Topics in Interference

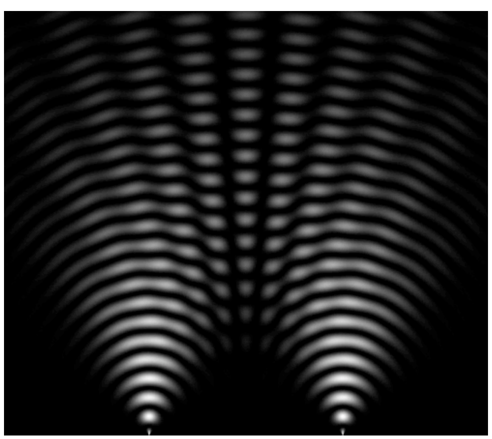
- Basics of Interference
  - Interference patterns
  - Vector and scalar mathematical descriptions
- Interferometers and classical experiments
  - Wavefront Splitting
  - Amplitude Splitting
- Applications
  - DVDs
  - Laser Interferometer Gravitational-Wave Observatory

Interference, Conceptually

Interference is the result of superposition, i.e. the vector sum of two or more fields.

Superposition itself is a result of the linearity of the wave equation.

Interference



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Basics of Optical Interference

- Recall principle of Superposition
- Scalar description:
  - Corresponds to interaction of lightwaves yielding a resultant intensity deviating from the sum of the component irradiances
- Vector description:

$$\vec{\mathbf{E}}^2 = \vec{\mathbf{E}}_1^2 + \vec{\mathbf{E}}_2^2 + 2\vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2$$

$$\vec{\mathbf{E}}_1(\vec{r},t) = \vec{\mathbf{E}}_{01} \cos(\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}} - \omega t + \varepsilon_1)$$

$$\vec{\mathbf{E}}_2(\vec{r},t) = \vec{\mathbf{E}}_{02} \cos(\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}} - \omega t + \varepsilon_2)$$

Combining Polarization States

$$I_{12} = \vec{\mathbf{E}}_{01} \cdot \vec{\mathbf{E}}_{02} \cos \delta$$

Phase difference, $\delta = \vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}} - \vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}} + \varepsilon_1 - \varepsilon_2$

- Perpendicular:

$I_{12} = 0$, therefore $I = I_1 + I_2$

- Parallel (useful for many applications):

$$I_1 = \langle \vec{\mathbf{E}}_1^2 \rangle_T = \frac{E_{01}^2}{2}$$

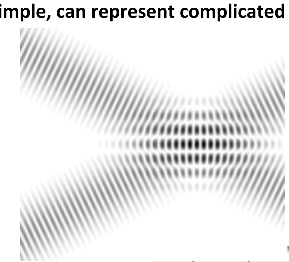
$$I_2 = \langle \vec{\mathbf{E}}_2^2 \rangle_T = \frac{E_{02}^2}{2}$$

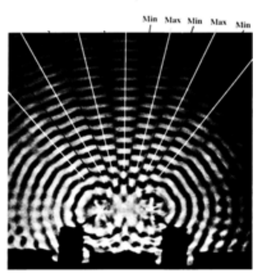
$$I_{12} = 2\sqrt{I_1 I_2} \cos \delta$$

Interference, Visually

Oil On Concrete
(complicated)

Plane and Spherical Waves
(simple, can represent complicated cases)





Interference in terms of Irradiance (scalar description)

$$I = \epsilon v \langle \vec{\mathbf{E}}^2 \rangle_T$$

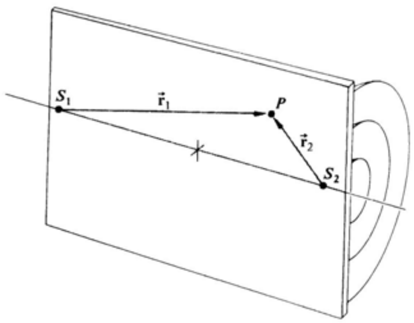
$$I = I_1 + I_2 + I_{12}$$

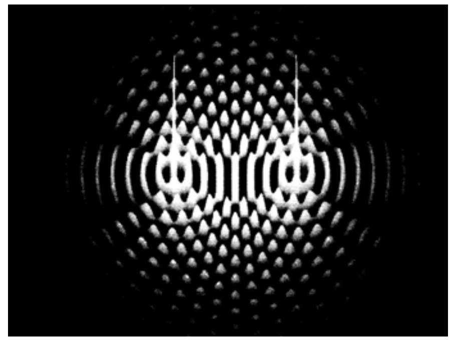
$$I_1 = \langle \vec{\mathbf{E}}_1^2 \rangle_T$$

$$I_2 = \langle \vec{\mathbf{E}}_2^2 \rangle_T$$

$$I_{12} = 2\langle \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 \rangle_T$$

