Handling Uncertainty in a Stochastic Wind Farm Simulator

Jack Kennedy
Kevin Wilson, Daniel Henderson

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SQuaRES
Newcastle University

Outline

Motivation

Code Uncertainty (Emulation)

Parameter Uncertainty in Computer Models

Athena Application

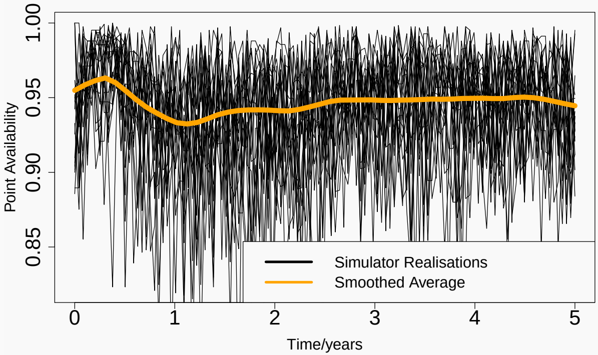
Motivation

Introduction

- Physical experiments with offshore wind farms at best impractical; possibly deadly
- Computer models predict the future when physical experimentation infeasible
- The output of a computer model $f(\cdot)$ depends on the (unknown) inputs $\boldsymbol{x}$
- In a wind farm context, novel designs may be implemented $\implies$ relevant data may not be available
- Perhaps only feasible solution is to **elicit** $\boldsymbol{x}$

Athena Simulator

- Inputs ($\boldsymbol{x}$): Component hazard parameters; maintenance strategy; wind farm topology (and more!)
- Key Outputs: Availability time series; Average availability ($\bar{A}$)



Uncertainties in computer modelling

- Input uncertainty $\pi(\boldsymbol{x}) \implies$ output uncertainty $\pi(f(\boldsymbol{x}))$
- Model is wrong (perhaps it is useful) $\implies$ model discrepancy
- Model might be stochastic $\implies$ output uncertainty EVEN IF inputs known
- Observation error: will typically observe $z = f(\boldsymbol{x}) + \varepsilon$
- If model complex, obtaining $f(\boldsymbol{x})$ for single $\boldsymbol{x}$ could take minutes, hours, weeks ...

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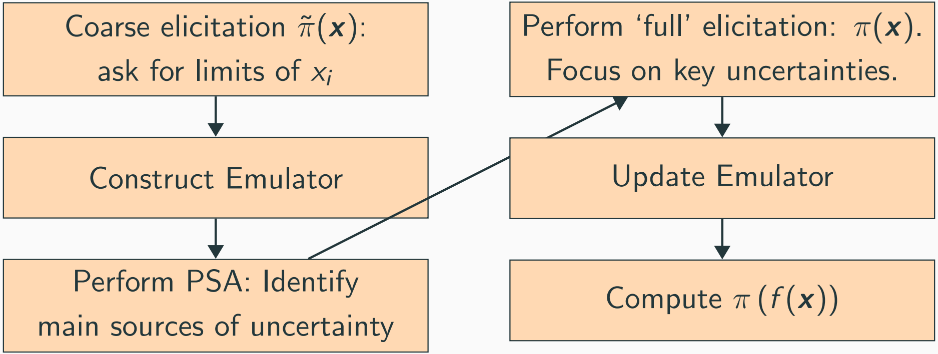
Subjectivism is going to solve all these problems!

Although we all know subjectivism is the best anyway :-)

Process for handling key uncertainties

Model complex $\implies$ not clear which x_i are most important in $f(\boldsymbol{x})$



Could replace $\pi(\boldsymbol{x})$ with a posterior $\pi(\boldsymbol{x}|\boldsymbol{z})$ IF relevant data available

Code Uncertainty (Emulation)

Emulators 101

- Suppose computer simulation is expensive to evaluate, possibly stochastic
- Although I have code to find $f(\boldsymbol{x})$, pragmatically $f(\boldsymbol{x})$ unknown
- Able to obtain a handful of runs $y_i = f(\boldsymbol{x}_i)$, $i = 1 \ldots, n$
- Hopefully $f(\boldsymbol{x})$ $(\mathrm{E}(f(\boldsymbol{x}))$ is "close" to $f(\boldsymbol{x}')$ $(\mathrm{E}(f(\boldsymbol{x}'))$ when $\boldsymbol{x}$ "close" to $\boldsymbol{x}'$

