I. E. S. " SAN ISIDRO "

Asignatura.................... Fecha ....

Alumno/a .... Curso.......... Nº..........

Apellidos Nombre

Calificación

$\Rightarrow f(z) = \operatorname{tg} z \sim z + \frac{2}{3!} z^3 = z + \frac{z^3}{3}$

d) $f(z) = \sqrt{z^2-1} = (z^2-1)^{1/2}$ $\quad f(0) = i$

$f'(z) = \frac{1}{2}(z^2-1)^{-1/2} \cdot 2z = z(z^2-1)^{-1/2}$; $f'(0) = 0$

$f''(z) = (z^2-1)^{-1/2} + z \cdot (z^2-1)^{-3/2} \cdot (-\frac{1}{2}) \cdot 2z =$

$= (z^2-1)^{-1/2} - z^2 (z^2-1)^{-3/2}$ $\quad f''(0) = i$

$f^{(3)}(0) = 0$

$f'''(z) = (z^2-1)^{-3/2} \cdot (-\frac{1}{2}) \cdot 2z - 2z(z^2-1)^{-3/2} - z^2 \cdot (z^2-1)^{-5/2} \cdot (-\frac{3}{2}) \cdot 2z =$

$= -z(z^2-1)^{-3/2} - 2z(z^2-1)^{-3/2} + 3z^3(z^2-1)^{-5/2} = -3z(z^2-1)^{-3/2} + 3z^3(z^2-1)^{-5/2}$

$f(z) = \sqrt{z^2-1} \sim i + \frac{i}{2!} z^2 = i + \frac{i}{2} z^2$

e) $f(z) = e^{\frac{1}{1-z}} = e^{(1-z)^{-1}}$ $\quad f(0) = e$

$f'(z) = e^{(1-z)^{-1}} \cdot (-1) \cdot (1-z)^{-2} \cdot (-1) = e^{(1-z)^{-1}} \cdot (1-z)^{-2}$; $f'(0) = e$

$f''(z) = e^{(1-z)^{-1}} \cdot (1-z)^{-2} \cdot (1-z)^{-2} + e^{(1-z)^{-1}} \cdot (1-z)^{-3} \cdot (-2) \cdot (-1) =$

$= e^{(1-z)^{-1}} (1-z)^{-4} + 2e^{(1-z)^{-1}} \cdot (1-z)^{-3}$; $f''(0) = 3e$

$f'''(z) = e^{(1-z)^{-1}} (1-z)^{-2} (1-z)^{-4} + e^{(1-z)^{-1}} \cdot (1-z)^{-5} \cdot (-4) \cdot (-1)$

$+ 2 e^{(1-z)^{-1}} (1-z)^{-2} \cdot (1-z)^{-3} + 2e^{(1-z)^{-1}} \cdot (1-z)^{-4} \cdot (-3)(-1) =$

$= e^{(1-z)^{-1}} (1-z)^{-6} + 4e^{(1-z)^{-1}} (1-z)^{-5} + 2e^{(1-z)^{-1}} (1-z)^{-5} + 6e^{(1-z)^{-1}} (1-z)^{-4}$

$f'''(0) = 13e$

$\Rightarrow f(z) = e^{\frac{1}{1-z}} \sim e + ez + \frac{3e}{2!} z^2 + \frac{13}{3!} e z^3 = e + ez + \frac{3e}{2} z^2 + \frac{13e}{6} z^3$