Interference Fringes for 2 Point Sources





Fresnel-Arago Laws describe conditions in which interference of polarized light occurs

1. Two orthogonal, coherent $\mathcal{P}$ -states cannot interfere in the sense that $I_{12} = 0$ and no fringes result
2. Two parallel, coherent $\mathcal{P}$ -states will interfere in the same way as natural light
3. The two constituent orthogonal $\mathcal{P}$ -states of natural light cannot interfere to form a readily observable fringe pattern even if rotated into alignment.

Total Constructive and Destructive Interference

- Irradiance is maximized and minimized as a function of phase angle.

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$\delta = 0, \pm 2\pi, \pm 4\pi, \ldots$$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$\delta = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots,$$

Light sources must be coherent for fringe patterns to be observed

- Recall that coherence means that waves have a constant relative phase
- Temporal Coherence
  - Interval of time in which wave resembles sinusoid
- Spatial Coherence
  - Space across wavefront with constant properties
  - Incoherence caused when different points on a source emit light of variable phase

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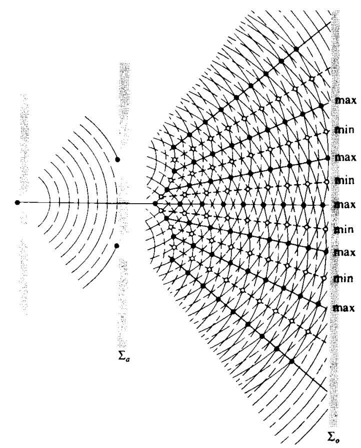
Wavefront-Splitting Interferometers

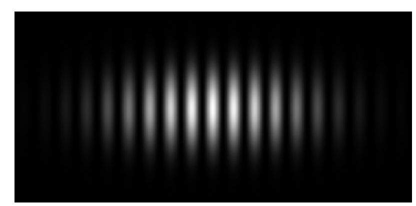
- Mechanism that splits coherent source wavefront into separate wavefronts that interfere with each other
- Single wavefront effectively split into multiple wavefronts
- Most well known example:

  Young's Double Slit Experiment

Young's Experiment

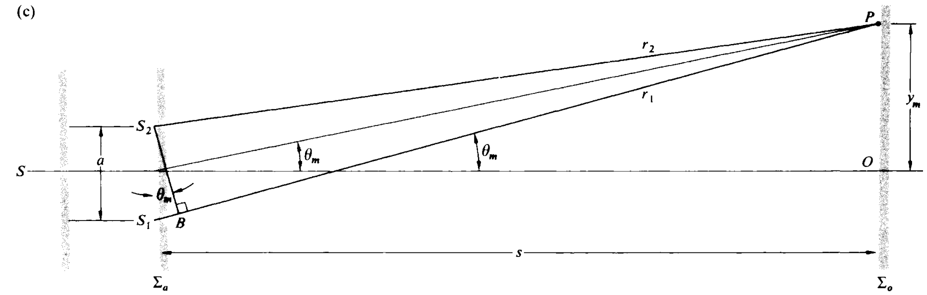
- Light interference experiment using double slits to split wavefront of single coherent source into two interfering wavefronts





Σ_a: Single source to screen distance
Σ_o: Double slit to screen distance

Young's Experiment-Quantitative



$(r_1 - r_2) = a\sin\theta \approx a\theta \approx \frac{a}{s}y$ Where: $\theta \approx \frac{y}{s}$

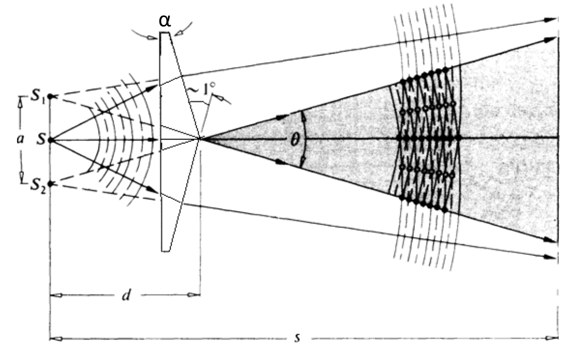
$r_1 - r_2 = m\lambda$ Where: m is an integer

$y_m \approx \frac{s}{a}m\lambda$ or $\theta_m = \frac{m\lambda}{a}$ Relation between slit separation and position of interference bright fringes!

