$\boxed{10.-}$ Calcula los cuatro primeros términos del desarrollo en serie de Taylor en $z=0$ de la función:

$\boxed{a)}$ $f(z) = \frac{1}{1+e^z} = (1+e^z)^{-1}$; $f(0) = 1/2$

$f'(z) = -(1+e^z)^{-2} \cdot e^z \qquad f'(0) = -\frac{1}{4}$

$f''(z) = -e^z(1+e^z)^{-2} + e^z \cdot 2(1+e^z)^{-3} \cdot e^z = 2e^{2z}(1+e^z)^{-3} - e^z(1+e^z)^{-2}$

$f''(0) = 0$

$f'''(z) = 2 \cdot e^{2z} \cdot 2(1+e^z)^{-3} - 2e^{2z} \cdot 3(1+e^z)^{-4} \cdot e^z - e^z(1+e^z)^{-2}$
$+ 2e^z(1+e^z)^{-3} \cdot e^z = 6e^{2z}(1+e^z)^{-3} - 6e^{3z}(1+e^z)^{-4} - e^z(1+e^z)^{-2}$

$f'''(0) = 1/8$

$\Rightarrow f(z) \sim \frac{1}{2} - \frac{1}{4}z + \frac{1}{8 \cdot 3!}z^3 = \frac{1}{2} - \frac{z}{4} + \frac{z^3}{48}$

$\boxed{b)}$ $f(z) = e^z \cos z \qquad f(0) = 1$

$f'(z) = e^z \cos z - e^z \operatorname{sen} z \qquad f'(0) = 1$

$f''(z) = e^z \cos z - e^z \operatorname{sen} z - e^z \operatorname{sen} z - \cancel{e^z \cos z} = -2e^z \operatorname{sen} z$

$f''(0) = 0$

$f'''(z) = -2e^z \operatorname{sen} z - 2e^z \cos z$

$f^{3)}(0) = -2$

$\Rightarrow f(z) \sim 1 + z - \frac{2}{3!}z^3 = 1 + z - \frac{z^3}{3}$

$\boxed{c)}$ $f(z) = \operatorname{tg} z = \frac{\operatorname{sen} z}{\cos z} \qquad f(0) = 0$

$f'(z) = \frac{\cos^2 z + \operatorname{sen}^2 z}{(\cos z)^2} = \frac{1}{\cos^2 z} = 1 + \operatorname{tg}^2 z \qquad f'(0) = 1$

$f''(0) = 0 \qquad f'''(0) = 2$

$f''(z) = 2\operatorname{tg} z \cdot (1 + \operatorname{tg}^2 z) = 2\operatorname{tg} z + 2\operatorname{tg}^3 z$

$f'''(z) = 2(1+\operatorname{tg}^2 z) + 2 \cdot 3 \cdot \operatorname{tg}^2 z(1+\operatorname{tg}^2 z) = 2 + 2\operatorname{tg}^2 z + 6\operatorname{tg}^2 z + 6\operatorname{tg}^4 z = 2 + 8\operatorname{tg}^2 z + 6\operatorname{tg}^4 z$

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Apellidos Nombre

Calificación

$$\Rightarrow f(z) = \operatorname{tg} z \sim z + \frac{2}{3!} z^3 = z + \frac{z^3}{3}$$

d) $f(z) = \sqrt{z^2-1} = (z^2-1)^{1/2}$ $\quad f(0) = i$

$$f'(z) = \frac{1}{2}(z^2-1)^{-1/2} \cdot 2z = z(z^2-1)^{-1/2} \quad ; f'(0) = 0$$

$$f''(z) = (z^2-1)^{-1/2} + z \cdot (z^2-1)^{-3/2} \cdot \left(-\tfrac{1}{2}\right) \cdot 2z =$$

$$= (z^2-1)^{-1/2} - z^2 (z^2-1)^{-3/2} \qquad f''(0) = i$$

$$f^{(3)}(0) = 0$$

$$f'''(z) = (z^2-1)^{-3/2} \cdot \left(-\tfrac{1}{2}\right) \cdot 2z - 2z(z^2-1)^{-3/2} - z^2 \cdot (z^2-1)^{-5/2} \cdot \left(-\tfrac{3}{2}\right) \cdot 2z =$$

$$= -z(z^2-1)^{-3/2} - 2z(z^2-1)^{-3/2} + 3z^3(z^2-1)^{-5/2} = -3z(z^2-1)^{-3/2} + 3z^3(z^2-1)^{-5/2}$$

