$$P(X=k|\lambda) = \frac{\lambda^{k} e^{-\lambda}}{k!}$$

b) $$L(\lambda | k_1 \dots k_n) = \prod_{j=1}^{n} \frac{\lambda^{k_j} e^{-\lambda}}{k_j!}$$

$$\ell(\lambda | k_1 \dots k_n) = \ln\left(\prod_{j=1}^{n} \frac{\lambda^{k_j} e^{-\lambda}}{k_j!} \right)$$

$$= \sum_{j=1}^{n} \ln \left(\frac{\lambda^{k} e^{-\lambda}}{k_j!} \right)$$

$$= \sum_{j=1}^{n} \left[\ln(\lambda^{k_j}) + \ln(e^{-\lambda}) - \ln(k_j!) \right]$$

$$= \sum_{j=1}^{n} \left[k_j \ln(\lambda) - \lambda - \ln(k_j!) \right]$$

$$= -n\lambda + \ln(\lambda) \sum_{j=1}^{n} k_j - \sum_{j=1}^{n} \ln(k_j!)$$

$$\frac{d}{d\lambda}(\lambda | k_1 \dots k_n) = \frac{d}{d\lambda}\left(-n\lambda + \ln(\lambda) \sum_{j=1}^{n} k_j - \sum_{j=1}^{n} \ln(k_j!)\right)$$

$$\frac{d}{d\lambda}\ell(\lambda | k_1 \dots k_n) = -n + \frac{1}{\lambda} \sum_{j=1}^{n} k_j$$

$$-n + \frac{1}{\lambda} \sum_{j=1}^{n} k_j = 0$$

c) $$\boxed{\hat{\lambda} = \frac{1}{n} \sum_{j=1}^{n} k_j}$$

$n=10$

$$\hat{\lambda} = \frac{1}{10}(55) = \frac{11}{5}$$

$$\frac{d}{d\lambda}\left(-n + \frac{1}{\lambda} \sum_{j=1}^{n} k_j\right) = -\frac{1}{\lambda^2} \sum_{j=1}^{n} k_j < 0 \text{ max}$$

negative everywhere b/c $\lambda > 0$ & $k > 0$