Bayesian solution takes f a priori to be a Gaussian Process

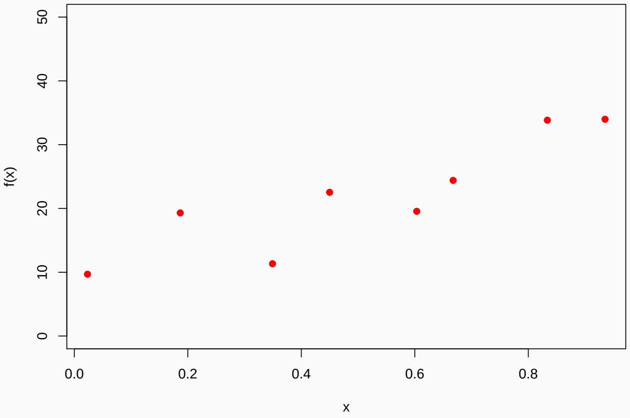
$$f(\cdot) \sim \mathcal{GP}\left\{m(\cdot), C(\cdot,\cdot) + \lambda^2(\cdot)\right\}$$

Emulators 101

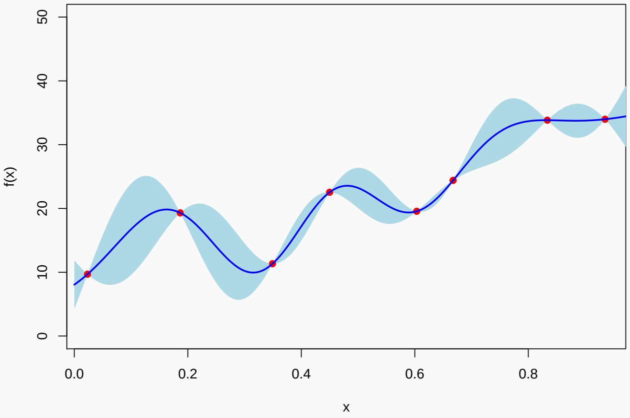
$$f(\cdot) \sim \mathcal{GP}\left\{m(\cdot), C(\cdot,\cdot) + \lambda^2(\cdot)\right\}$$

- $\lambda = 0$ when $f(\cdot)$ deterministic
- $m(\boldsymbol{x}) = \boldsymbol{h}(\boldsymbol{x})^T\beta$: captures global variation in f
- $C(\boldsymbol{x}, \boldsymbol{x}') = \sigma^2 \exp\left\{-(\boldsymbol{x} - \boldsymbol{x}')^T D^{-1}(\boldsymbol{x} - \boldsymbol{x}')\right\}$: local variation, epistemic uncertainty, differentiability
- $\log \lambda^2(\cdot) \sim \mathcal{GP}\left\{m_\lambda(\cdot), C_\lambda(\cdot,\cdot) + \omega^2\right\}$: aleatory uncertainty (noise)
- Emulator is the posterior predictive: $f(\boldsymbol{x})|\boldsymbol{y} \sim \mathcal{N}(M(\boldsymbol{x}), V(\boldsymbol{x}))$

