15-150 Assignment 10

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Section K
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2: Better Sequences

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1. `update` is implemented in `betterseq.sml`.
2. `mapIdx` is implemented in `betterseq.sml`.
3. `extractSomes` is implemented in `shrubseq.sml`.

6: Implementing the Game

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1. `apply_effect` is implemented in `pokemon.sml`.
2. `apply_damage` is implemented in `pokemon.sml`.
3. `apply_attack` is implemented in `pokemon.sml`.
4. `reset_stats` is implemented in `pokemon.sml`.
5. `make_move` is implemented in `pokemon.sml`.
6. `available_pokemon` is implemented in `pokemon.sml`.
7. `moves` is implemented in `pokemon.sml`.
8. `status` is implemented in `pokemon.sml`.

7: Sequence Thingies

1. The value of `reduce (op +) 5 s` $= 5 * 2^n$ when the value of `s` is the all-zero integer sequence $\langle 0, 0, ..., 0\rangle$ of length 2^n.

   Proof. By strong induction on `n`.
   Let the proposition stated above be denoted by $P(n)$. We will show that $\forall n \geq 0. P(n)$ holds.

   Base case: $\mathbf{n = 0} \Rightarrow$ `length s =` $2^0 = 1$. Considering $P(0)$...
   With $n = 0$, we have $s = \langle 0\rangle \Rightarrow$
   `reduce (op +) 5 s => reduce (op +) 5 <0> => (op +) (nth 0 <0>, 5)`, by inspection of `reduce`. The operation `op +` is defined to be a total function, so we may say that the above evaluates to: `0 + 5 => 5 x` 2^0 (0th position of `s` is 0). Hence, $P(0)$ holds.

   IH: Assume $P(k)$ holds $\forall k$ such that $1 \leq k \leq n$. Also, for convenience, let `++` denote `(op +)`.

   IS: WWTS that $P(n)$ holds, so consider $P(k + 1)$...
   `reduce ++ 5 s => ++ (reduce ++ 5 s1, reduce ++ 5 s2)`, by inspection of `reduce`.
   Note that the length of `s` is 2^{k+1}, which means that the call to `split` implies that `s1,s2` are of length 2^k. Therefore, `(++)(reduce ++ 5 s1, reduce ++ 5 s2) => (++)(5 x` 2^k`, 5 x` 2^k`)`, by our IH. Evaluating further by use of the aforementioned totality of `(op +)`, we have: `5 x` 2^k `+ 5 x` 2^k `=> 5 x` 2^{k+1}, and we're done.

   Hence, by the Strong Principle of Mathematical Induction, we may conclude that $\forall n \geq 0. P(n)$ holds. □

2. Work and span of `reduce g z s`, when `g` is a constant-time function value, `z` is a value, and `s` is a sequence value of length 2^n.

   $W(0) = c_0$
   $W(1) = c_1$ (`g` is constant-time)
   $W(n) = c_0 + c_1 + 2 * W(n - 1)$
   $\Rightarrow W(n) = c_2 + 2 * W(n - 1)$
   $\Rightarrow W(n)$ is O(2^n)

   $S(0) = k_0$
   $S(1) = k_1$ (`g` is constant-time)
   $S(n) = k_0 + k_1 + 2 * W(n - 1)$
   $\Rightarrow S(n) = k_2 + S(n - 1)$
   $\Rightarrow S(n)$ is O(n)

3. **Prove** by induction on the length of `s` that:

   ```
   mapreduce f g z s = reduce g z (map f s)
   ```

   Proof. By strong induction on length of `s`.
   Let the above proposition be denoted by $P(n)$, where n is the length of `s`. We will show $\forall n \geq 0. P(n)$.

   Base case: $\mathbf{n = 0} \Rightarrow$ length `s = 0`. Considering $P(0)$...
   LHS: `mapreduce f g z s => z`, by the definition of `mapreduce`.
   RHS: `reduce g z (map f s) => reduce g z s`, by definition of `map`, and `reduce g z s => z`,

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by definition of `reduce`. Hence, `mapreduce f g z s = reduce g z (map f s) = z`, and thus $P(0)$ holds.

Inductive Step: Let k be an arbitrary integer; assume $P(k)$ holds $\forall k$ such that$1 \leq k \leq n-1$, and assume. WWTS $P(k+1)$ is also true. Considering $P(k+1)$...
LHS: `mapreduce f g z s \Rightarrow g(mapreduce f g z s1, mapreduce f g z s2)`, by observation of the wildcard case ($k \geq 1 => k+1 \geq 2$, and hence the length is not 0 or 1).

By the definition of `split`, `val (s1,s2) = split s`, where the lengths of `s1` and `s2` $\leq n$, which means that the induction hypothesis applies ($k \leq n-1$). Then notice, by our IH,
`g(mapreduce f g z s1, mapreduce f g z s2) =>`
`g(reduce g z (map f s1), reduce g z (map f s2))`.

RHS: Let `split s = (s1,s2)`, and notice that `map f s = join(map f s1, map f s2)`, since `map` is a total function. Using this, we have:
`reduce g z (map f s) => reduce g z join(map f s1, map f s2)`.
Finally, by the definition of `reduce`, this then evaluates to
`g(reduce g z (map f s1), reduce g z (map f s2))`. Hence, the LHS = RHS, and thus $P(k+1)$ holds.

By the Strong Principle of Mathematical Induction, we may conclude that $\forall n \geq 0. P(n)$ holds. □

8: Alphabeta Pruning

1. `F2`, `G2`, and `next_move` are defined in `alphabeta.sml`.

9: Jamboree

1. `splitMoves`, `Fjam`, `GJam`, and `next_move` are defined in `jamboree.sml`.