gate count all over other algorithms
[1,0,3,2,5,7,4,6]	4	4	5	3	3	4	4	4	5	-	3	0
[7,0,1,2,3,4,5,6]	3	3	3	3	3	3	3	3	3	-	3	0
[0,1,2,3,4,6,5,7]	3	3	5	3	3	3	3	3	3	-	3	0
[0,1,2,4,3,5,6,7]	5	7	6	5	4[1]	4[1]	5	5	5	-	5	0
[1,2,3,4,5,6,7,0]	3	3	3	3	3	4	3	3	3	-	3	0
[3,6,2,5,7,1,0,4]	7	8	8	6	6	6	7	7	8	-	6	0
[1,2,7,5,6,3,0,4]	7	8	7	5	6	6	6	7	6	-	5	0
[4,3,0,2,7,5,6,1]	6	6	7	5	5	5	6	6	6	-	5	0
[7,5,4,2,0,1,6,3]	-	-	-	5	-	-	-	-	-	-	4	-1
[7,5,2,4,6,1,0,3]	7	6	7	-	5	6	7	9	-	-	5	0
ex-1_82	-	-	-	-	-	-	-	-	4	8	3	-1
ham3	5	-	9	-	-	-	-	5	5	10	5	0
miller_5	-	-	-	-	-	-	-	-	5	10	5	0
3_17	6	-	14	-	-	-	-	6	6	12	4	-2
peres_4	-	-	-	-	-	-	-	-	2	5	2	0
[0,1,14,15,4,5,10, 11,7,9,6,8,12,13,2,3]	-	-	-	7	-	-	-	-	-	-	5	-2
[1,2,3,4,5,6,7,8,9, 10,11,12,13,14,15,0]	4	4	4	4	3[2]	-	-	6	-	-	4	0
[0,7,6,9,4,11,10,13, 8,15,14,1,12,3,2,5]	4	4	4	4	4	4	-	-	-	-	4	0
[6,3,14,13,2,11,7, 10,0,5,8,1,12,15,9,4]	-	-	-	10	-	-	-	-	-	-	8	-2
[0,9,10,5,4,15,14,8, 11,2,6,3,12,7,1,13]	-	-	-	10	-	-	-	-	-	-	7	-3
[6,4,11,0,9,8,12,2, 15,5,3,7,10,13,14,1]	17	21	-	10	13	11	-	-	-	-	10	0
[13,1,14,0,9,2,15,6, 12,8,11,3,4,5,7,10]	14	29	14	9	10	10	-	-	-	-	9	0
[9,7,13,10,4,2,14,3, 0,12,6,8,15,11,1,5]	14	23	-	-	11	10	-	-	-	-	10	0
[6,2,14,13,3,11,10, 7,0,5,8,1,15,12,4,9]	14	19	14	-	11	12	-	-	-	-	9	-2
[0,1,2,3,4,5,6,8,7,9, 10,11,12,13,14,15]	7	9	10	-	7	7	-	-	-	-	7	0
4_49	-	-	36	-	-	-	-	-	-	29	10	-19
aj-e11_81	-	-	-	-	-	-	-	-	-	29	6	-23
hwb5	-	-	-	-	-	-	-	-	-	64	52	-12
mod5d2_17	-	-	-	-	-	-	-	-	-	17	6	-11

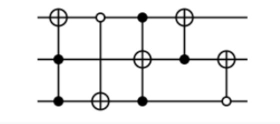
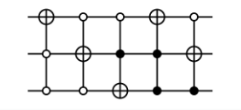
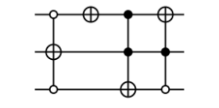
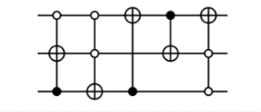
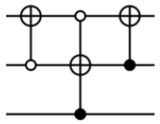
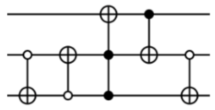
[1] We have experiment on this function by exhaustive method, the minimum gate count is 5, [2] We have experiment on this function by exhaustive method, the minimum gate count is 4

The cost of the KNQTS synthesis circuit is **at least the same** as or **even lower** than its predecessors.

