Intro to Modern Optics uBook

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Fall 2012

Contents

Part I

Physics of Light

Chapter 1

Wave Motion

Light acts in some ways as a wave and in other ways like a particle (does that make it a wavicle?). Understanding the basics of waves is useful for studying light. We'll examine these first as one dimensional waves to develop some first principles and then expand these fundamentals to apply to two dimensional and three dimensional waves.

1.1   One-Dimensional Waves

This is the easiest way to consider a wave. Consider perturbing a string or a spring by suddenly jerking one end upward and then back to it's original position. This will cause a perturbation to travel through the object. The actual material does not permanently deform as it returns to its original state and thus the disturbance advances and not the medium. The disturbance of a wave is a function of position and time and is denoted by the symbol ψ:

$$\psi(x,t) = f(x,t)$$

The profile of the wave is determined by setting time equal to zero as in:

$$\psi(x,t)|_{t=0} = f(x,0) = f(x)$$

When considering time, a wave travels at a specific velocity v. The distance a wave travels is simply vt. An alternate frame of reference, S' can be used which travels along with the pulse in time. In this frame, the pulse always looks identical to the profile when $t=0$. Here, the coordinate is x' rather than x such that $\psi = f(x')$. The relationship between x and x' is: $x' = x - vt$. To describe the wave as someone would observe from the original reference frame, S, we can now write that:

$$\psi(x,t) = f(x - vt)$$

This is the general form of the one-dimensional wavefunction.

1.1.1　The Differential Wave Equation

The differential wave equation is a linear, homogeneous, second order, partial differential equation that is usually taken as the defining expression for physical waves in a lossless medium. The one-dimensional form of the wave equation is derived from the initial relation ship of $\psi(x,t) = f(x')$. Derivatives are taken twice (see Hecht, 4th edition pages 13-14 for details) to bring to the final result of:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \tag{1.1}$$

This is the wave equation for undamped systems that do not contain sources in the region under consideration. Damping effects would be considered using a $\frac{\partial \psi}{\partial t}$ term.

1.2　Harmonic Waves

Harmonic waves have a sinusoidal profile. Any wave shape can be synthesized by a superposition of harmonic waves, so they're pretty useful. The simplest profile of a harmonic wave can be expressed as:

$$\psi(x,t)|_{t=0} = \psi(x) = A\sin kx = f(x)$$

where A is the amplitude of the wave and k is a positive constant known as teh propagation number. Transforming this into a progressive wave yields:

$$\psi(x,t) = A\sin k(x - vt) = f(x - vt) \tag{1.2}$$

The symbol λ represents the wavelength (also known as spatial period) of the wave and is related to k by the following equation:

$$k = 2\pi/\lambda \tag{1.3}$$

The amount of time it takes for one complete wave to pass a stationary observer is defined as the temporal period, τ. Propagation number, wave velocity, and temporal period are related by the following relationship:

$$kv\tau = 2\pi$$

it also follows that

$$\tau = \lambda/v$$

The temporal frequency, ν, is the number of waves per unit time (often measured in Hertz) and is related to the above terms under the following equations:

$$\nu \equiv 1/\tau \tag{1.4}$$

$$v = \nu\lambda \tag{1.5}$$

Other related useful terms are the angular temporal frequency, ω, and the wave number (spatial frequency), κ, which are defined respectively as:

$$\omega \equiv 2\pi/\tau = 2\pi\nu \tag{1.6}$$

$$\kappa \equiv 1/\lambda \tag{1.7}$$

Another important note is that no wave is monochromatic, meaning that it has perfect frequency. All waves fall into a band of frequencies. When that band is small, the wave is termed quasimonochromatic.

1.3 Phase and Phase Velocity

Wave equations are often written in the form:

$$\psi(x,t) = A\sin(kx - \omega t + \epsilon) \tag{1.8}$$

Wherein the portion inside the sine term consists of the position of the wave, kx, the time state of the wave, ωt, and a constant, ϵ that defines the initial phase of the wave. Without the initial phase, the function would always be zero at the origin of space and time.

Note once again that ω is the rate of change of phase with time:

$$|(\frac{\partial \psi}{\partial t})_x| = \omega$$

the rate of change of phase with distance keeping t constant is k:

$$|(\frac{\partial \psi}{\partial x})_t| = k$$

and the phase velocity, v, is the speed at which the wave propagates in space:

$$(\frac{\partial x}{\partial t})_\psi = \pm\frac{\omega}{k} = \pm v$$

1.4 The Superposition Principal (basics)

Since the differential wave equation is a linear partial differential equation, it holds that the sum of two individual solutions to the wave equation is also a solution to the wave equation. When two separate waves overlap in space, the resulting disturbance at each point in the region of overlap is the algebraic sum of the individual constituent waves at that location.

Waves are said to be in-phase when their phase angles are identical and can be out of phase to a limit of having a phase angle difference of π. Out of phase waves give rise to interference.

1.5   The Complex Representation

Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$, is often a mathematically optimal way to express harmonic waves since operations such as taking a derivative and multiplying functions is much easier. It is often most convenient to express the harmonic wave as:

$$\psi(x,t) = Ae^{i(\omega t - kx + \epsilon)} \tag{1.9}$$

An important note is that while the imaginary portion of the function is kept out of convenience through calculations, the real part of the equation is the actual expression of the wave. This is only done after obtaining the final result of all calculations.