$$f(z) = \sqrt{z^2-1} \sim i + \frac{i}{2!} z^2 = i + \frac{i}{2} z^2$$

e) $f(z) = e^{\frac{1}{1-z}} = e^{(1-z)^{-1}}$ $\quad f(0) = e$

$$f'(z) = e^{(1-z)^{-1}} \cdot (-1) \cdot (1-z)^{-2} \cdot (-1) = e^{(1-z)^{-1}} \cdot (1-z)^{-2} \quad ; f'(0) = e$$

$$f''(z) = e^{(1-z)^{-1}} \cdot (1-z)^{-2} \cdot (1-z)^{-2} + e^{(1-z)^{-1}} \cdot (1-z)^{-3} \cdot (-2) \cdot (-1) =$$

$$= e^{(1-z)^{-1}} (1-z)^{-4} + 2e^{(1-z)^{-1}} \cdot (1-z)^{-3} \quad ; \quad f''(0) = 3e$$

$$f'''(z) = e^{(1-z)^{-1}} (1-z)^{-2} (1-z)^{-4} + e^{(1-z)^{-1}} \cdot (1-z)^{-5} \cdot (-4) \cdot (-1)$$

$$+ 2e^{(1-z)^{-1}} (1-z)^{-2} \cdot (1-z)^{-3} + 2e^{(1-z)^{-1}} \cdot (1-z)^{-4} \cdot (-3)(-1) =$$

$$= e^{(1-z)^{-1}} (1-z)^{-6} + 4e^{(1-z)^{-1}} (1-z)^{-5} + 2e^{(1-z)^{-1}} (1-z)^{-5} + 6e^{(1-z)^{-1}} (1-z)^{-4}$$

$$f'''(0) = 13e$$

$$\Rightarrow f(z) = e^{\frac{1}{1-z}} \sim e + ez + \frac{3e}{2!} z^2 + \frac{13}{3!} e z^3 = e + ez + \frac{3e}{2} z^2 + \frac{13e}{6} z^3$$

f) $f(z)=\log(1+e^{-s})$ cte. del. principal.

$$f(z)=\log(1+e^{-5z}) \qquad f(0)=\log 2 = \log 2$$

$$f'(z)=\frac{1}{1+e^{-5z}}\cdot e^{-5z}\cdot(-5) \qquad f'(0)=-5/2$$

$$\parallel$$

$$-5\cdot\frac{e^{-5z}}{1+e^{-5z}}$$

$$f''(z)=-5\cdot\frac{e^{-5z}(-5)(1+e^{-5z})-e^{-5z}\cdot e^{-5z}\cdot(-5)}{(1+e^{-5z})^2}=-5\,\frac{-5e^{-5z}\cancel{-5e^{-10z}}+\cancel{5e^{-10z}}}{(1+e^{-5z})^2}=$$

$$=25\,\frac{e^{-5z}}{(1+e^{-5z})^2}\ ; \quad f''(0)=25/2$$

$$f'''(z)=25\cdot\frac{e^{-5z}(-5)(1+e^{-5z})^2-e^{-5z}\cdot 2(1+e^{-5z})\cdot e^{-5z}(-5)}{(1+e^{-5z})^4}=$$

$$\Rightarrow f'''(0)=25\cdot\frac{-5\cdot 2^2+10\cdot 2}{2^4}=0$$

$$f(z)=\log(1+e^{-5z})\sim\log 2-5/2\,z+\frac{25}{2\cdot 2!}z^2=\log 2-\frac{5}{2}z+\frac{25}{4}z^2$$

g) $f(z)=\log(1+\cos z)$ $\qquad f(0)=\log 2$

$$f'(z)=\frac{-\operatorname{sen} z}{1+\cos z} \qquad f'(0)=0$$

$$f''(z)=\frac{-\cos z(1+\cos z)+\operatorname{sen} z(-\operatorname{sen} z)}{(1+\cos z)^2}=\frac{-\cos z-\cos^2 z-\operatorname{sen}^2 z}{(1+\cos z)^2}=-\frac{1+\cos z}{(1+\cos z)^2}=$$

$$=-(1+\cos z)^{-1} \qquad f''(0)=-\frac{1}{2}$$

$$f'''(z)=+(1+\cos z)^{-2}\cdot(-\operatorname{sen} z) \qquad f^{3)}(0)=0$$

$$\Rightarrow f(z)=\log(1+\cos z)\sim\log 2-\frac{1}{2\cdot 2!}z^2=\log 2-\frac{z^2}{4}$$