Uncertainty Quantification for Offshore Wind Energy

Jack Kennedy
Kevin Wilson, Daniel Henderson

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Introduction

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- **Example**: stochastic windfarm simulation, used to model large, offshore windfarms [1]

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- Sophisticated simulators often computationally intensive
- **Example**: stochastic windfarm simulation, used to model large, offshore windfarms [1]
  - Walney Extension - 87 turbines / 659 MW capacity
  - London Array - 175 turbines / 630 MW capacity
  - Floating offshore wind farms

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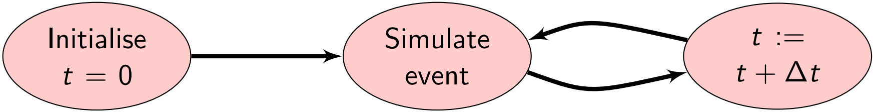

- Inputs: Hundreds - some known; many uncertain

Known	Uncertain
Windfarm topology	Cable failure rates
Simulation length	Hazard function parameters
	Learning Rate

Simulator Description

- Output: Availability trajectories

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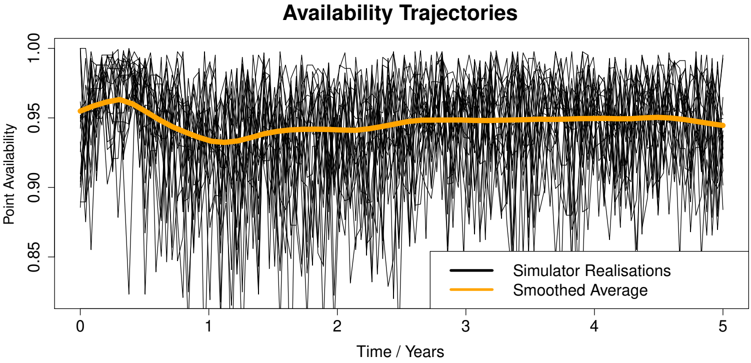

Simulator Description

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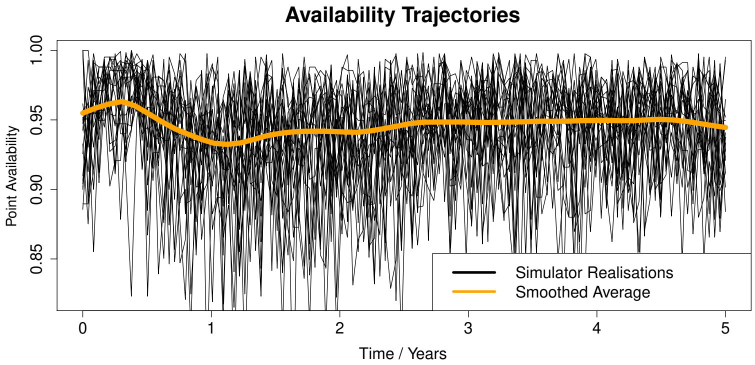

- Average over time period gives "fixed term availability" or $A(\boldsymbol{x})$.
- Profitable windfarm satisfies $A(\boldsymbol{x}) > 97\%$

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No. Turbines	1 Run (s)	10^6 Runs (years)
9	15	0.5
200	120	3.8

- Performing an uncertainty analysis is seemingly impossible!

Elicitation

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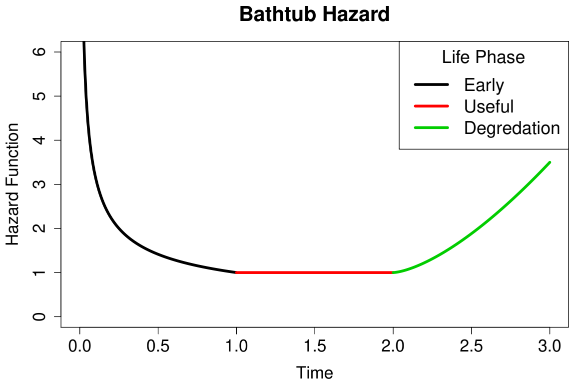
- Lifetimes are known to have certain forms at certain times

$$T_i \sim \mathit{Weibull}(\tau_i, \lambda_i, \kappa_i)$$



Emulators

Recall ... 10^6 runs takes ~ 4 years

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- An emulator is a statistical (Bayesian) **surrogate** model of the simulator or **"model of the model"**.