Deterministic Example



Deterministic Example



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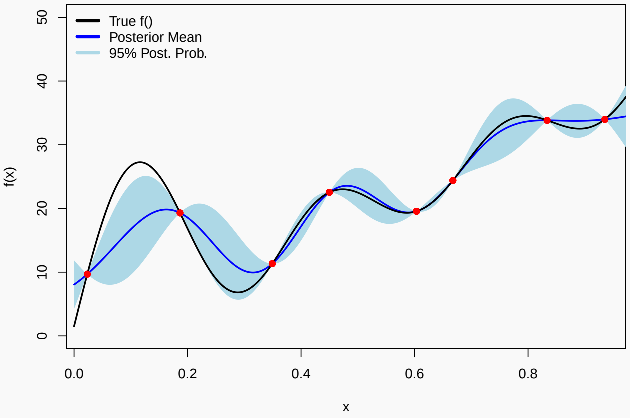
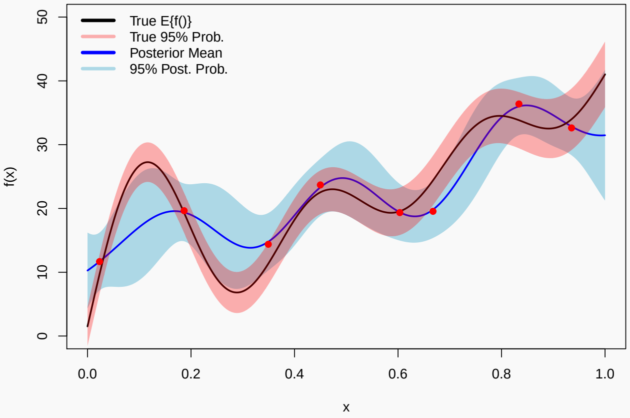
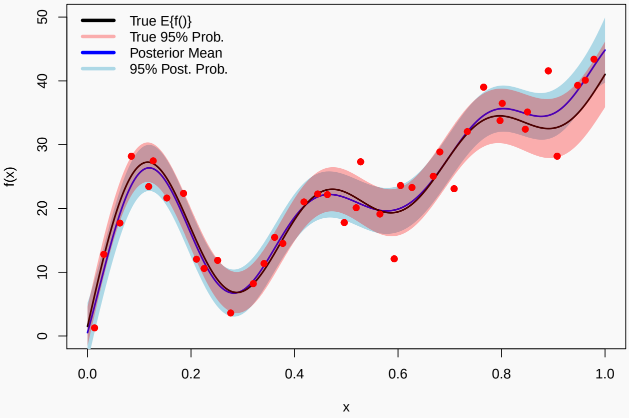


Stochastic Example



Stochastic Example



Stochastic Example

- Many complex stochastic simulators have high noise with low signal
- HetGPs require a lot of "information" to separate signal from noise
- Stochastic simulation is of increasing use to people who aren't statisticians
- Highly topical example: variants of stochastic SIR being used so that humanity survives a massive pandemic
- A few different approaches to signal/noise problem Baker et al. [2020]

Parameter Uncertainty in Computer Models

Parameter Uncertainty

- $f(\cdot)$ effectively "unknown", but I have emulator
- $\boldsymbol{x}$ (parameters of f) also unknown for many complex projects
- E.g. in Athena
  - Generator WT = Nobody Knows
  - An Expert might say:
    The time at which a typical generator begins degrading $\sim \mathit{Gamma}(2, 4)$
- Formulate expert opinion as prior density $\pi(\boldsymbol{x})$
- **How does uncertainty in $\boldsymbol{x}$ induce uncertainty in $f(\boldsymbol{x})$?**

Scenario Analysis

Figure 1: “what if” predictions of a coastal town given different plans for energy generation

“Scenarios” are evaluations of $f(\boldsymbol{x}^{(i)})$ $i = 1, 2, 3, 4$.

How does this take into account $\pi(\boldsymbol{x})$?

Spoiler alert: it doesn’t!

Saltelli and Annoni [2010] explain other dubious practices e.g. OAT

Probabilistic Sensitivity Analysis (Means)

Consider a complex (deterministic) function $Y = f(X)$; $X \sim \pi(x)$

$f(X)$ has a unique ANOVA decomposition

$$f(X) = f_0 + \sum_i f_i(X_i) + \sum_{i<j} f_{ij}(X_{ij}) + \cdots + f_{1\ldots K}(X)$$

$$f_i(x_i) = \mathrm{E}_{X_{-i|i}}\{Y|X_i = x_i\} - f_0$$

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Certain choices of $\pi(\boldsymbol{x}) \implies$ tractable approximation [Oakley and O'Hagan, 2004]

Probabilistic Sensitivity Analysis (Variances)

- Means "decompose" shape of f
- Want to decompose variance of f: assume f deterministic

$$
\begin{aligned}
S_i &= \frac{\mathrm{Var}_{X_i}\left\{f_i(X_i)\right\}}{\mathrm{Var}(Y)} \\
S_{T_i} &= \frac{\mathrm{Var}(Y) - \mathrm{Var}_{X_{-i}}\left\{\mathrm{E}(Y|X_{-i})\right\}}{\mathrm{Var}(Y)} \\
S_{T_1} &= S_1 + S_{12} + S_{13} + S_{123} \text{ for example}
\end{aligned}
$$