1.6   Phasors and the Addition of Waves

Phasors are a useful abstraction for understanding waves. Phasor notation contains the amplitude and current phase angle of the wave. Phase angle is the angle by which the wave is offset from its reference state. Phasors are expressed as

$$A\angle\phi$$

where A is the maximum amplitude of the wave and ϕ is its phase angle.

When combining wavefunctions, phasors can be used similarly to vectors. The wavefunctions being summed are added head to tail in order to determine the resultant vector.

1.7   Plane Waves

A plane wave is the simplest example of a three-dimensional wave, but it is extremely useful as all other three-dimensional waves can be described as a combination of plane waves.

Plane waves travel along a propagation vector $\vec{k}$ whose magnitude is the propagation number, k, which has already been described in terms of harmonic waves. The equation of a plane, r, which is perpendicular to $\vec{k}$ is:

$$\vec{k} \cdot \vec{r} = a$$

where a is a constant.

The general equation of a harmonic plane wave in Cartesian coordinates is:

$$\psi(x,y,z,t) = Ae^{i(k_x x + k_y y + k_z z \mp \omega t)}$$

1.8   The Three-Dimensional Wave Equation

The three-dimensional wave equation is extremely similar to the 1-dimensional version. The only difference is that three spatial variables are taken into account. The 3-D wave equation takes on the form:

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial\psi}{\partial t^2} \tag{1.10}$$

where ∇ is the Laplacian operator: $\nabla \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

1.9 Spherical Waves

Spherical waves represent a point source radiating outward or a spherical shell radiating inward. The harmonic spherical wave equation is given as:

$$\psi(r,t) = \left(\frac{\mathcal{A}}{r}\right) \cos k(r \mp vt) \tag{1.11}$$

1.10 Cylindrical Waves

Cylindrical waves do not have a clean solution to their differential wave equation. Bessel's equation can be used to approximate cylindrical waves of large radii:

$$\psi(r,t) \approx \frac{\mathcal{A}}{\sqrt{r} \cos k(r \mp vt)}$$

This equation best approximates what happens to a plane wave that encounters a long, narrow slit.

1.11 Glossary

amplitude: The maximum disturbance of a wave.

angular temporal frequency: The number of phase angle changes per unit time, denoted as ω.

harmonic waves: A wave that can be represented using sine or cosine curves.

in-phase: Multiple waves having a phase-angle difference of zero are in phase. The disturbance of the waves sums maximally causing a much greater intensity resultant wave.

initial phase (ε): The angle which is the constant contribution to the phase arising at the wave generator. This is independent of how far in space or how long in time the wave has traveled.

longitudinal wave: A wave in which the medium is displaced in the direction of the motion of the wave.

monochromatic: A wave which travels at constant frequency.

out-of-phase: Multiple waves having a phase-angle difference of 180° are said to be out of phase. The waves interfere with each other such that the resultant wave disturbance is minimized.

phase velocity: The speed at which a wave profile moves. Denoted as $v = \frac{\omega}{k}$.

phasor: An abstraction useful in expressing a harmonic wave in terms of its amplitude, A and phase, ϕ as $A \angle \phi$.

plane wave: A planar wave that is perpendicular from a direction vector, $\vec{\mathbf{k}}$.

propagation number: A positive constant denoted as k used in studying harmonic waves which ensures correct units inside the sine function and can change the period of the sine wave. When λ is defined as the wavelength, $k = 2\pi/\lambda$.

spacial frequency: The number of waves per unit length, denoted by κ. Synonymous with wave number.

superposition principle: The principle in which multiple waves traveling along the same path are summed.

temporal frequency: The number of waves per unit time, denoted as ν. Often takes on units of Hertz (Hz).

temporal period: Denoted as τ. This is the amount of time it takes for one complete wave to pass a stationary observer.

transverse wave: A wave in which the medium is displaced in a direction perpendicular to that of the motion of the wave

traveling wave: A wave whose crest travels across particles.

wavefront: The surfaces of a three-dimensional wave that join all points of equal phase.

wave number: The number of waves per unit length, denoted by κ. Synonymous with spacial frequency.

1.12   Important Equations

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\psi(x, t) = A \sin k(x - vt) = f(x - vt)$$

$$k = 2\pi/\lambda$$

$$\nu \equiv 1/\tau$$

$$v = \nu\lambda$$

$$\omega \equiv 2\pi/\tau = 2\pi\nu$$

$$\kappa \equiv 1/\lambda$$

$$\psi(x,t) = A\sin(kx - \omega t + \epsilon)$$

$$\psi(x,t) = Ae^{i(\omega t - kx + \epsilon)}$$

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial \psi}{\partial t^2}$$

$$\psi(r,t) = \left(\frac{\mathcal{A}}{r}\right)\cos k(r \mp vt)$$

1.13   Homework Problems

Due October 4, 2012.

Problems from Hecht Optics Chapter 2: numbers: 4, 13, 17, 18, 22, 32 Solutions were hand written and are shown on the following pages.

2.4) Given: • Waves (harmonic)

    • 0.5s between crests

    • disturbance takes 1.5s to travel 4.5m

Find: • Frequency

    • period (assume temporal period.)