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- Utilise Gaussian Process (GP) emulators [2] facilitate computation
- An emulator is a statistical (Bayesian) **surrogate** model of the simulator or **"model of the model"**.
- Crucially: emulators are incredibly cheap to evaluate

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$$C_0(\boldsymbol{x}_i, \boldsymbol{x}_j) = \sigma^2 \exp\left\{ -\sum_{k=1}^{K} \left(\frac{x_i^k - x_j^k}{\theta_k}\right)^2 \right\} + \lambda^2 \delta_{ij}$$

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$$\begin{pmatrix} \boldsymbol{Y}_t \\ \boldsymbol{Y}_p \end{pmatrix} \sim \mathcal{N}\left\{ \begin{pmatrix} \boldsymbol{m}_0(x_t) \\ \boldsymbol{m}_0(x_p) \end{pmatrix}, \begin{pmatrix} \Sigma_{TT} & \Sigma_{TP} \\ \Sigma_{PT} & \Sigma_{PP} \end{pmatrix} \right\}$$

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Then

$$\boldsymbol{Y}_p | \boldsymbol{Y}_t = \boldsymbol{y}_t \sim \mathcal{N}\{\boldsymbol{m}^*(\boldsymbol{x}), \Sigma^*\}$$

Where

$$\boldsymbol{m}^*(\boldsymbol{x}) = \boldsymbol{m}_0(\boldsymbol{x}_p) + \Sigma_{PT}\Sigma_{TT}^{-1}(\boldsymbol{y}_t - \boldsymbol{m}_0(\boldsymbol{x}_t))$$

$$\Sigma^* = \Sigma_{PP} - \Sigma_{PT}\Sigma_{TT}^{-1}\Sigma_{TP}$$

Emulation (design)

- Experimental Design: Latin Hypercube over 6 inputs, 50 datapoints:

Farm Characteristics	Initial Hazard Function Parameters
Learning rate	Generator Wearout Onset
Cable Failure rate	Gearbox Wearout Onset
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- Will emulate the mean and variance of $A(\boldsymbol{x})$