Certain choices of $\pi(x) \implies$ tractable approximation [Oakley and O'Hagan, 2004]

Probabilistic Sensitivity Analysis (Variances)

Marrel et al. [2012] propose adjustment of PSA to accommodate stochastic modelsConsider a complex stochastic function $Y = f(X, X_\varepsilon)$

$$\begin{aligned} \mathrm{Var}(Y) &= \mathrm{E}\left\{Y_d(X)\right\} + \mathrm{Var}\left\{Y_m(X)\right\} \\ S_i &= \frac{\mathrm{Var}_{X_{-i}}\left\{Y_m(X_i)\right\}}{\mathrm{Var}(Y)} \\ S_{T_\varepsilon} &= \frac{\mathrm{E}\left\{Y_d(X)\right\}}{\mathrm{Var}(Y)} \end{aligned}$$

S_{T_ε} is "uncontrollable" uncertainty in simulations (c.f. $1 - R^2$)

Athena Application

Emulator of Athena Model

- Varied 6 inputs using Latin Hypercube design to construct emulator
- We aim to model an offshore wind farm with 200 turbines

Parameter	Lower	Upper
Learning Rate	1.5	10
Cable Failure Rate	0.01	1
Cable Repair Rate	0.15	10
Gearbox WT	0.1	1.5
Freq. Conv. WT	0.1	1.5
Generator WT	0.1	1.5

WT = time to wear out/wear out time

Emulator of Athena Model

- $\bar{A} \in (0, 1) \implies$ emulate probit($\bar{A}$)
- In fact, used multilevel emulator

$$\text{probit}(\bar{A}_{200}(x)) = \rho \mathrm{E} \left\{ \text{probit}(\bar{A}_{20}(x)) \right\} + \delta(x) + \varepsilon(x)$$

- Useful when simulator is “very” expensive and cheap approximations are available
- See previous talk for more details

Emulator of Athena Model

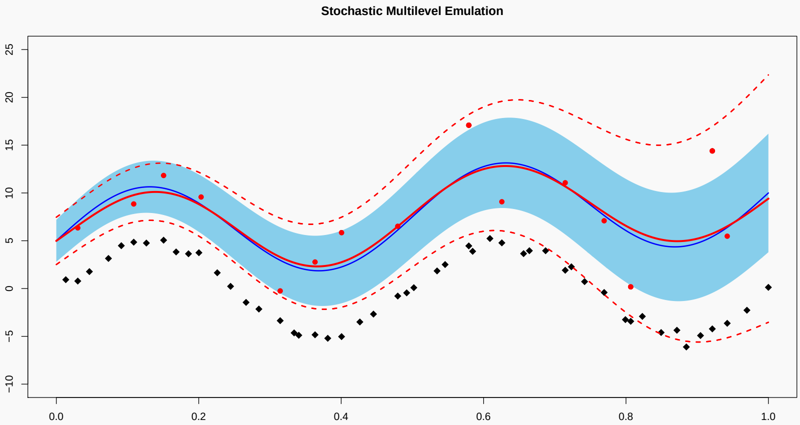


Figure 2: Quick graphical representation of stochastic multilevel emulator.

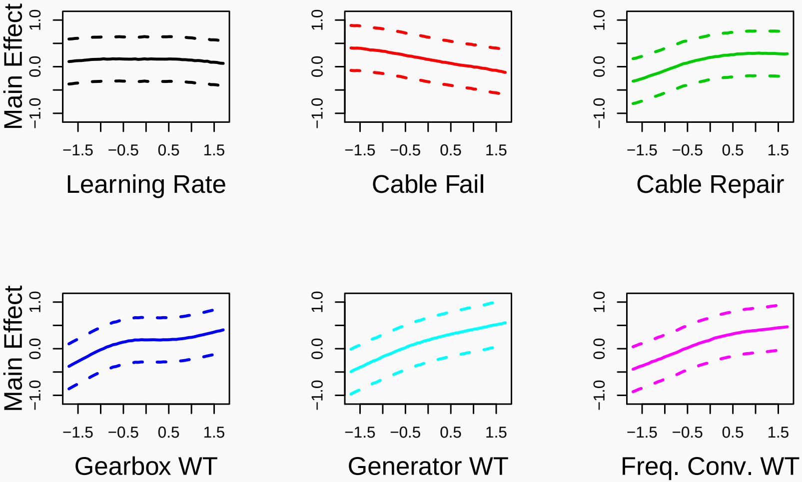
Stochastic PSA with Athena (Variances)