    • wavelength. (λ)

velocity

$$V = \frac{4.5m}{1.5s} = 3.0 \, m/s$$

temporal period

$$\tau = 0.5s$$

$$\tau = \frac{\lambda}{V} \rightarrow \quad \lambda = \tau V = 0.5s * 3.0 \, m/s = 1.5m$$

$$\lambda = 1.5m$$

frequency

$$\nu = 1/\tau = \frac{1}{0.5s} = 2/s \quad \nu = 2/s = 2Hz$$

~~Frequency: 2s~~

Frequency: 2Hz
Wavelength: 1.5m
Period: 0.5s

2.13) Given: • Figure of transverse wave @ $t=0$

• $V = 20.0 \, m/s$

• $A = 0.020 \, m$ (inspection from figure)

• $\lambda = 4.0 \, m$ (" " ")

(a) Find: Wavelength

Wavelength, $\lambda = 4.0 \, m$

(b) Find: Frequency $V = \nu\lambda \Rightarrow \nu = \frac{V}{\lambda}$

$\nu = \frac{20.0 \, m/s}{4.0 \, m} = 5 \, Hz$ $\nu = 5.0 \, Hz$

(c) Write down wave function of disturbance.

$\omega = 2\pi\nu = \cancel{10 Hz}\pi$ $\frac{10\pi \, rad}{s}$ $k = \frac{2\pi}{\lambda} = \frac{1}{2m}\pi$

$\psi(x,t) = \underbrace{(0.020 \, m)}_{A} \cos\left(\underbrace{\tfrac{1}{2m}x}_{kx} - \underbrace{10\pi\tfrac{rad}{s} t}_{\omega} - \underbrace{\pi}_{\varepsilon \text{ (phase determined by inspection)}} \right)$
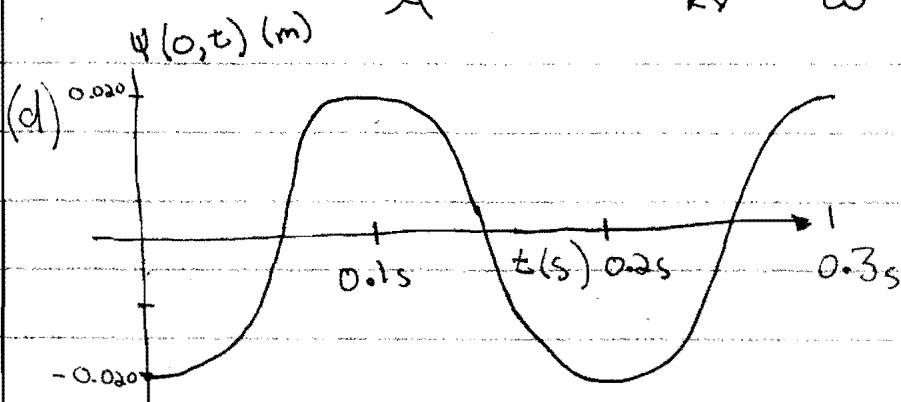
(d)

2.17

Given: $\Psi(x,t) = (30.0 \text{ cm}) \cos[(6.29 \frac{rad}{m})x - (20.0 \frac{rad}{s})t]$

with labels: A (pointing to 30.0 cm), k (pointing to 6.29 rad/m), +direction ω (pointing to 20.0 rad/s)

Find: • Frequency (a) (ν)

• Wavelength (b) (λ)

• Period (c) (τ)

• Amplitude (d) (A)

• Phase velocity (e) (v)

• direction of motion (F) (sign on omega)

(a) $\omega = 2\pi\nu$ $\qquad \nu = \frac{\omega}{2\pi} = \frac{20 \text{ rad/s}}{2\pi \text{ rad}} = \frac{10}{\pi} \text{ Hz}$ $\qquad \nu = \frac{10}{\pi} \text{ Hz}$

(b) $k = \frac{2\pi}{\lambda}$ $\qquad \lambda = \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{6.28 \text{ rad/m}} = 1 \text{ m}$ $\qquad \lambda = 1 \text{ m}$

(c) $\tau = \frac{1}{\nu} = \frac{1}{10/\pi \text{ Hz}} = \frac{\pi}{10} \text{ s}$ $\qquad \tau = \frac{\pi}{10} \text{ s}$

(d) $A = 30.0 \text{ cm}$ (by inspection) $\qquad A = 30.0 \text{ cm}$

(e) $v = \nu\lambda = \frac{10}{\pi \text{ s}} \times 1 \text{ m} = \frac{10 \text{ m}}{\pi \text{ s}}$ $\qquad v = \frac{10}{\pi} \frac{\text{m}}{\text{s}}$

(F) The wave moves in the positive x-direction.

2.18 Show that: $\Psi_{(x,t)} = A \sin[k(x-vt)]$ is a solution to the differential wave equation.

$$\Psi(x,t) = A \sin[kx - kvt]$$

$$\frac{\partial \Psi}{\partial x} = Ak \cos(kx - kvt) \qquad \frac{\partial \Psi}{\partial t} = Akv \cos(kx - kvt)$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -Ak^2 \sin(kx - kvt) \qquad \frac{\partial^2 \Psi}{\partial t^2} = -Ak^2 v^2 \sin(kx - kvt)$$

Assuming $\frac{\partial^2 \Psi}{\partial t^2} \neq 0$:

$$\frac{\frac{\partial^2 \Psi}{\partial x^2}}{\frac{\partial^2 \Psi}{\partial t^2}} = \frac{-Ak^2 \sin(kx-kvt)}{-Ak^2 v^2 \sin(kx-kvt)} = \frac{1}{v^2}$$

$$\boxed{\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2}}$$

Q.E.D.