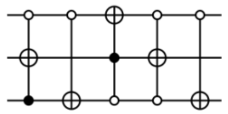
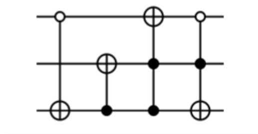
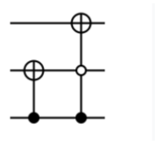
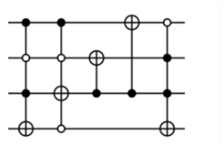
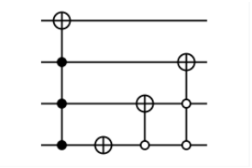
There is a significant improvement when the number of bits increases.

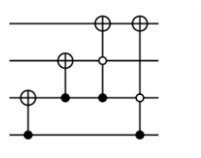
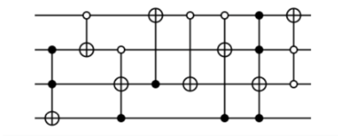
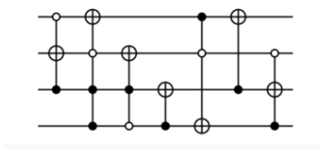
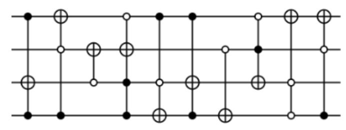
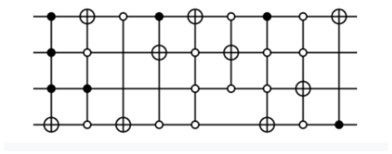
The circuits synthesized by the KNQTS algorithm

[1,0,3,2,5,7,4,6]	
[7,0,1,2,3,4,5,6]	
[0,1,2,3,4,6,5,7]	
[0,1,2,4,3,5,6,7]	
[1,2,3,4,5,6,7,0]	
[3,6,2,5,7,1,0,4]	

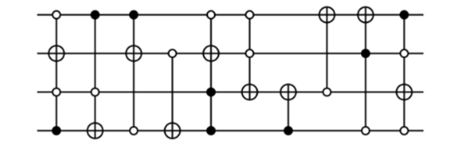
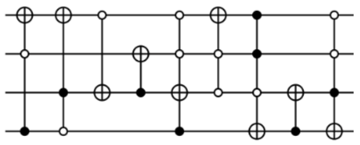
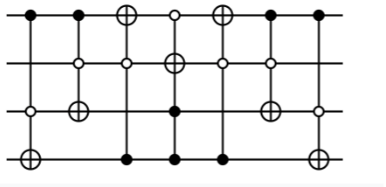
[1,2,7,5,6,3,0,4]	
[4,3,0,2,7,5,6,1]	
[7,5,4,2,0,1,6,3]	
[7,5,2,4,6,1,0,3]	
[4,5,6,1,0,7,2,3]	
ham3	

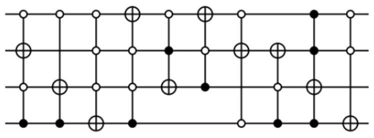
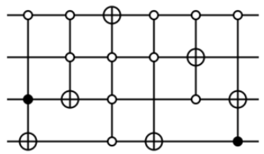
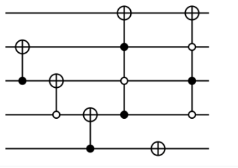
The circuits synthesized by the KNQTS algorithm

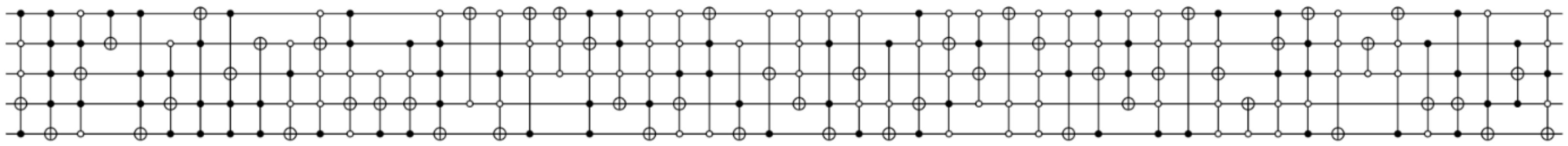
miller_5	
3_17	
peres_4	
[0,1,14,15,4,5,10,11,7,9,6,8,12,13,2,3]	
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,0]	

