10.- Calcula los cuatro primeros términos del desarrollo en serie de Taylor en $z=0$ de la función:

a) $f(z) = \dfrac{1}{1+e^z} = (1+e^z)^{-1}$; $f(0) = \frac{1}{2}$

$f'(z) = -(1+e^z)^{-2} \cdot e^z$ $f'(0) = -\dfrac{1}{4}$

$f''(z) = -e^z(1+e^z)^{-2} + e^z \cdot 2(1+e^z)^{-3} \cdot e^z = 2e^{2z}(1+e^z)^{-3} - e^z(1+e^z)^{-2}$

$f''(0) = 0$

$f'''(z) = 2 \cdot e^{2z} \cdot 2(1+e^z)^{-3} - 2e^{2z} \cdot 3(1+e^z)^{-4} \cdot e^z - e^z(1+e^z)^{-2}$

$\qquad + 2e^z(1+e^z)^{-3} \cdot e^z = 6e^{2z}(1+e^z)^{-3} - 6e^{3z}(1+e^z)^{-4} - e^z(1+e^z)^{-2}$

$f'''(0) = \frac{1}{8}$

$\Rightarrow f(z) \sim \dfrac{1}{2} - \dfrac{1}{4}z + \dfrac{1}{8 \cdot 3!}z^3 = \dfrac{1}{2} - \dfrac{z}{4} + \dfrac{z^3}{48}$

b) $f(z) = e^z \cos z$ $f(0) = 1$

$f'(z) = e^z \cos z - e^z \operatorname{sen} z$ $f'(0) = 1$

$f''(z) = e^z \cos z - e^z \operatorname{sen} z - e^z \operatorname{sen} z - e^z \cos z = -2e^z \operatorname{sen} z$

$f''(0) = 0$

$f'''(z) = -2e^z \operatorname{sen} z - 2e^z \cos z$

$f^{3)}(0) = -2$

$\Rightarrow f(z) \sim 1 + z - \dfrac{2}{3!}z^3 = 1 + z - \dfrac{z^3}{3}$

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c) $f(z) = \operatorname{tg} z = \dfrac{\operatorname{sen} z}{\cos z}$ $\qquad\qquad\qquad\qquad$ $f(0) = 0$

$f'(z) = \dfrac{\cos^2 z + \operatorname{sen}^2 z}{(\cos z)^2} = \dfrac{1}{\cos^2 z} = 1 + \operatorname{tg}^2 z$ $\qquad$ $f'(0) = 1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $f''(0) = 0$ $\qquad$ $f'''(0) = 2$

$f''(z) = 2\operatorname{tg} z \cdot (1 + \operatorname{tg}^2 z) = 2\operatorname{tg} z + 2\operatorname{tg}^3 z$

$f'''(z) = 2(1 + \operatorname{tg}^2 z) + 2 \cdot 3 \cdot \operatorname{tg}^2 z(1 + \operatorname{tg}^2 z) = 2 + 2\operatorname{tg}^2 z + 6\operatorname{tg}^2 z + 6\operatorname{tg}^4 z = 2 + 8\operatorname{tg}^2 z + 6\operatorname{tg}^4 z$

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Alumno/a... Curso................... Nº.............

Apellidos   Nombre

Calificación

$\Rightarrow f(z) = tg\, z \sim z + \dfrac{2}{3!} z^3 = z + \dfrac{z^3}{3}$

d) $f(z) = \sqrt{z^2-1} = (z^2-1)^{1/2}$ $f(0) = i$

$f'(z) = \dfrac{1}{2}(z^2-1)^{-1/2}\cdot 2z = z(z^2-1)^{-1/2}\; ; \; f'(0) = 0$

$f''(z) = (z^2-1)^{-1/2} + z\cdot(z^2-1)^{-3/2}\cdot(-\tfrac{1}{2})\cdot 2z =$

$\qquad = (z^2-1)^{-1/2} - z^2(z^2-1)^{-3/2}$ $f''(0) = i$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad f^{3)}(0) = 0$

$f'''(z) = (z^2-1)^{-3/2}\cdot(-\tfrac{1}{2})\cdot 2z - 2z(z^2-1)^{-3/2} - z^2\cdot(z^2-1)^{-5/2}\cdot(-\tfrac{3}{2})\cdot 2z =$

$\qquad = -z(z^2-1)^{-3/2} - 2z(z^2-1)^{-3/2} + 3z^3(z^2-1)^{-5/2} = -3z(z^2-1)^{-3/2} + 3z^3(z^2-1)^{-5/2}$

$f(z) = \sqrt{z^2-1} \sim i + \dfrac{i}{2!} z^2 = i + \dfrac{i}{2} z^2$

e) $f(z) = e^{\frac{1}{1-z}} = e^{(1-z)^{-1}}$ $f(0) = e$

$f'(z) = e^{(1-z)^{-1}}\cdot(-1)\cdot(1-z)^{-2}\cdot(-1) = e^{(1-z)^{-1}}\cdot(1-z)^{-2}\; ; \; f'(0) = e$

$f''(z) = e^{(1-z)^{-1}}\cdot(1-z)^{-2}\cdot(1-z)^{-2} + e^{(1-z)^{-1}}\cdot(1-z)^{-3}\cdot(-2)\cdot(-1) =$

$\qquad = e^{(1-z)^{-1}}(1-z)^{-4} + 2e^{(1-z)^{-1}}\cdot(1-z)^{-3}\; ; \; f''(0) = 3e$

$f'''(z) = e^{(1-z)^{-1}}(1-z)^{-2}(1-z)^{-4} + e^{(1-z)^{-1}}\cdot(1-z)^{-5}\cdot(-4)\cdot(-1)$

$\qquad + 2e^{(1-z)^{-1}}(1-z)^{-2}\cdot(1-z)^{-3} + 2e^{(1-z)^{-1}}\cdot(1-z)^{-4}\cdot(-3)(-1) =$

$\qquad = e^{(1-z)^{-1}}(1-z)^{-6} + 4e^{(1-z)^{-1}}(1-z)^{-5} + 2e^{(1-z)^{-1}}(1-z)^{-5} + 6e^{(1-z)^{-1}}(1-z)^{-4}$

$f^{3)}(0) = 13e$

$\Rightarrow f(z) = e^{\frac{1}{1-z}} \sim e + ez + \dfrac{3e}{2!} z^2 + \dfrac{13}{3!} e z^3 = e + ez + \dfrac{3e}{2} z^2 + \dfrac{13e}{6} z^3$