9.16 A stream of electrons, each having an energy of 0.5 eV, impinges on a pair of extremely thin slits separated by 10^{-2} mm. What is the distance between adjacent minima on a screen 20 m behind the slits? ($m_e = 9.108 \times 10^{-31}$ kg, 1 eV $= 1.602 \times 10^{-19}$ J.)

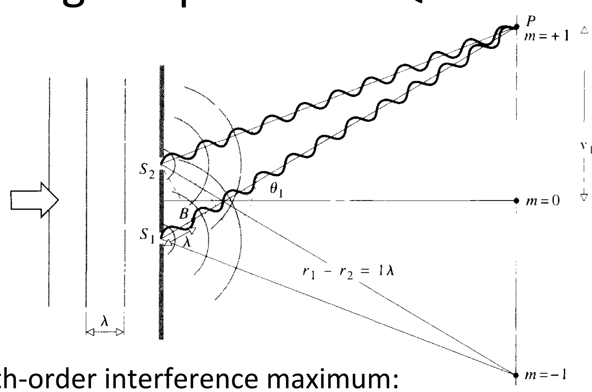
Other Wavefront-Splitting Interferometers

Fresnel's Double Prism:



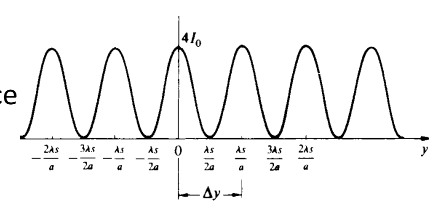
S_1 and S_2 are virtual sources of S generating the interference pattern

Young's Experiment-Quantitative



For the mth-order interference maximum:
$a \sin \theta_m = m\lambda$ or $\theta_m \approx m\lambda/a$

Maxima Separation: $\Delta y \approx \frac{s}{a}\lambda$

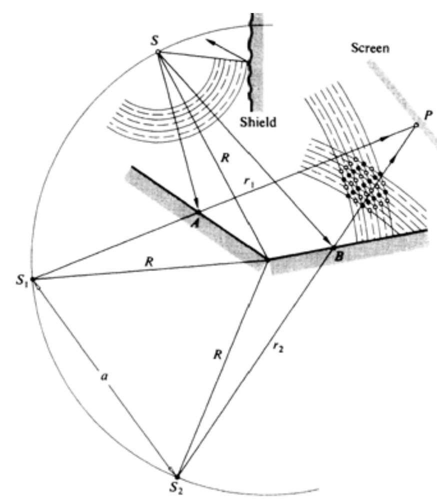


Resultant Irradiance
(w/o diffraction)

$$I = 4I_0 \cos^2 \frac{ya\pi}{s\lambda}$$

Other Wavefront-Splitting Interferometers

Fresnel's Double Mirror:



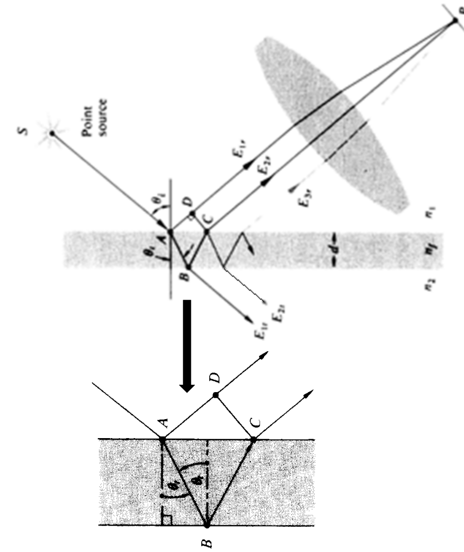
S_1 and S_2 are images of source S generated by the two mirrors

Virtual sources S_1 and S_2 generate the interference wavefronts

Amplitude-Splitting Interferometers

- Mechanism that splits an incoming lightwave into separate waves of lower amplitude that recombine at different location
- Interference results when lower amplitude waves are brought back together

Dielectric Films-Fringes of Equal Inclination



Optical path length difference: $\Lambda = 2n_f d \cos\theta_t$

For films immersed in SINGLE medium (n-n_{film}-n setup): Additional phase shift of π arises from reflection, going from low to high index!

