

3) Parametrizamos S_3

$$\Phi_3: (0, \sqrt{3}) \times (0, 2\pi) \longrightarrow \mathbb{R}^3$$

$$(r, \theta) \longrightarrow (r \cos \theta, r \sin \theta, 0)$$

Φ_3 es de clase C^1 e inyectiva y si $D_3 = (0, \sqrt{3}) \times (0, 2\pi)$ entonces $\Phi_3(D_3) = S_3$

$$\left. \begin{aligned} \frac{\partial \Phi_3}{\partial r} &= (\cos \theta, \sin \theta, 0) \\ \frac{\partial \Phi_3}{\partial \theta} &= (-r \sin \theta, r \cos \theta, 0) \end{aligned} \right\} \frac{\partial \Phi_3}{\partial r} \times \frac{\partial \Phi_3}{\partial \theta} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos \theta & \sin \theta & 0 \\ -r \sin \theta & r \cos \theta & 0 \end{vmatrix} =$$

$$= 0\vec{i} + 0\vec{j} + r(\cos^2 \theta + \sin^2 \theta)\vec{k} = (0, 0, r).$$

Esta es la normal interior y como estamos considerando la exterior, reescribimos

$$\tilde{\Phi}_3: \tilde{D}_3 \longrightarrow \mathbb{R}^3$$

$$(\theta, r) \longrightarrow (r \cos \theta, r \sin \theta, 0)$$

$$\text{con } \tilde{D}_3 = (0, 2\pi) \times (0, \sqrt{3})$$

$$\tilde{\Phi}_3(\tilde{D}_3) = S_3$$

$$\text{y } \frac{\partial \tilde{\Phi}_3}{\partial \theta} \times \frac{\partial \tilde{\Phi}_3}{\partial r} = - \frac{\partial \Phi_3}{\partial r} \times \frac{\partial \Phi_3}{\partial \theta} = -(0, 0, r) = (0, 0, -r)$$

Por tanto

$$\iint_{S_3} \nabla f \cdot d\vec{S} = \iint_{\tilde{D}_3} (\nabla f \circ \tilde{\Phi}_3) \cdot \left(\frac{\partial \tilde{\Phi}_3}{\partial \theta} \times \frac{\partial \tilde{\Phi}_3}{\partial r} \right) d\theta dr =$$

$$= \iint_{\tilde{D}_3} \nabla f(r \cos \theta, r \sin \theta, 0) \cdot (0, 0, -r) dr = \iint_{\tilde{D}_3} (2r \cos \theta + 2r \sin \theta - 3, 2r \cos \theta, 0) \cdot (0, 0, -r) dr =$$

$$= \iint_{\tilde{D}_3} 0 dr d\theta = 0$$