2.22) Write expression for wave f⁰ of harmonic wave:

Given:
- $A = 10^3 \, V/m$
- $\tau = 2.2 \times 10^{-15} \, s$
- $V = 3 \times 10^8 \, m/s$
- Propagates in −(ive) x-direction
- $\Psi(0,0) = 10^3 \, V/m$

$\lambda = V\tau = 3 \times 10^8 \, m/s \times 2.2 \times 10^{-15} \, s = 6.6 \times 10^{-7} \, m$

$$\boxed{\Psi = 10^3 \frac{V}{m} \cos\left(2\pi \left(\frac{x}{6.6 \times 10^{-7} \, m} + \frac{t}{2.2 \times 10^{-15} \, s}\right)\right)}$$

2.32) · Determine which of the following describe traveling waves.

· Where appropriate: draw profile and find speed + direction of motion.

(a) $\psi(y,t) = e^{-(a^2 y^2 + b^2 t^2 - 2abty)}$ → this is a traveling wave

since it is twice differentiable in y ✗ and t.

The velocity is b/a in the positive y direction.

(b) $\psi(z,t) = A\sin(az^2 - bt^2)$ → this is a traveling wave. Twice differentiable

in z and t.

$$\frac{\partial\psi}{\partial z} = 2Aa \overset{\cos}{\sin}(az^2 - bt^2) \qquad \frac{\partial\psi}{\partial t} = -2Abcos(az^2 - bt^2)$$

$$\frac{\partial^2\psi}{\partial z^2} = 2Aa^2 \sin(az^2 - bt^2) \qquad \frac{\partial^2\psi}{\partial t^2} = -2Ab^2\overset{sin}{\cos}(az^2 - bt^2)$$

$$\frac{\partial^2\psi}{\partial z^2} \Big/ \frac{\partial^2\psi}{\partial t^2} = \frac{a^2}{b^2} \Rightarrow \frac{\partial^2\psi}{\partial z^2} = \boxed{\frac{a^2}{b^2}} \frac{\partial^2\psi}{\partial t^2}$$

$$\hookrightarrow \frac{a^2}{b^2} = \frac{1}{v^2} \quad v = b/a$$

The velocity is b/a in the positive z direction.

(c) $\psi(x,t) = A\sin 2\pi\left(\frac{x}{a} + \frac{t}{b}\right)^2$ $\qquad \frac{\partial^2\psi}{\partial x^2} = \frac{1/a^2}{1/b^2} \frac{\partial^2\psi}{\partial t^2} \Rightarrow v = \frac{a}{b}$

Traveling wave b/c twice differentiable in space + time

travels at velocity $\frac{a}{b}$ in negative x-direction.

(d) $\psi(x,t) = A\cos^2 2\pi(t-x)$ $\qquad \frac{\partial\psi}{\partial x^2} = \dfrac{-8\pi^2 \cos(4\pi(x-t))}{-8\pi^2 \cos(4\pi(x-t))} \qquad \frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi}{\partial t^2}$

$\frac{\partial^2\psi}{\partial t^2}$

· This is a traveling wave because it is twice differentiable in x and t.

· It travels in the positive x-direction at a velocity of unity.

Chapter 2

Superposition of Waves

For certain types of waves, specifically small-amplitude linear systems, the Principle of Superposition is able to be used as a convenient method for determining how multiple waves will interact in a system. The principle of superpositions suggests that the resultant disturbance at any point in a medium is the algebraic sum of the separate constituent waves.

2.1 Addition of Waves of the Same Frequency

2.1.1 The Algebraic Method

Take two harmonic electromagnetic waves, E_1 and E_2, of the form

$$E(x,t) = E_0 \sin[\omega t + \alpha(x, \epsilon)]$$

(where $\alpha(x, \epsilon) = -(kx + \epsilon)$ is used to separate the spatial terms from the temporal terms) are occupying the same place in space such that they interact with one another. By grouping terms the equations can be written out as:

$$E_1 = E_{01} \sin(\omega t + \alpha_1)$$

$$E_2 = E_{02} \sin(\omega t + \alpha_2)$$

The resulting disturbance, $E = E_1 + E_2$ can be written as:

$$E = E_0 \sin(\omega t + \alpha) \tag{2.1}$$

where

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1) \tag{2.2}$$

$$\tan \alpha = \frac{E_{01} \sin \alpha_1 + E_{02} \sin \alpha_2}{E_{01} \cos \alpha_1 + E_{02} \cos \alpha_2} \tag{2.3}$$

The term: $2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$ is the interference term which contains the crucial phase difference term:

$$\delta \equiv (\alpha_2 - \alpha_1)$$

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which can arise from a difference in path length as well as a difference in initial phase angle such that:

$$\delta = (kx_1 + \epsilon_1) - (kx_2 + \epsilon_2)$$

$$\delta = \frac{2\pi}{\lambda}(x_1 - x_2) + (\epsilon_1 - \epsilon_2)$$

where x_1 and x_2 are distances from the sources of the two waves to the point of observation. If the waves are initially in-phase at their sources, then $\epsilon_1 = \epsilon_2$.