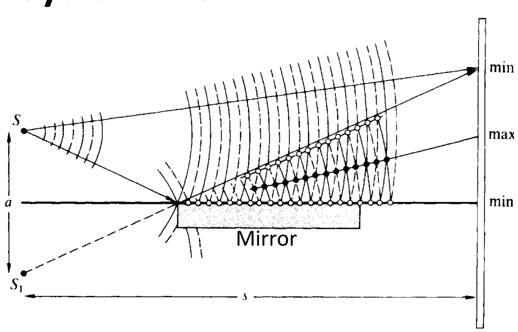
Interference Maximum: $d \cos\theta_t = (2m+1)\frac{\lambda_f}{4}$

Interference Minimum: $d \cos\theta_t = 2m\frac{\lambda_f}{4}$

$\lambda_f = \lambda_0/n_f$

Other Wavefront-Splitting Interferometers

Lloyd's Mirror:



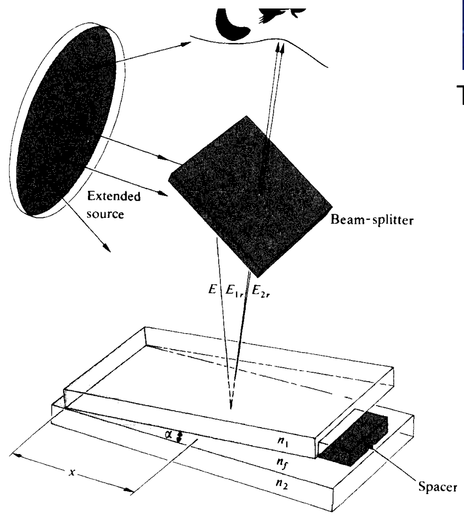
Interference between S and its reflected virtual source image, S_1

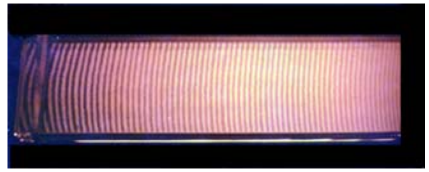
Dielectric Films

- **Thin Film**: Film with thickness on the order of given wavelength from incident EM wave
- **Optical Flat**: An extremely flat piece of glass, used with monochromatic light to determine the flatness of other optical surfaces by interference

Dielectric Films-Fringes of Equal Thickness

Air Wedge:





Thickness at any particular point:
d=xtan(α)=xα

Interference Max:

$$\left(m + \tfrac{1}{2}\right)\lambda_0 = 2n_f d_m = 2\alpha x_m n_f$$

$$x_m = \left(\frac{m + 1/2}{2\alpha}\right)\lambda_f$$

Interference Minimum:

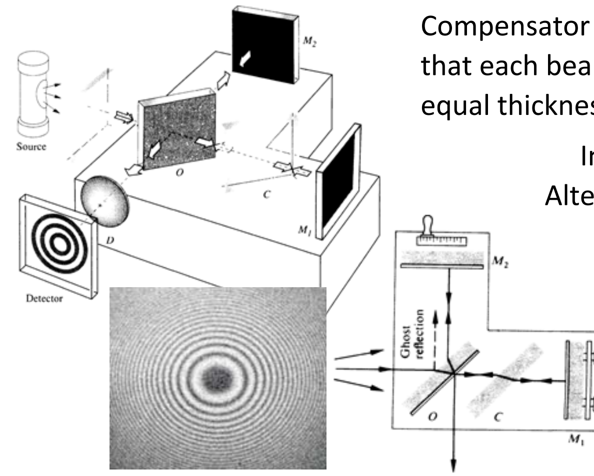
$$x_m = \left(\frac{m + ½}{2\alpha}\right)\lambda_f$$

Fringe Separation:

$$\Delta x = \lambda_f / 2\alpha$$

Mirrored Interferometers

Michelson Interferometer: Interferometer that divides waves using a beam splitter into mirrors. Waves are then reflected and recombined, forming an interference pattern



Compensator plate C is inserted so that each beam will pass through equal thicknesses of glass!

