

$$3. \quad L(\sigma^2 | x) = \prod \left(\frac{1}{\sqrt{2\pi\sigma^2}} \right)^{1/2} \exp\left[-\frac{x^2}{2\sigma^2}\right]$$

$$L = \left(\frac{1}{2\pi\sigma^2} \right)^{1/2} \exp\left[-\frac{1}{2\sigma^2} \sum x^2\right]$$

$$\begin{aligned} \ell(\sigma^2 | x) &= -\frac{1}{2} \ln\left(\frac{1}{2\pi\sigma^2}\right) - \frac{1}{2\sigma^2} \sum x^2 \\ &= -\frac{1}{2} \ln(1) - \left[\frac{1}{2} \ln(2\pi) + \ln(\sigma^2) \right] - \frac{1}{2\sigma^2} \sum x^2 \\ &= \frac{1}{2} \ln(2\pi) + \frac{1}{2} \ln(\sigma^2) - \frac{1}{2\sigma^2} \sum x^2 \end{aligned}$$

$$\frac{d}{d\sigma} \ell(\sigma^2 | x) = 2\left(\frac{1}{2}\right) \frac{1}{\sigma} - \frac{-4\sigma}{4\sigma^3} \sum x^2$$

$$0 = -\frac{1}{\sigma} + \frac{1}{\sigma^3} \sum x^2$$

$$\frac{1}{\sigma^3} \sum x^2 = \frac{1}{\sigma}$$

$$\sigma = \frac{1}{\sigma^2} \sum x^2$$

$$\sigma^2 = \frac{1}{n} \sum x^2$$