

$$\begin{aligned}
 \pi(\theta=1 | x_1, \dots, x_n) &= \frac{P(\theta=1) \cdot f(x_1, \dots, x_n | \theta=1)}{P(\theta=1) \cdot f(x_1, \dots, x_n | \theta=1) + P(\theta=2) \cdot f(x_1, \dots, x_n | \theta=2)} \\
 &= \frac{\frac{1}{2} \prod_{i=1}^n f(x_i | \theta=1)}{\frac{1}{2} \prod_{i=1}^n f(x_i | \theta=1) + \frac{1}{2} \prod_{i=1}^n f(x_i | \theta=2)} = \frac{\prod_{i=1}^n 1 \cdot x_i^0}{\prod_{i=1}^n 1 \cdot x_i^0 + \prod_{i=1}^n 2 \cdot x_i} \\
 &= \frac{1}{1 + 2^n \prod_{i=1}^n x_i}
 \end{aligned}$$

Análogamente

$$\pi(\theta=2 | x_1, \dots, x_n) = \frac{2^n \prod_{i=1}^n x_i}{1 + 2^n \prod_{i=1}^n x_i}$$

La región de rechazo es

$$R = \{ (x_1, \dots, x_n) \mid \pi(\theta=1 | x_1, \dots, x_n) < \pi(\theta=2 | x_1, \dots, x_n) \}$$

La condición  $\pi(\theta=1 | x_1, \dots, x_n) < \pi(\theta=2 | x_1, \dots, x_n)$  equivale a

$$\frac{1}{1 + 2^n \prod_{i=1}^n x_i} < \frac{2^n \prod_{i=1}^n x_i}{1 + 2^n \prod_{i=1}^n x_i} \Leftrightarrow 1 < 2^n \prod_{i=1}^n x_i \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{2^n} < \prod_{i=1}^n x_i$$

Esto es

$$R = \left\{ (x_1, \dots, x_n) \mid \frac{1}{2^n} < \prod_{i=1}^n x_i \right\}$$