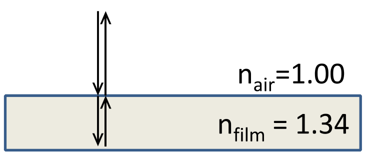
Interference Pattern:
Alternate bright/dark rings

Can determine distance traveled by mirror!

$$\Delta d = N(\lambda_0/2)$$

N: Number of fringes past reference point

9.26 A soap film surrounded by air has an index of refraction of 1.34. If a region of the film appears bright red ($\lambda_0 = 633$ nm) in normally reflected light, what is its minimum thickness there?



The lowest **non-zero** thickness occurs at **m=0** for the **maximum interference** condition:

$$d \cos \theta_t = (2m + 1)\frac{\lambda_f}{4}$$

At Normal Incidence: θ_t=0°

So:

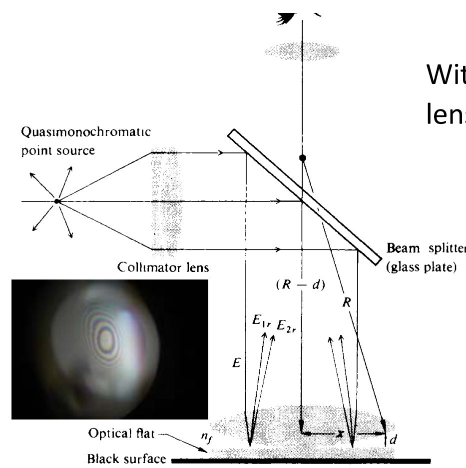
$$d_{min} = (0+1)\frac{\lambda_0}{4n_f}$$

$$d_{min} = (0+1)\frac{633\ nm}{4 * 1.34}$$

$$\boxed{d_{min} = 118.10\ nm \text{ or } 118 * 10^{-9}\ m}$$

Dielectric Films-Fringes of Equal Thickness

Newton's Rings: Series of concentric, nearly circular fringes formed from interference



With R = Radius of curvature of convex lens:

$$x^2 = R^2 - (R-d)^2 = 2Rd - d^2$$

R >> d: $x^2 = 2Rd$

Interference Max/ Bright Ring:

$$2n_f d_m = \left(m + \tfrac{1}{2}\right)\lambda_0$$

$$x_m = \left[\left(m + \tfrac{1}{2}\right)\lambda_f R\right]^{1/2}$$

Interference Min/ Dark Ring:

$$x_m = (m\lambda_f R)^{1/2}$$

9.37* Suppose we place a chamber 10.0 cm long with flat parallel windows in one arm of a Michelson Interferometer that is being illuminated by 600-nm light. If the refractive index of air is 1.000 29 and all the air is pumped out of the cell, how many fringe-pairs will shift by in the process?

Optical Pathlength Difference:

$$\Delta d = (n_{air} - n_{vacuum}) * 10.0\ cm$$

$$\Delta d = (1.00029 - 1) * 10.0\ cm$$

$$\Delta d = 2.9 * 10^{-3}\ cm\ or\ 2.9 * 10^{-5}\ m$$

Again use: $\Delta d = N(\lambda_0/2)$

Rearrange in terms of N: $N = \frac{\Delta d * 2}{\lambda_0}$

$$N = \frac{2.9 * 10^{-5}\ m\ * 2}{600 * 10^{-9}\ m}$$

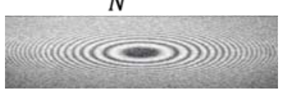
$$\boxed{N = 96.6 \text{ fringes or } 97 \text{ whole fringes}}$$

9.35 A Michelson Interferometer is illuminated with monochromatic light. One of its mirrors is then moved 2.53×10^{-5} m, and it is observed that 92 fringe-pairs, bright and dark, pass by in the process. Determine the wavelength of the incident beam.