When two disturbances from the same source travel different routes before arriving at the point of observation,

$$\delta = \frac{2\pi}{\lambda_0}n(x_1 - x_2)$$

where n is the index of refraction of the medium. $n(x_1 - x_2)$ is known as the optical path difference (OPD or Λ).

Also, when $\epsilon_1 - \epsilon_2$ is a constant value, the waves are said to be coherent. For multiple in-phase, coherent sources:

$$E_0^2 = \left(\sum_{i=1}^{N} E_{0i}\right)^2$$

which simplifies to the following when all the amplitudes are the same value of E_{01}:

$$E_0^2 = N^2 E_{01}^2$$

whereas for incoherent light:

$$E_0^2 = N E_{01}^2$$

2.1.2 Standing Waves

A standing wave consists of 2 harmonic waves (possibly a reflection of a wave from the same source) which have the same frequency and period, but travel in opposite directions. A standing wave, or stationary wave, has a profile that does not move through space. Considering a case with two waves, the incident wave: E_I, and the reflected wave: E_R:

$$E_I = E_{0I}\sin(kx + \omega t + \epsilon_I)$$

$$E_R = E_{0R}\sin(kx + \omega t + \epsilon_R)$$

The boundary condition of the standing wave requires that $\epsilon_1 = \epsilon_2$. Assuming that the amplitudes are the same (that is, $E_{0I} = E_{0R}$) the following equation will represent the resultant wave:

$$E(x,t) = 2E_{0I}\sin kx \cos \omega t \tag{2.4}$$

The point of lowest intensity on a standing wave is called a node, and the point of highest amplitude is an amplitude. These are significant concepts in electromagnetic theory.

2.2 Addition of Waves of Different frequency

2.2.1 Beats

Beats contain 2 waves at different frequencies traveling in the same direction. They have the same initial phase angles (can assume this to be zero):

$$E_1 = E_{01} \cos(k_1 x - \omega_1 t)$$

$$E_2 = E_{02} \cos(k_2 x - \omega_2 t)$$

It can be shown that the total disturbance will take the form if the waves have equal amplitudes and zero initial phase angles:

$$E = 2E_{01} \cos(k_m x - \omega_m t) \cos(\bar{k} x - \bar{\omega} t) \tag{2.5}$$

where

$$\bar{\omega} \equiv \frac{1}{2}(\omega_1 + \omega_2)$$

$$\omega_m \equiv \frac{1}{2}(\omega_1 - \omega_2)$$

$$\bar{k} \equiv \frac{1}{2}(k_1 + k_2)$$

$$k_m \equiv \frac{1}{2}(k_1 - k_2)$$

The difference terms are dominated by the low frequency component, whereas the average terms of ω and k are dominated by the high frequency component. The low frequency wave profile essentially encompasses the high frequency wave. The low frequency wave moves at the group velocity, $v_g = \frac{\omega_m}{k_1} = \left(\frac{\partial \omega}{\partial k}\right)_{\bar{\omega}}$ whereas the high frequency carrier wave travels at a phase velocity $v_c = \frac{\bar{\omega}}{\bar{k}}$. $\omega(k)$ is the dispertion relation which is a property of the medium.

2.3 Anharmonic Periodic Waves

Some waves are not harmonic. These are hard to analyzed, so they are instead represented as superpositioning harmonic waves so that they're continuous and differentiable. The Fourier series is commonly used for this. A periodic function $f(x)$ can be represented by the following series:

$$f(x) = \frac{A_0}{2} + \sum_{m=1}^{\infty} A_m \cos mkx + \sum_{m=1}^{\infty} B_m \sin mkx \tag{2.6}$$

where

$$A_0 = \frac{2}{\lambda} \int_0^{\lambda} f(x) dx \tag{2.7}$$

$$A_m = \frac{2}{\lambda} \int_0^{\lambda} f(x) \cos mkx dx \tag{2.8}$$

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$$B_m = \frac{2}{\lambda} \int_0^\lambda f(x) \sin mkx dx \tag{2.9}$$

This Fourier Series Analysis business can be extrapolated into two dimensions for the discrete Fourier transform which is commonly used in image processing.

2.4   Nonperiodic Waves

Nonperiodic waves do not repeat continuously. This makes Fourier series somewhat of an awkward tool to use for analysis. The Fourier integral was designed to handle such signals which are often caused by pulses and wave packets. The governing equations of the Fourier integral are as follows:

$$f(x) = \frac{1}{\pi} \left[\int_0^\infty A(k) \cos kx dk + \int_0^\infty B(k) \sin kx dk \right] \tag{2.10}$$

where:

$$A(k) = \int_{-\infty}^\infty f(x) \cos kx dx \tag{2.11}$$

$$B(k) = \int_{-\infty}^\infty f(x) \sin kx dx \tag{2.12}$$

2.5   Glossary

Absorption band: The wavelength band within which materials absorb electromagnetic energy.

Anomalous dispersion: When the group velocity is greater than the carrier velocity in a system of waves with multiple frequencies.

Coherence length: The spatial length in which a wave remains within its frequency bandwidth.

Coherence time: The time in which a wave remains in its allocated frequency bandwidth.

