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EE 268 HW1

Hecht: ch2: 4, 13, 17, 18, 22, 32

2.4) Given: • Waves (harmonic)

• 0.5s between crests

• disturbance takes 1.5s to travel 4.5m

Find: • Frequency

• period (assume temporal period.)

• wavelength. (λ)

velocity

$$v = \frac{4.5m}{1.5s} = 3.0 m/s$$

temporal period

$$\tau = 0.5s$$

$$\tau = \frac{\lambda}{v} \rightarrow \quad \lambda = \tau v = 0.5s * 3.0 m/s = 1.5m$$

$$\lambda = 1.5m$$

Frequency

$$\nu = 1/\tau = \frac{1}{0.5s} = 2/s \quad \nu = 2/s = 2Hz$$

~~Frequency: 2s~~

Frequency: 2Hz
Wavelength: 1.5m
Period: 0.5s

2.13) Given: • Figure of transverse wave @ $t=0$

• $v = 20.0\ m/s$

• $A = 0.020\ m$ (inspection from figure)

• $\lambda = 4.0\ m$ (" " ")

(a) Find: wavelength

Wavelength, $\lambda = 4.0\ m$

(b) Find: Frequency $\quad v = \nu\lambda \Rightarrow \nu = \frac{v}{\lambda}$

$$\nu = \frac{20.0\ m/s}{4.0\ m} = 5\ Hz \qquad \nu = 5.0\ Hz$$

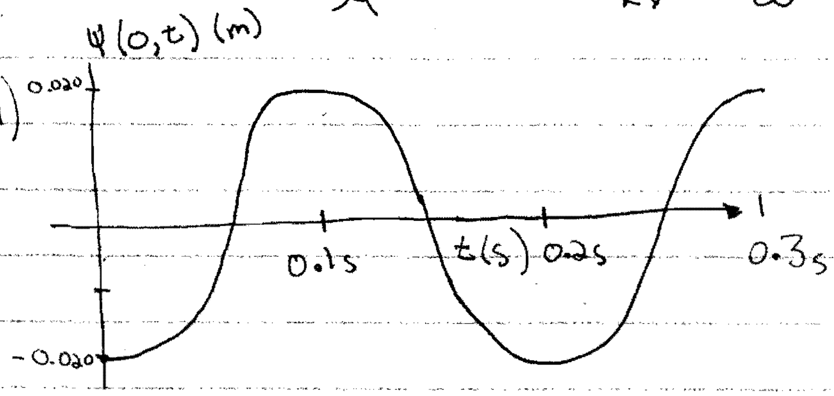
(c) Write down wave function of disturbance

$$\omega = 2\pi\nu = \cancel{10\,Hz\,\pi}\ 10\pi\ \frac{rad}{s} \qquad k = 2\pi/\lambda = \frac{1}{2m}\pi$$

$$\psi(x,t) = \underbrace{(0.020\,m)}_{A} \cos\left(\underbrace{\tfrac{1}{2m}x}_{kx} - \underbrace{10\pi\tfrac{rad}{s}t}_{\omega} - \underbrace{\pi}_{\varepsilon}\right)$$

ε (phase determined by inspection)

(d)



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$$\text{2.17) Given: } \Psi(x,t) = \underset{A}{(30.0\,\text{cm})} \cos\left[\underset{k}{\left(6.29\,\tfrac{\text{rad}}{\text{m}}\right)} x \underset{+\text{direction}}{-} \underset{\omega}{\left(20.0\,\tfrac{\text{rad}}{\text{s}}\right)} t\right]$$

Find:
- Frequency (a) (ν)
- Wavelength (b) (λ)
- Period (c) (τ)
- Amplitude (d) (A)
- Phase velocity (e) (v)
- direction of motion (F) (-sign on omega)

(a) $\omega = 2\pi\nu \qquad \nu = \frac{\omega}{2\pi} = \frac{20\,\text{rad/s}}{2\pi\,\text{rad}} = \frac{10}{\pi}\,\text{Hz} \qquad \nu = \frac{10}{\pi}\,\text{Hz}$

(b) $k = 2\pi/\lambda \qquad \lambda = \frac{2\pi}{k} = \frac{2\pi\,\text{rad}}{6.28\,\text{rad/m}} = 1\,\text{m} \qquad \lambda = 1\,\text{m}$

(c) $\tau = \frac{1}{\nu} = \frac{1}{10/\pi\,\text{Hz}} = \frac{\pi}{10}\,\text{s} \qquad \tau = \frac{\pi}{10}\,\text{s}$

(d) $A = 30.0\,\text{cm}$ (by inspection) $\qquad A = 30.0\,\text{cm}$

(e) $v = \nu\lambda = \frac{10}{\pi\,\text{s}} \ast 1\,\text{m} = \frac{10\,\text{m}}{\pi\,\text{s}} \qquad v = \frac{10}{\pi}\,\frac{\text{m}}{\text{s}}$

(F) The wave moves in the positive x-direction.