Emulation

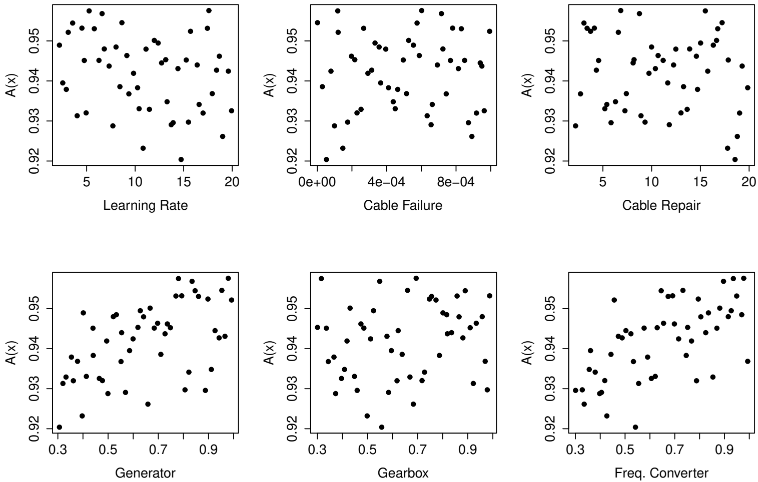

Figure 1: Mean of $A(\boldsymbol{x})$ against inputs

Emulation

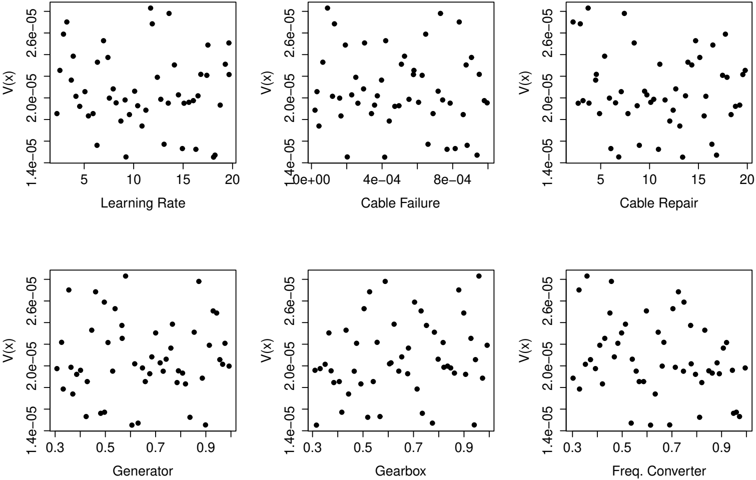

Figure 2: Variance of $A(\boldsymbol{x})$ against inputs

Emulation

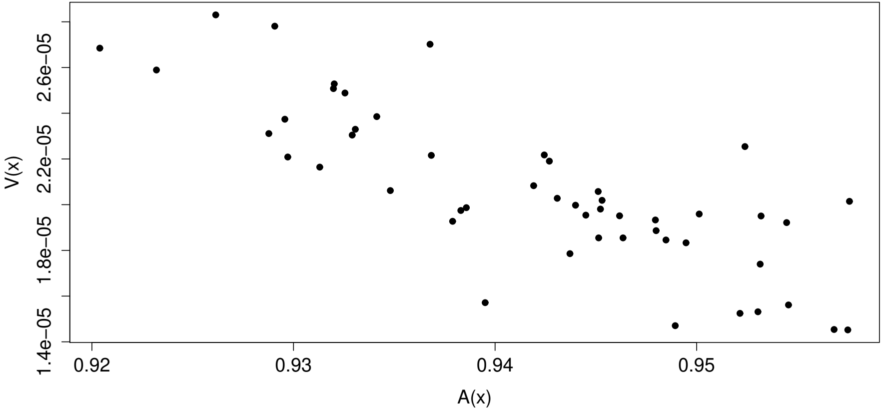

Figure 3: Variance of $A(\boldsymbol{x})$ against the mean of $A(\boldsymbol{x})$

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Result: Super cheap way to obtain realisations of windfarm's fixed term availability

Emulator Validation

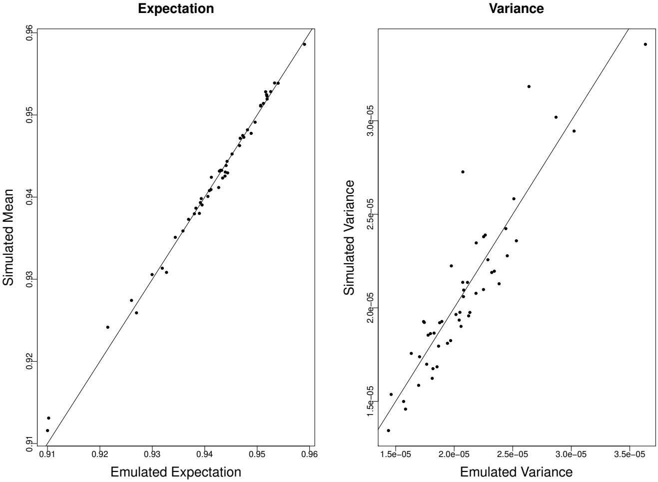

Figure 4: Observed vs predicted values of simulator mean and variance on independently generated validation data

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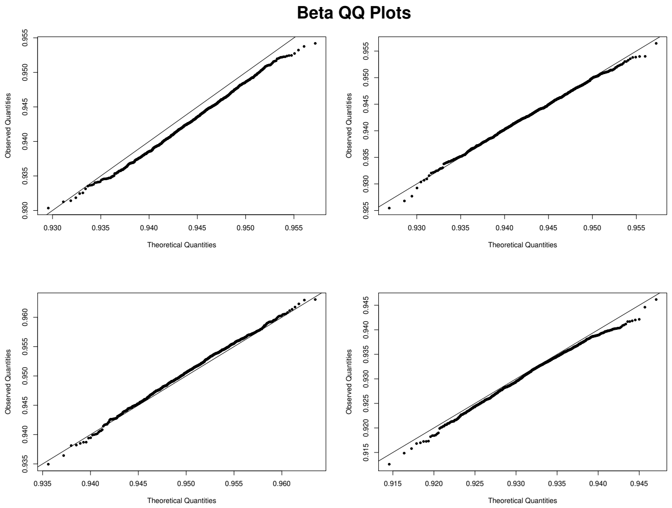

Figure 5: Beta QQ plots based on emulated mean and variance of validation data

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- Perform an elicitation on input parameters to assess $\mathrm{P}(A > 97\%)$ - probably for floating offshore wind turbines
- In performing the UQ, we need to identify important inputs and elicit uncertainty over these inputs; this can be done using expert knowledge and mathematical techniques

References

MC Kennedy and A O'Hagan. Bayesian calibration of computer models. *Journal Of The Royal Statistical Society Series B-Statistical Methodology*, 63:425–450, 2001. ISSN 1369-7412.

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