Coherent: Waves are coherent when their initial phase difference $\varepsilon_1 - \varepsilon_2 = a$ where a is a constant.

Constructive interference: Occurs when interference among waves causes an overall increase in the intensity of the disturbance.

Destructive interference: When the interference between waves causes an overall decrease in intensity of the distrubance.

Dispersive medium: A medium in which the phase velocity of a wave or group of waves depends on its frequency.

Group velocity: The velocity of some shape or leading edge of a pulse, it is taken as the rate at which a feature moves to be the velocity of the group of waves as a whole.

Normal dispersion: When group velocity is less than the carrier velocity of waves with multiple frequencies.

Optical path difference (OPD): The difference in two optical paths, $n(x_1 - x_2)$

Power spectrum: A measure of the distribution of energy, or power, at each and every component frequency that has been analyzed using a discrete Fourier transform.

Standing or Stationary wave: A wave whose profile does not move through space.

Wave packet: A small part of a wavetrain. One pulse of a wavetrain specifically that assemble together as a continuous range of spatial frequencies.

2.6   Useful Equations

Sum of 2 waves:

$$E = E_0 \sin(\omega t + \alpha) \tag{2.13}$$

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1) \tag{2.14}$$

$$\tan\alpha = \frac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2} \tag{2.15}$$

Fourier Series:

$$f(x) = \frac{1}{\pi}\left[\int_0^\infty A(k)\cos kx\,dk + \int_0^\infty B(k)\sin kx\,dk\right] \tag{2.16}$$

where:

$$A(k) = \int_{-\infty}^\infty f(x)\cos kx\,dx \tag{2.17}$$

$$B(k) = \int_{-\infty}^\infty f(x)\sin kx\,dx \tag{2.18}$$

Chapter 3

Electromagnetic Theory, Photons, and Light

Chapter 4

Polarization

4.1 The nature of polarized light

Light in this text has previously been considered to be linearly polarized or plane-polarized. In this case, the orientation of the electrical field is constant, but it's magnitude as sign can vary in time. This disturbance occurs in a plane called a plane of vibration which is a fixed plane containing the electric field and propagation vectors.

Light waves can also act in such a way that their electric field orientations are mutually perpendicular. The superposition of these waves may not be linearly polarized as they are when 2 linearly polarized waves interact.

4.1.1 Linear Polarization

Two orthogonal optical waves can be represented as:

$$\vec{E}_x(z,t) = \hat{i}E_{0x}\cos(kz - \omega t)$$

$$\vec{E}_y(z,t) = \hat{j}E_{0y}\cos(kz - \omega t + \varepsilon)$$

where ε is the relative phase difference between the waves which are both traveling in the z-direction. The resultant wave when the two are in phase is:

$$\vec{E} = \left(\hat{i}E_{0x} + \hat{j}E_{0y}\right)\cos(kz - \omega t)$$

This wave is linearly polarized on a plane that is some angle between the x-z and y-z planes.

If ε is an odd integer multiple of π such that the waves are 180 degrees out of phase then the resulting equation becomes:

$$\vec{E} = \left(\hat{i}E_{0x} - \hat{j}E_{0y}\right)\cos(kz - \omega t)$$

which is a rotated version of the previous equation.

4.1.2   Circular Polarization

When both waves have the same amplitude, yet vary in phase by some form of $\pm\frac{\pi}{2}$, they are said to be circularly polarized. The two waves resemble:

$$\vec{E}_x(z,t) = \hat{i}E_{0x}\cos(kz - \omega t)$$

$$\vec{E}_x(z,t) = \hat{i}E_{0x}\sin(kz - \omega t)$$

and form a consequent waves:

$$\vec{E} = E_0\left[\hat{i}\cos(kz - \omega t) + \hat{j}\sin(kz - \omega t)\right]$$

When light moving toward an observer is rotating clockwise, the wave is called right-circularly clockwise. Similarly, a wave rotating counter clockwise as it travels toward an observer is left-circularly polarized. In circularly polarized light, the endpoint travels in a circle around the axis that the wave is traveling at a rate of 1 rotation per wavelength.

4.1.3   Elliptical Polarization

Both linear and circular polarized light are types of elliptical polarization (they're at the extremes of the light). This type of polarization is when the end location of light travels in an ellipse on a plane perpendicular to the propagation vector. This is the case when the equation describing the two waves are independent in of space or time (the kz - ωt is gone). The equations mentioned so far in this chapter can be altered to the form:

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\varepsilon = \sin^2\varepsilon \tag{4.1}$$

Which is a plane making angle αwith the (E_x, E_y)-coordinate system such that:

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2} \tag{4.2}$$

The state of polarization can be linearly/planarly plarized in a $\mathcal{P}$-state, it can be right or left circularly polarized in an $\mathcal{R}$ or $\mathcal{L}$ state, and finally is can be in a state of elliptical polarization which is designated as an $\mathcal{E}$ state.

4.1.4   Natural light

Light sources which emit random rapidly varying successions of different polarization states are referred to as natural light (also as unpolarized light). Light found in nature and man-made light are neither made completely of polarized or completely unpolarized lights. The electric field vector usually varies in what is known as being partially polarized. Do note that the infinite nature of monochromatic light causes it to be perfectly polarized.

4.1.5 Angular Momentum and Photon Picture

Electromagnetic waves apply energy and linear momentum to bodies they encounter. Power delivered to system by an electromagnetic wave is measured in energy transferred per unit time, $d\mathcal{E}/dt$. The power generated by a torgue, Γ, acting on a rotating body is $\omega\Gamma$ (or vF for linear motion), such that:

$$\frac{d\mathcal{E}}{dt} = \omega\Gamma$$

Torque is equal to the time rate-of change of angular momentum, L, so that on average:

$$\frac{d\mathcal{E}}{dt} = \omega\frac{dL}{dt}$$

A charge that absorbs a quantity of energy from the incident circular wave simultaneously absorbs angular momentum L so:

$$L = \frac{\mathcal{E}}{\omega}$$

4.2 Polarizers

Any device that takes an input of unpolarized light and outputs a form of polarized light is called a polarizer. Polarizers use one of the following physical mechanisms: dichroism (selective absorption), reflection, scattering, and birefrengence (double refraction). There is asymmetry involved with all of these processes.

4.2.1 Malus's Law

This is involved in experimentally determining if a device is a (linear) polarizer.

The $\mathcal{P}$-state of a wave emerging from a linear polarizer will have an orientation parallel to what is defined as the transmission axis of a polarizer. Only the component of the optical field parallel to the transmission axis will pass through the device unaffected. An analyzer is a second polarizer that is put into an optical train to test the polarization of the other polarizing device. The irradiance reaching a detector behind both devices is given by:

$$I(\theta) = \frac{c\epsilon_0}{2}E_{01}^2\cos^2\theta$$

Maximum irradiance occurs when the angle between the transmission axes of the analyzer and polarizer is zero, I(0). This leads to Malus's Law which is:

$$I(\theta) = I(0)\cos^2\theta \tag{4.3}$$

where I(0) is the irradiance arriving at the analyzer:

$$I(0) = c\epsilon_0 E_{01}^2/2 \tag{4.4}$$

Be aware that nonideal polarizers only work within a certain range of wavelengths.

4.2.2 Dichroism

Dichroism is the selective absorption of one of the two orthogonal $\mathcal{P}$-state components of an incident beam. Dichroic polarizers are physicially anisotropic producing an asymmetric absorption of one field component and remaining transparent to the other.

4.2.3 Birefringence

An anisotropy in the binding force between atoms in a crystal lattice will be manifest in an anisotropy in the refractive index. This is because the speed of the wave (which is related to index of refraction) is the difference between the frequency of the $\vec{E}$-field and the natural frequency of the atoms. A material that displays two different indices of refraction is defined to be birefringent.

   The optic axis is a direction through which a material has one index of refraction and is perpendicular to 2 other directions of another index of refraction. The direction of the optic axis corresponds to a special crystallographic orientation which is an axis of 3-fold symmetry.

   In a birefrengent crystal such as calcite. Rays traveling through it as if they had passed through a plate of glass are known as ordinary rays (o-rays). Ordinary rays will not rotate along with a crystal. Rays that are affected by rotation of a crystal are extraordinary rays. Ordinary and extraordinary waves are both polarized and are perpendicular to one another.

4.2.4 Scattering and polarization

Particles around the same size as light scatter it in many different directions. Unpolarized light entering a medium with scattering particles can be scattered at angles normal to the incident light's propagation vector. The scattered light in the forward direction is completely unpolarized.

4.2.5 Polarization by reflection

Reflecting from dielectric (insulating) media is a common source of polarized light. Forms of glare from windows, paper, and other insulating types of material are partially polarized. For an electron-oscillator: the incoming polarized wave made up of 2 incoherent orthogonal $\mathcal{P}$-states, only the component polarized normal to the incident plane and therefore parallel to the surface will be reflected. The angle of incidence for which this occurs is θ_p which is the polarization angle or Brewster's angle where $\theta_p + \theta_t = 90°$. From Snell's Law:

$$n_i \sin \theta_p = n_t \sin \theta_t \tag{4.5}$$

The fact that $\theta_t = 90° - \theta_p$ leads to Brwster's Law:

$$\tan \theta_p = n_t / n_i \tag{4.6}$$

where the subscript "i" refers to the incident beam and the properties of its medium, the subscript "t" refers to the same for the transmitted beam, and the subscript "p" refers to polarization.

The Fresnel equations lead to a desirable concept of the degree of polarization V, defined as:

$$V = \frac{I_p}{I_p + I_n} \tag{4.7}$$

where I_p and I_n are the constituent flux densities of the polarized and "unpolarized" natural light. This gives a quantitative way to described how polarized light is.

4.3 Retarders

A retarder is an optical element which serves to change the polarization of an incident wave. A wave of a certain polarization goes in and a wave with a different type of polarization goes out. This is accomplished by changing the relative phase of the incident beam.

If an optic axis of a uniaxial crystal (ie calcite) is arranged to be parallel to the front and back surfaces and the incident monochromatic plane wave's $\vec{E}$-field has components parallel and perpendicular to the optic axis, then two separate plane waves will propagate through the crystal. Since $v_\parallel > v_\perp$ and $n_o > n_e$: the e-wace will move across the specimen faster than the o-wave. After traveling a plate of thickness, d, the resultant wave has a relative phase difference of $\Delta\varphi$ (these are harmonic waves of the same frequency with orthogonal electric fields.