Easy enough, start with:

$$\Delta d = N(\lambda_0/2)$$

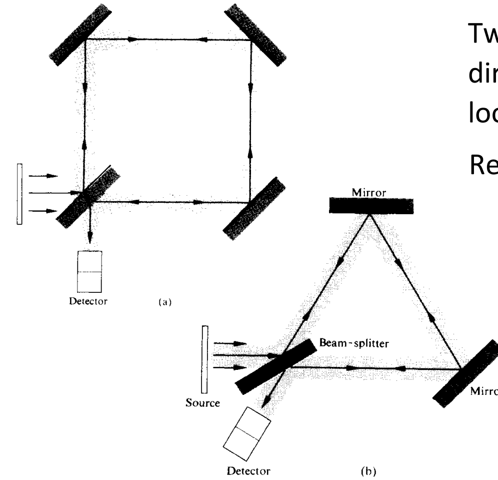
Rearrange in terms of λ_0: $\lambda_0 = \frac{\Delta d * 2}{N}$



$$\boxed{\lambda_0 = 5.50 * 10^{-7} m \text{ or } 550\ nm}$$

Mirrored Interferometers

Sagnac Interferometer: Interferometer that consists of beam splitters and totally reflecting mirrors that allow waves to travel identical opposite paths before recombining

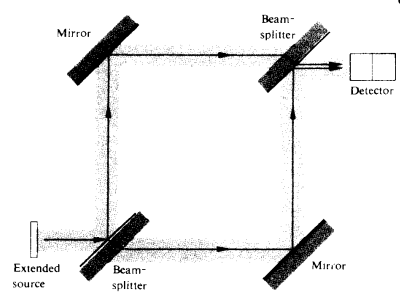


Two identical but oppositely directed paths that form closed loops before recombining

Relatively easy to align

Mirrored Interferometers

Mach-Zehnder Interferometer: Interferometer that consists of two beam splitters and two totally reflecting mirrors. Waves travel along two separate paths before

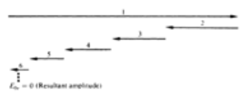


Relatively difficult to align

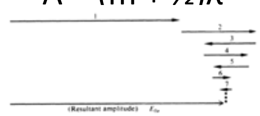
- Optical path difference can be introduced by slight tilt of one of the beamsplitters
- Object interposed in one beam will alter optical path length difference

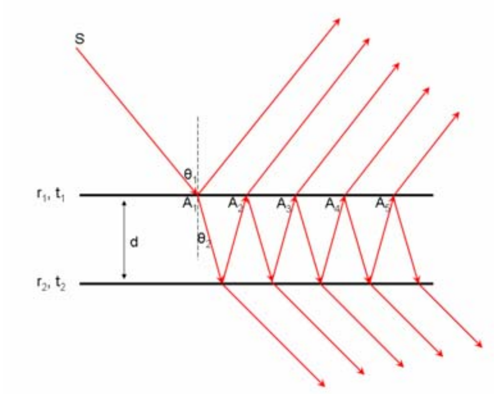
Multiple Beam Interference

- Two special cases:
  - $\Lambda = m\lambda$

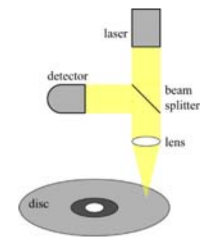


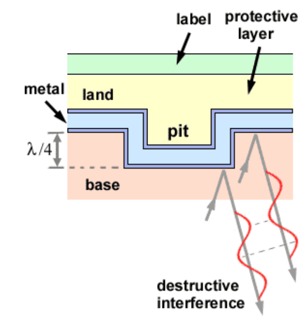
  - $\Lambda = (m + ½)\lambda$





DVD/Blu-Ray





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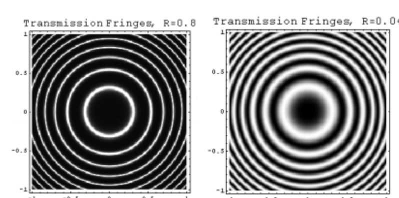
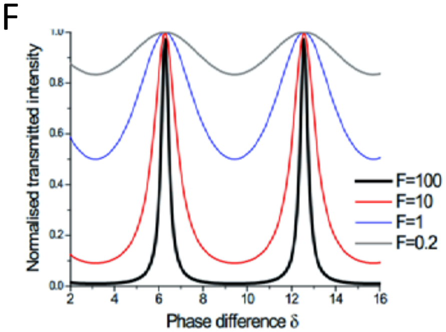
  - Wavelength Multiplexing, DBR Mirrors
  - Gravitational Wave detection