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2.18 Show that: $\Psi_{(x,t)} = A\sin[k(x-vt)]$ is a sollution to the differential wave equation.

$$\Psi(x,t) = A\sin[kx - kvt]$$

$$\frac{\partial\Psi}{\partial x} = Ak\cos(kx-kvt) \qquad \frac{\partial\Psi}{\partial t} = Akv\cos(kx-kvt)$$

$$\frac{\partial^2\Psi}{\partial x^2} = -Ak^2\sin(kx-kvt) \qquad \frac{\partial^2\Psi}{\partial t^2} = -Ak^2v^2\sin(kx-kvt)$$

Assuming $\frac{\partial^2\Psi}{\partial t^2} \neq 0$:

$$\frac{\frac{\partial^2\Psi}{\partial x^2}}{\frac{\partial^2\Psi}{\partial t^2}} = \frac{\cancel{-A}\,\cancel{k^2}\,\cancel{\sin(kx-kvt)}}{\cancel{-A}\,\cancel{k^2}v^2\,\cancel{\sin(kx-kvt)}} = \frac{1}{v^2}$$

$$\boxed{\frac{\partial^2\Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}}$$

Q.E.D.

2.22) Write expression for wave f^n of harmonic wave:

Given:
- $A = 10^3\ V/m$
- $\tau = 2.2\times10^{-15}\ s$
- $v = 3\times10^8\ m/s$
- Propagates in $-$(ive) x-direction
- $\Psi(0,0) = 10^3\ V/m$

$\lambda = v\tau = 3\times10^8\ m/s * 2.2\times10^{-15}\ s = 6.6\times10^{-7}\ m$

$$\boxed{\Psi = 10^3\frac{V}{m}\cos\left(2\pi\left(\frac{x}{6.6\times10^{-7}\ m} + \frac{t}{2.2\times10^{-15}\ s}\right)\right)}$$

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2.32) • Determine which of the Following describe traveling waves.

• Where appropriate: draw profile and find speed + direction of motion.

(a) $\Psi(y,t) = e^{-(a^2y^2 + b^2t^2 - 2abty)}$ → this is a traveling wave since it is twice differentiable in y and t.

The velocity is b/a in the positive y direction.

(b) $\Psi(z,t) = A\sin(az^2 - bt^2)$ → this is a traveling wave. Twice differentiable in z and t.

$$\frac{\partial\psi}{\partial z} = 2Aa\,\overset{\cos}{\cancel{\sin}}(az^2 - bt^2) \qquad \frac{\partial\psi}{\partial t} = 2Ab\cos(az^2 - bt^2)$$

$$\frac{\partial^2\psi}{\partial z^2} = \cancel{4}Aa^2\sin(az^2 - bt^2) \qquad \frac{\partial^2\psi}{\partial t} = \cancel{4}Ab^2\overset{\sin}{\cancel{\cos}}(az^2 - bt^2)$$

$$\frac{\dfrac{\partial^2\psi}{\partial z^2}}{\dfrac{\partial^2\psi}{\partial t^2}} = \frac{a^2}{b^2} \Rightarrow \frac{\partial^2\psi}{\partial z^2} = \frac{a^2}{b^2}\frac{\partial^2\psi}{\partial t^2}$$

$$\hookrightarrow \frac{a^2}{b^2} = \frac{1}{v^2} \quad v = \frac{b}{a}$$

The velocity is b/a in the positive z direction.

(c) $\Psi(x,t) = A\sin 2\pi\left(\frac{x}{a} + \frac{t}{b}\right)^2$ $\qquad \frac{\partial^2\psi}{\partial x^2} = \frac{\frac{1}{a^2}}{\frac{1}{b^2}}\frac{\partial^2\psi}{\partial t^2} \Rightarrow v = \frac{a}{b}$

Traveling wave b/c twice differentiable in space + time.

travels at velocity $\frac{a}{b}$ in negative x-direction.

(d) $\Psi(x,t) = A\cos^2 2\pi(t - x)$ $\qquad \frac{\dfrac{\partial^2\psi}{\partial x^2}}{\dfrac{\partial^2\psi}{\partial t^2}} = \frac{-8\pi^2\cos(4\pi(x-t))}{-8\pi^2\cos(4\pi(x-t))} \qquad \frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2\psi}{\partial t^2}$

• This is a traveling wave because it is twice differentiable in x and t.

• It travels in the positive x-direction at a velocity of unity.