The relative optical path length difference is:

$$\Lambda = d(|n_o - n_e|)$$

since $\Delta\varphi = k_0\Lambda$, the phase difference (measured in radians) is:

$$\Delta\varphi = \frac{2\pi}{\lambda_0}d(|n_o - n_e|) \tag{4.8}$$

where λ_0 is the wavelength in vacuum.

Some types of retarders: zero order retarder has the minimum thickness necessary to produce a phase difference. Multiple order retarder has a thickness corresponding to a whole number of 2π shifts plus the phase difference. Compound zero-order wave plates combine the fast axis of one retarder with the slow axis of another to compensate for temperature variations.

4.3.1 The Full-Wave Plate

When the relative retardation or retardance, $\Delta\varphi = 2\pi$, the optical component is called a full-wave plate or full-wave retarder. Retarders of this sort only work for a narrow range of wavelengths. When the velocity of light parallel to the optic axis is faster than in the

perpendicular direction, the optic axis is then referred to as the fast axis. The slow axis is then the perpendicular direction to the optic direction. For materials in which $v_\perp > v_\parallel$, the names are switched (since the perpendicular direction is now faster).

These are used to eliminate inadvertent changes in polarization state of light passing through an optical system.

4.3.2   The Half-Wave Plate

A retardation plate with retardance of π radians between the o-waves and e-waves. These are often made of polyvinyl alcohol sheets or cellophane tape on a microscope slide. For this case, the thickness of the material satisfies:

$$d(|n_o - n_e|) = (2m + 1)\lambda_0/2$$

where m is an integer.

4.3.3   The Quarter-Wave Plate

A retarder with retardance, $\Delta\varphi = \pi/2$. Crude quarter wave plates can be made from plastic food wrap. Commercial quarter-wave plates are used for their linear retardation. For this case, the thickness of the material must satisfy:

$$d(|n_o - n_e|) = (4m + 1)\lambda_0/4$$

4.3.4   Fresnel Rhomb

The Fresnel rhomb causes a beam to be internally reflected twice, thus imparting a 90° relative phase shift. These are almost achromatic 90 degree retarders.

4.3.5   Compensators and Variable Retarders

A compensator is an optical device that is capable of impressing a controllable retardance on a wave. A common type of compensator, the Babinet compensator uses two triangular wedges to vary the thickness of calcite that a wave will travel through. The total phase difference (retardance) of these devices is:

$$\Delta\varphi = \frac{2\pi}{\lambda_0}(d_1 - d_2)(|n_o - n_e|)$$

4.4   Circular polarizers

A series combination of an appropriately oriented linear polarizer and a 90 degree retarder will perform as a circular polarizer. both $\mathcal{L}$ and $\mathcal{R}$ states can be achieved by varying the transmission axis of the linear polarizer at positive 45 degrees or negative 45 degrees measured to the fast axis of the retarder.

4.5   Optical Activity

Any material that causes an $\vec{E}$-field of an incident linear plane wave to appear to rotate is deemed optically active.

4.6   Glossary

Analyzer: A second polarizer in an optical train used to test another polarizer. Often a mostly ideal linear polarizer is used for this purpose.

Birefringence: the material property referring the the display of two different indices of refraction. Mathematically defined as $\Delta n = n_e - n_o$ where n_e and n_o are the index of refraction of the extraordinary arrays and the ordinary rays, respectively.

Brewster's angle: The polarization angle θ_p in which the reflected angle and transmitted angle from an incident beam onto a surface is equal to 90 degrees.

Circular birefringence:

Circularly polarized: A result of multiple waves traveling to an observer that appear to rotate circularly on a plane perpendicular to the axis of propagation.

Cleavage plane: Smooths surfaces on which a material can be easily split.

Compensator: An optical device that is capable of impressing a controllable retardance on a wave.

Dichroism: The selective absorption of one of the two orthogonal $\mathcal{P}$-state components of an incident beam.

Extraordinary ray: A ray that rotates with a birefringent crystal.

Faraday effect:

Fast axis: The axis in a wave plate in which waves travel fastest.

Half-wave plate: A wave plate with retardance of π radians.

Jones vector:

Kerr effect:

Liquid Crystal:

Malus's law: $I(\theta) = I(0) \cos^2 \theta$

Mueller matrix:

Optic axis: The direction in a birefrengent crystal through which light travels through one index of refraction and has a crystal such that the 2 perpendicular directions to the Optic axis direction are of another index of refraction.

Optical activity: the phenomenon in which a material causes the elctric field of an incident linear plane wave to appear to rotate.

Ordinary ray: A ray that is unaffected by the rotation of a birefringent crystal.

Polarization state: describes if light is in a: linear or planar $\mathcal{P}$-state; right or left circularly polarized in an $\mathcal{R}$ or $\mathcal{L}$ state; or in a state of elliptical polarization which is designated as an $\mathcal{E}$ state.

Retarder: An optical element that changes the polarization of an incident wave.

Stress birefringence:

Stokes parameters:

Transmission axis: The axis to which the light traveling out of a polarizer is parallel.

Twisted nematic cell:

Uniaxial crystal: A crystal in which there is only one direction about which atoms are arranged symmetrically.

Unpolarized light: Also known as natural light. A disturbance from a source that emits randomly polarized wave packets.

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