ME 351A: Inviscid Fluid Mechanics Notes

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Fall 2012

Contents

Chapter 1

What is a fluid? - 9/25/2012

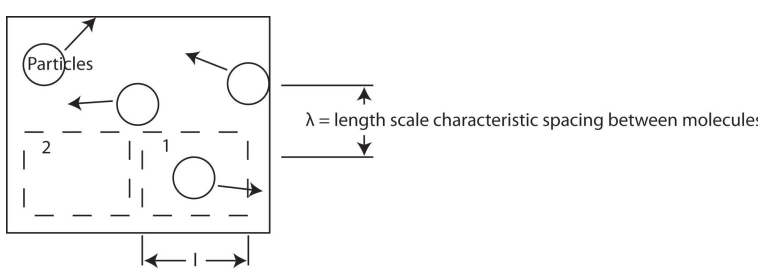
- Commonsense definition: fluids ”fill their container”, fluids ”takes shape of container
- Studying fluid mechanics requires more mathematical precision.

1.1 Mathematically precise definition of fluid

(1) A fluid is a substance that deforms* continuously under the action of any shear* stress.

We will use continuum assumption to describe fluid flows. Namely, fluid/flow properties are continous and can be defined pointwise in space.

Consider: A microscopic view of actual fluids



- Individual particles undergo random motion in a fluid

Suppose You want to quantify the fluid density, ρ: $\rho(\bar{x}, t)$ which is a function of space and time.

If I use sampling windows spanning lxl:

- $\rho \neq 0$ in window 1
- $\rho = 0$ in window 2

Result: To study fluids at the macro level, assume that sampling windows span size l such that:

$$\lambda << l << \eta$$

where η represents the smallest important scale of the flow system. This will keep my system continuous.

- Note that η is set by flow geometry
- η_{Gas} is between 10^{-7} and 10^{-8} meters
- We need a $\lambda << l$ so properties vary continuously
- $l << \eta$ is sued so we can discriminate variations in properties.

In practice: do not need to actually define l for most engineering problems. Usually can be confident that such a scale exists.

!Warning Might not be able to apply continuum assumption under the following cases:

- Microfluidics can be problematic if η is very small
- Rarified systems (ie: in space), intermolecualar spacing, λ, might get very large. This can be a problem too.
- Study of fluid at molecular scale is field on its own called Kinetic Theory or Physical Fluid Dynamics

1.2 Fluid Properties

Scalars:

- Zero order tensor
- no directional information
- time, t; pressure, p; density, ρ; kinematic viscosity ν

Vectors:

- Order 1 tensor
- 1 item of directional information
- 1 subscript associated with it.
- Position $\vec{x} = (\underline{\mathrm{x}}) = x_1\hat{e}_1 + x_2\hat{e}_2 + x_3\hat{e}_3 = x_i\hat{e}_i$; velocity $\vec{u}$, vorticity $\vec{\omega}$

  Note: Einstein summation convention: sum over repeated indicies.

  Therefore: $x_i\hat{e}_i = \sum\limits_i x_i\hat{e}_i$

Unit Vectors: Note that in Cartesian coordinates, unit vectors are: $\hat{i}, \hat{j}, \hat{k}$ for x,y,z directions respectively. For velocity, components are oriented in the u,v,w direction.

2nd Order Tensors

- 2 directional components
- 2 subscripts
- Velocity gradient tensor $\nabla\vec{u} = (\nabla\vec{u})_{ij}$ the ∇ operator creates a tensor of order 1 higher than the initial variable.

Note on Tensors: $(\nabla\vec{u})_{ij}$ contains an i and a j component. i refers to the row and derivative operator. j refers to the column and velocity component. Expanded, the operator looks like this:

$$\nabla\vec{u} = \begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_2}{\partial x_1} & \frac{\partial u_3}{\partial x_1} \\ \frac{\partial u_1}{\partial x_2} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_3}{\partial x_2} \\ \frac{\partial u_1}{\partial x_3} & \frac{\partial u_2}{\partial x_3} & \frac{\partial u_3}{\partial x_3} \end{pmatrix}$$

!Fact: for any general 2nd order tensor $\underline{\underline{A}}$ can decompose as:

$$\underline{\underline{A}} = \frac{1}{2}\left(\underline{\underline{A}} + \underline{\underline{A}}^T\right) + \frac{1}{2}\left(\underline{\underline{A}} - \underline{\underline{A}}^T\right)$$

- For $\nabla\vec{u}$: we write: $\nabla\vec{u} = \frac{1}{2}\left(\nabla\vec{u} + (\nabla\vec{u})^T\right) + \frac{1}{2}\left(\nabla\vec{u} - (\nabla\vec{u})^T\right) = \underline{\underline{S}} + \underline{\underline{\Omega}}$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} - \frac{\partial u_i}{\partial x_j}\right)$$

- Where S is the train rate tensor and Ω is the rotation rate tensor.
- Note that KUNDU defines these with a different notation. He has a different sign and multiplier.

Chapter 2

Types of Fluid Motion - 9/27

2.1 Understanding Velocity gradient tensor