- 6 inputs $X_i^* \sim \mathcal{N}(0, 0.65^2)$
- Want to use more realistic prior in future (expert's "coarse" opinion)
- With emulator computation takes $\sim$ 2 hours (one run Athena $\sim$ 30 mins!)

Param	Learning Rate	Cable Failure Rate	Cable Repair Rate	Gearbox WT	Generator WT	Freq. Conv. WT	S_{T_ε}
$\hat{S}_i$	0.005	0.080	0.113	0.082	0.274	0.252	0.142

Interactions amongst controllable variables account for $\approx 5\%$ variation

Stochastic PSA with Athena (Expectations)



Sensitivity to the Decision

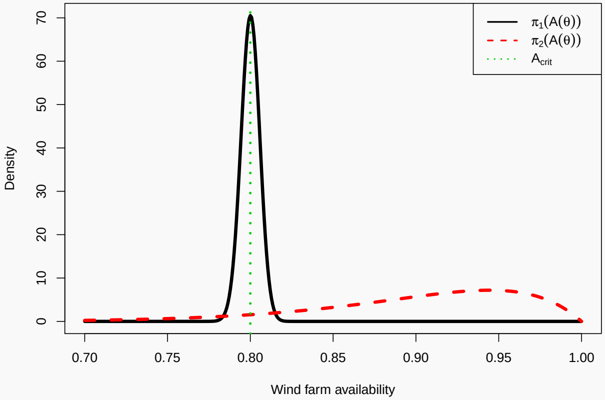
- I can choose to implement a tried & tested component (d_1) or implement a novel technology (d_2)
- I know a lot about the tried & tested component $\implies \pi_1$ "tight"
- I know relatively little about the novel component $\implies \pi_2$ diffuse
- Suppose I need $\bar{A} > A_{\text{crit}} = 0.8$ (say)
- The choice of component depends on the decision criterion

$$P(\bar{A} > 0.8)$$

Sensitivity to the Decision

Clearly d_2 is best although the particular outcome is more uncertain!

Variance is not always the best description of uncertainty



Sensitivity to the Decision

In fact, the S_i from before have a decision theoretic justification

$$EVPI = \mathrm{E}_{x_i}\left[\max_{d\in\mathcal{D}} \mathrm{E}_{\boldsymbol{x}_{-i|i}}\left[U\{d, f(\boldsymbol{x})\}\right]\right] - U^*$$

$$U^* = \max_{d\in\mathcal{D}} \mathrm{E}_{\boldsymbol{x}}\left[U(d, f(\boldsymbol{x})\right]$$

$U(d, f(\boldsymbol{x})) = -(d - f(\boldsymbol{x}))^2 \implies$ recover PSA (variance) and $d^* = \mathrm{E}(f(\boldsymbol{x})|\boldsymbol{y})$

i.e. best guess of $f(\boldsymbol{x})$ is its posterior expectation

If I want to improve guess of $Y = f(\boldsymbol{x})$ I need to "learn" x_i with max S_i

Another one of Oakley [2009]'s brainy ideas

Conclusions

- Emulator reduced computation time from absolutely ages to under a week (most of this was training emulators)
- Athena seems to be approximately additive in mean response and quite smooth
- When I get round to it I will be doing PSA on the variance too
- My uncertainty in turbine performance (especially generator) is driving majority of my uncertainty in wind farm performance
- $\implies$ In elicitation, spend most time/resources on this (because has largest $\hat{S}_i$)

Further Work

- Over the next year(ish) plan to perform a series of elicitations to quantify expert uncertainty w.r.t. an actual problem (using Athena)
- i.e. get $\tilde{\pi}(\boldsymbol{x})$ then use PSA to guide elicitation of $\pi(\boldsymbol{x})$
- Might even throw some decision analysis in there if I have the time

Thanks!

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