[0,7,6,9,4,11,10,13,8,15,14,1,12,3,2,5]	
[6,3,14,13,2,11,7,10,0,5,8,1,12,15,9,4]	
[0,9,10,5,4,15,14,8,11,2,6,3,12,7,1,13]	
[6,4,11,0,9,8,12,2,15,5,3,7,10,13,14,1]	
[13,1,14,0,9,2,15,6,12,8,11,3,4,5,7,10]	

The circuits synthesized by the KNQTS algorithm

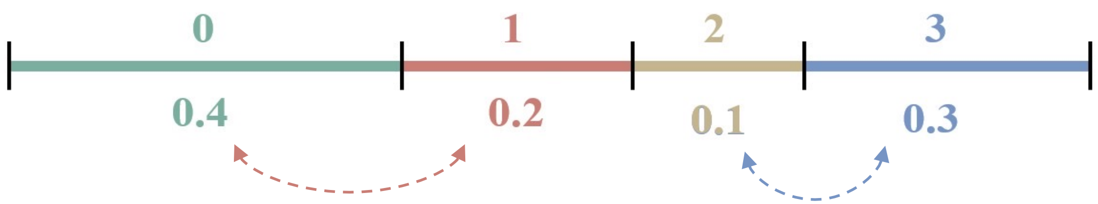
[9,7,13,10,4,2,14,3,0,12,6,8,15,11,1,5]	
[6,2,14,13,3,11,10,7,0,5,8,1,15,12,4,9]	
[0,1,2,3,4,5,6,8,7,9,10,11,12,13,14,15]	

4_49	
[1,2,4,8,0,3,5,6,7,9,10,11,12,13,14,15]	
mod5d2_17	

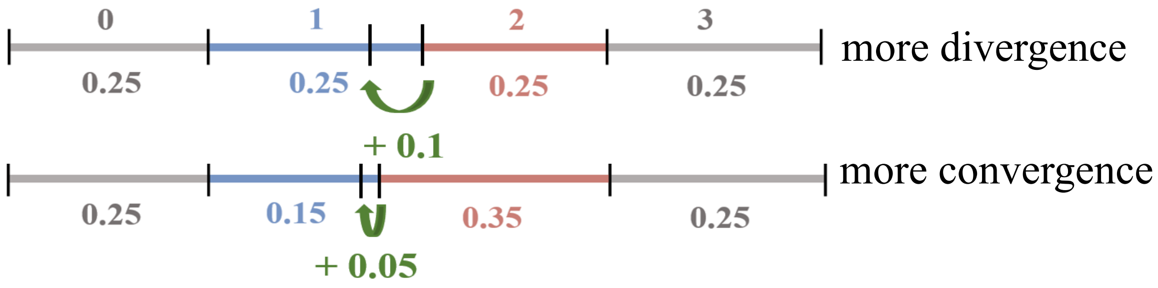
hwb5	

Conclusion

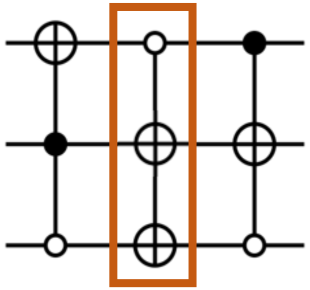
Quantum-Not gate can help escape from local optimum



Knowledge navigation strategy can adaptively adjust the updated parameter



We improves the repair mechanism to make it more reasonable



Conclusion

KNQTS algorithm performs at least the same as or even better than its predecessors.

Stability of over 85% means that our proposed algorithm is sufficiently stable.

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Thank you for listening

KNQTS will be able to avoid premature convergence and prevent falling into a local optimum.

KNQTS is easy to implement and stable.

Our proposed algorithm outperforms the previous algorithms.

We are the first to invent an innovative KNQTS algorithm to significantly optimize reversible circuit synthesis.