Multiple Beam Interference

- Phase difference: $\delta = 2\pi/\lambda * \Lambda$
- $A(\delta) = I/I_{max} = 1/(1 + F \sin^2(\delta/2))$
  - $F = 4R^2/(1-R^2)^2$
- FWHM: $\gamma = 2\delta_{1/2} = 4/\sqrt{F}$

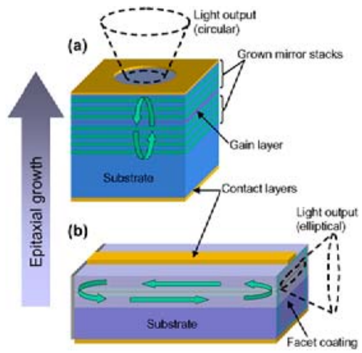


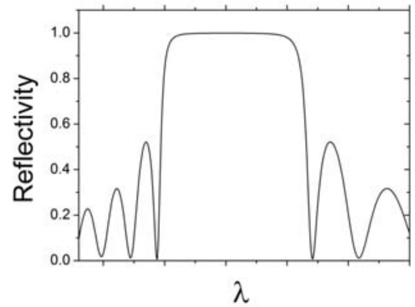


Distributed Bragg Reflector

- Extremely high reflectivity mirrors made of dielectrics

- Makes an excellent cavity



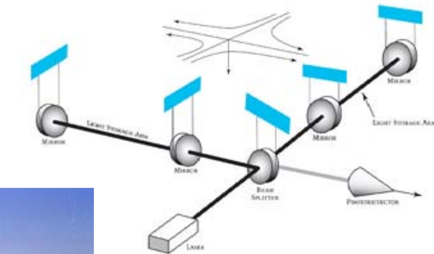


LIGO:

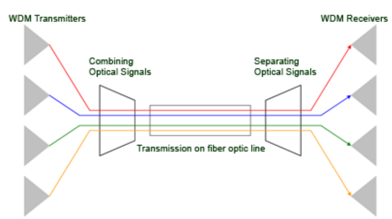
Michelson Interferometer + Fabry-Perot Cavity

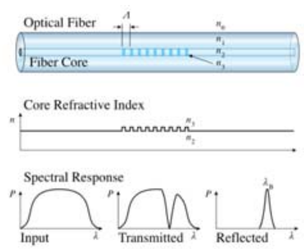
- 4 km long arms, x75 reflections

- Sensitive to 1 part in 10^{21}



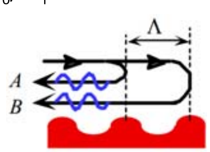
Wave-Division Multiplexing

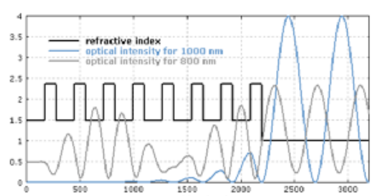




- Series of Fresnel reflections

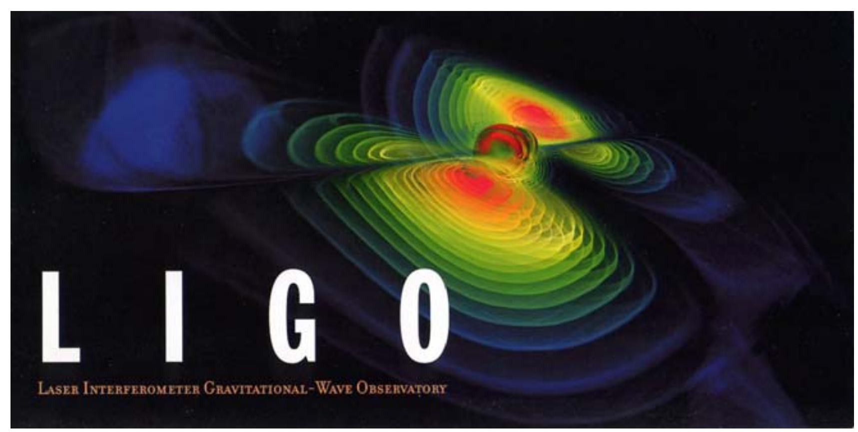
- $t_i = \lambda_0/4n_i$





LIGO:

Interferometry taken to another level



LIGO: Miserable Failure

But there's hope: