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Computer Graphics
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HW2, Q1

1. 1.1. In order to scale this vector, we first need to align it with the line $y = x$. To do so, we perform a rotation of 45 degrees:

$$\begin{bmatrix} cos(x) & -sin(x) \\ sin(x) & cos(x) \end{bmatrix}$$

Now that the vector lays along the line we want, we can scale it along the y-axis (which is now the line $y = x$):

$$\begin{bmatrix} 1 & 0 \\ 0 & \sigma_{diagonal} \end{bmatrix}$$

Now we have to rotate back to our original orientation:

$$\begin{bmatrix} cos(x) & sin(x) \\ -sin(x) & cos(x) \end{bmatrix}$$

In all of these, $x = 45$ degrees, and so $sin(x)$ and $cos(x) = \frac{\sqrt{2}}{2}$. Therefore, I substitute x for both $sin(x)$ and $cos(x)$. So we multiply these three matrices and get:

$$\begin{bmatrix} x^2(1+\sigma) & x^2(-1+\sigma) \\ x^2(-1+\sigma) & x^2(1+\sigma) \end{bmatrix}$$

$x^2 = \frac{1}{2}$, so we can substitute and pull out the constant.

$$\frac{1}{2}\begin{bmatrix} (1+\sigma) & (-1+\sigma) \\ (-1+\sigma) & (1+\sigma) \end{bmatrix}$$

1.2. Geometrically, **q** represents the axis of rotation.

q is the eigenvector because it is “the one axis that is not changed by the rotation” (pg. 126, Marschner).

If $\lambda \neq 1$, then the vector will be stretched in some dimension and so the matrix will no longer represent solely a rotation.