Sistemas no lineales

$$\begin{cases} F_1(x_1, x_2, \dots, x_n) = 0 \\ F_2(x_1, x_2, \dots, x_n) = 0 \\ \quad\cdots \\ F_n(x_1, x_2, \dots, x_n) = 0. \end{cases}$$

$$F(x) = 0 \ \text{ donde } \ F = (F_1, F_2, \dots, F_n)^{\mathrm{T}} \ \text{ y } \ x = (x_1, x_2, \dots, x_n)^{\mathrm{T}}$$

1. Método de Newton

$$\boxed{\begin{cases} x^0 \in \mathbb{R}^n \ \text{ dado} \\ J_F(x^{k-1})x^k = J_F(x^{k-1})x^{k-1} - F(x^{k-1}), \quad k \in \mathbb{N} \end{cases}}$$

En cada iteración se resuelve el sistema

$$\boxed{A_{k-1}u = b_{k-1}}$$

donde

$$A_{k-1} = J_F(x^{k-1}) \ \text{ y } \ b_{k-1} = J_F(x^{k-1})x^{k-1} - F(x^{k-1}).$$

Para ello, se considera la descomposición $D - E - F$ por puntos de A_{k-1}:

$$A_{k-1} = D_{k-1} - E_{k-1} - F_{k-1}$$

1.1. Método de Newton–Jacobi de m pasos

$$\boxed{\begin{cases} u^0 = x^{k-1} \\ D_{k-1}u^p = (E_{k-1} + F_{k-1})u^{p-1} + b_{k-1}, \quad 1 \le p \le m \\ x^k = u^m \end{cases}}$$

$$\boxed{m = 1} \qquad \Longrightarrow \qquad x_i^k = x_i^{k-1} - \frac{F_i(x^{k-1})}{\dfrac{\partial F_i}{\partial x_i}(x^{k-1})}, \quad i = 1, 2, \dots, n.$$

1.2. Método de Newton–relajación de m pasos

$$\boxed{\begin{cases} u^0 = x^{k-1} \\ \left(\dfrac{D_{k-1}}{w} - E_{k-1}\right)u^p = \left(\dfrac{1-w}{w}D_{k-1} + F_{k-1}\right)u^{p-1} + b_{k-1}, \quad 1 \le p \le m \\ x^k = u^m \end{cases}}$$

$$\boxed{m = 1} \quad \Longrightarrow \quad x_i^k = x_i^{k-1} - \frac{w}{\dfrac{\partial F_i}{\partial x_i}(x^{k-1})}\left(F_i(x^{k-1}) - \sum_{j=1}^{i-1} \frac{\partial F_i}{\partial x_j}(x^{k-1})\left(x_j^{k-1} - x_j^k\right)\right), \quad i = 1, 2, \dots, n.$$

2. Generalización de métodos lineales

2.1. Método de Jacobi no lineal

$$F_i(x_1^{k-1}, x_2^{k-1}, \dots, x_{i-1}^{k-1}, u, x_{i+1}^{k-1}, \dots, x_n^{k-1}) = 0$$

$$\boxed{\begin{cases} u^0 = x_i^{k-1} \\ u^p = u^{p-1} - \dfrac{F_i(x_1^{k-1}, x_2^{k-1}, \dots, x_{i-1}^{k-1}, u^{p-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}{\dfrac{\partial F_i}{\partial x_i}(x_1^{k-1}, x_2^{k-1}, \dots, x_{i-1}^{k-1}, u^{p-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}, \quad 1 \le p \le m \\ x_i^k = u^m \end{cases}}$$

$$\boxed{m = 1} \quad \Longrightarrow \quad x_i^k = u^1 = x_i^{k-1} - \frac{F_i(x^{k-1})}{\dfrac{\partial F_i}{\partial x_i}(x^{k-1})}, \quad i = 1, 2, \dots, n.$$

2.2. Método de Gauss–Seidel no lineal

$$F_i(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, u, x_{i+1}^{k-1}, \dots, x_n^{k-1}) = 0$$

$$\boxed{\begin{cases} u^0 = x_i^{k-1} \\ u^p = u^{p-1} - \dfrac{F_i(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, u^{p-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}{\dfrac{\partial F_i}{\partial x_i}(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, u^{p-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}, \quad 1 \le p \le m \\ x_i^k = u^m \end{cases}}$$

$$\boxed{m = 1} \quad \Longrightarrow \quad x_i^k = u^1 = x_i^{k-1} - \frac{F_i(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, x_i^{k-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}{\dfrac{\partial F_i}{\partial x_i}(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, x_i^{k-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}, \quad i = 1, 2, \dots, n.$$

2.3. Método de relajación no lineal

$$\begin{cases} x_i^k = (1-w)x_i^{k-1} + wu \\ F_i(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, u, x_{i+1}^{k-1}, \dots, x_n^{k-1}) = 0 \end{cases}$$

$$\boxed{\begin{cases} u^0 = x_i^{k-1} \\ u^p = u^{p-1} - \dfrac{F_i(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, u^{p-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}{\dfrac{\partial F_i}{\partial x_i}(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, u^{p-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}, \quad 1 \le p \le m \\ x_i^k = (1-w)x_i^{k-1} + wu^m \end{cases}}$$

$$\boxed{m = 1} \quad \Longrightarrow \quad x_i^k = x_i^{k-1} - w\frac{F_i(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, x_i^{k-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}{\dfrac{\partial F_i}{\partial x_i}(x_1^{k}, x_2^{k}, \dots, x_{i-1}^{k}, x_i^{k-1}, x_{i+1}^{k-1}, \dots, x_n^{k-1})}, \quad i = 1, 2, \dots, n.$$