An alternative method for calculating the natural logarithm of a determinant that can end up as a very large (or very small) number.

In our case, we need to solve

$$\frac{1}{2}\ln\left(\det\left(\mathbf{A}\right)\right) \tag{1}$$

where $\mathbf{A}$ is an $n \times n$ matrix.

First, we note that the determinant can be solved for as follows

$$\det\left(\mathbf{A}\right) = \det\left(\mathbf{L}\right)\prod_{i=1}^{n}\mathbf{U}_{ii} \tag{2}$$

where the matrices $\mathbf{L}$ and $\mathbf{U}$ are the lower permuted and upper triangular matrices obtained through $LU$ factorization of $\mathbf{A}$. Note that always $\det\left(\mathbf{L}\right) = \pm 1$. Substituting Eq. 2 into Eq. 1

$$\frac{1}{2}\ln\left(\det\left(\mathbf{A}\right)\right) = \frac{1}{2}\ln\left(\det\left(\mathbf{L}\right)\prod_{i=1}^{n}\mathbf{U}_{ii}\right) \tag{3}$$

We can drop the $\det\left(\mathbf{L}\right)$ term if we have another way to keep track of the sign of the resulting value, so

$$\frac{1}{2}\ln\left(\det\left(\mathbf{L}\right)\prod_{i=1}^{n}\mathbf{U}_{ii}\right) = \frac{1}{2}\left(\sum_{i=1}^{n}\ln\left(\mathbf{U}_{ii}\right)\right) \tag{4}$$

We can keep track of the sign by evaluating the real part of Eq. 4 as the magnitude of the answer, and the complex part will indicate the sign. The final answer is therefore

$$\frac{1}{2}\ln\left(\det\left(\mathbf{A}\right)\right) = \frac{1}{2}\operatorname{real}\left(\sum_{i=1}^{n}\ln\left(\mathbf{U}_{ii}\right)\right) \tag{5}$$

Now, we must examine the complex part for the sign. Using MATLAB, it is possible to obtain a complex result for $\ln\left(\cdot\right)$ if the argument is either complex or negative. For a complex number $z = x + iy$

$$\ln\left(z\right) = \ln\left|x\right| + i\theta \tag{6}$$

where $\theta$ is the angle of $re^{i\theta}$ expression of $z$ (recall the old favorite equation $e^{i\pi} = -1$). For any negative real number, $\theta = \pi$ or any other odd integer multiple of $\pi$ $\left(\pi, 3\pi, 5\pi, \ldots\right)$. Similarly, for any positive real number, $\theta = 0$ or any even multiple of $\pi$ $\left(2\pi, 4\pi, 6\pi, \ldots\right)$. So, for a positive real number

$$\ln\left(z\right) = \ln\left|z\right| \tag{7}$$

whereas for a negative real number

$$\ln\left(z\right) = \ln\left|z\right| + i\pi \tag{8}$$

Returning to the issue of the sign in Eq. 5, covariance matrices as we will evaluate are positive definite, so the determinant should be positive. For the determinant to be positive, it must be the case that any negative terms on the diagonal of $\mathbf{U}$ occur in pairs. Therefore, while it is possible for Eq. 4 to yield an imaginary component, upon summing nay terms with imaginary terms equal to $i\pi$, the result will always be a multiple of $2\pi$ which indicates that $\det\left(\mathbf{A}\right)$ would be positive. We are thus justified in simply taking the real portion of Eq. 4, as expressed in Eq. 5.