

2) Sea Φ_2 una parametrización de S_2

$$\Phi_2: (0, 1) \times (0, 2\pi) \longrightarrow \mathbb{R}^3$$

$$(r, \theta) \longrightarrow (r \cos \theta, r \sin \theta, 2 - \sqrt{1-r^2})$$

Si $D_2 = (0, 1) \times (0, 2\pi) \Rightarrow \Phi_2(D_2) = S_2$ y Φ_2 es C^1 e inyectiva.

$$\frac{\partial \Phi_2}{\partial r} = \left(\cos \theta, \sin \theta, \frac{r}{\sqrt{1-r^2}} \right) \quad \frac{\partial \Phi_2}{\partial \theta} = \left(-r \sin \theta, r \cos \theta, 0 \right)$$

$$\Rightarrow \frac{\partial \Phi_2}{\partial r} \times \frac{\partial \Phi_2}{\partial \theta} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos \theta & \sin \theta & \frac{r}{\sqrt{1-r^2}} \\ -r \sin \theta & r \cos \theta & 0 \end{vmatrix} =$$

$$= -\frac{r^2 \cos \theta}{\sqrt{1-r^2}} \vec{i} - \frac{r^2 \sin \theta}{\sqrt{1-r^2}} \vec{j} + r(\cos^2 \theta + \sin^2 \theta) \vec{k} =$$

$$= \left(-\frac{r^2 \cos \theta}{\sqrt{1-r^2}}, -\frac{r^2 \sin \theta}{\sqrt{1-r^2}}, r \right)$$

Esta normal es la interior en la esfera

pero la exterior considerando la superficie ∂V .

Así:

$$\iint_{S_2} \nabla f \cdot d\vec{S} = \iint_{D_2} (\nabla f \circ \Phi_2) \cdot \left(\frac{\partial \Phi_2}{\partial r} \times \frac{\partial \Phi_2}{\partial \theta} \right) dr d\theta =$$

$$= \iint_{D_2} \nabla f(r \cos \theta, r \sin \theta, 2 - \sqrt{1-r^2}) \cdot \left(-\frac{r^2 \cos \theta}{\sqrt{1-r^2}}, -\frac{r^2 \sin \theta}{\sqrt{1-r^2}}, r \right) dr d\theta =$$

$$= \iint_{D_2} (2r \cos \theta + 2r \sin \theta - 3, 2r \cos \theta, 4 - 2\sqrt{1-r^2}) \cdot \left(-\frac{r^2 \cos \theta}{\sqrt{1-r^2}}, -\frac{r^2 \sin \theta}{\sqrt{1-r^2}}, r \right) dr d\theta =$$

$$= \iint_{D_2} \left(\frac{2r^3 \cos^2 \theta}{\sqrt{1-r^2}} - \frac{2r^3 \sin \theta \cos \theta}{\sqrt{1-r^2}} + \frac{3r^2 \cos \theta}{\sqrt{1-r^2}} - \frac{2r^3 \sin \theta \cos \theta}{\sqrt{1-r^2}} + 4r - 2r\sqrt{1-r^2} \right) dr d\theta$$