4.- Sea f una función entera. Si $R>0$ $a, b \in \mathbb{C}$ $a \neq b$, $|a|, |b| < R$

calcula

$$\int_{|z|=R} \frac{f(z)}{(z-a)(z-b)} \, dz \, . \quad \text{Deduces el } T^a \text{ de Liouville.}$$

$$\frac{1}{(z-a)(z-b)} = \frac{A}{z-a} + \frac{B}{z-\beta} = \frac{A(z-b) + B(z-a)}{(z-a)(z-b)}$$

$$1 = A(z-b) + B(z-a)$$

Si $z=b$ $\quad 1 = A \cdot 0 + B(b-a) \implies B = \frac{1}{b-a}$

$z=a$ $\quad 1 = A(a-b) \quad \iff A = \frac{1}{a-b}$

$$\implies \int_{|z|=R} \frac{f(z)}{(z-a)(z-b)} \, dz = \frac{1}{a-b} \int_{\gamma} \frac{f(z)}{z-a} \, dz + \frac{1}{b-a} \int_{\gamma} \frac{f(z)}{z-b} =$$