Examen CI

Pregunta 2.-

Para calcular el volumen de $B_{n,\beta} = \{ \pi/2 - \beta < \phi_1 < \pi/2 + \beta \}$ realizamos la integral

$$\int_0^1 \int_0^{2\pi} \int_0^{\pi} \cdots \int_0^{\pi} \int_{\pi/2-\beta}^{\pi/2+\beta} |J_g| \cdot d\phi_1 \cdot d\phi_2 \cdots d\phi_{n-1} =$$

$$\left(\int_0^1 r^{n-1} dr\right) \cdot \left(\int_{\pi/2-\beta}^{\pi/2+\beta} sen^{n-2}\phi_1 \, d\phi_1\right) \cdot \prod_{i=2}^{n-2} \int_0^{\pi} sen^{n-i-1} \phi_i \, d\phi_i \cdot \int_0^{2\pi} d\phi_{n-1} =$$

$$\underset{\text{Simetria}}{=} \frac{1}{n} \cdot \left(\int_{\pi/2-\beta}^{\pi/2+\beta} sen^{n-2}\phi_1 \, d\phi_1\right) \cdot \prod_{i=2}^{n-2} 2 \cdot \int_0^{\pi/2} sen^{n-i-1}\phi_i \, d\phi_i \cdot 4 \int_0^{\pi/2} d\phi_{n-1}$$

$$= \frac{2}{n} \cdot \left(\int_{\pi/2-\beta}^{\pi/2+\beta} sen^{n-2}\phi_1 \, d\phi_1\right) \cdot \prod_{i=2}^{n-1} 2 \cdot \int_0^{\pi/2} sen^{n-i-1}\phi_i =$$

$$B(x,y) = 2\int_0^{\pi/2} \cos^{2x-1}(\theta) \cdot sen^{2y-1}(\theta) \, d\theta$$

$$= \frac{2}{n} \left(\int_{\pi/2-\beta}^{\pi/2+\beta} sen^{n-2}\phi_1 \, d\phi_1\right) \cdot \prod_{i=2}^{n-1} B\left(\frac{1}{2}, \frac{n-i}{2}\right) =$$

Juan Carlos Llamas Nuñez DNI - 11867802-D

$$= \frac{2}{n} \int_{\pi/2-\beta}^{\pi/2+\beta} \operatorname{sen}^{n-2}\phi_1 \, d\phi_1 \cdot \prod_{i=2}^{n-1} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{n-i}{2}\right)}{\Gamma\left(\frac{n-i+1}{2}\right)} =$$

$$= \frac{2\,\Gamma\left(\frac{1}{2}\right)^{n-1}}{n} \cdot \int_{\pi/2-\beta}^{\pi/2+\beta} \operatorname{sen}^{n-2}\phi_1 \, d\phi_1 \cdot \prod_{i=2}^{n-1} \Gamma\left(\frac{n-i}{2}\right) \cdot \frac{1}{\prod_{i=1}^{n-2} \Gamma\left(\frac{n-i}{2}\right)} =$$

$$= \frac{2}{n}\, \Gamma\left(\frac{1}{2}\right)^{n-2} \cdot \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \cdot \int_{\pi/2-\beta}^{\pi/2+\beta} \operatorname{sen}^{n-2}\phi_1 \, d\phi_1 =$$

$$= \frac{2\,\Gamma\left(\frac{1}{2}\right)^{n-1}}{n\,\Gamma\left(\frac{n-1}{2}\right)} \int_{\pi/2-\beta}^{\pi/2+\beta} \operatorname{sen}^{n-2}\phi_1 \, d\phi_1$$

Ahora $$\frac{Vol(B_{n,\beta})}{Vol(B_n)} = \frac{\frac{\cancel{2\Gamma\left(\frac{1}{2}\right)^{n-1}}}{\cancel{n}\,\Gamma\left(\frac{n-1}{2}\right)} \cdot \int_{\pi/2-\beta}^{\pi/2+\beta} \operatorname{sen}^{n-2}\phi_1 \, d\phi_1}{\frac{\cancel{2\Gamma\left(\frac{1}{2}\right)^{n}}}{\cancel{n}\,\Gamma\left(\frac{n}{2}\right)}} =$$

$$= \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \cdot \int_{\pi/2-\beta}^{\pi/2+\beta} \operatorname{sen}^{n-2}\phi_1 \, d\phi_1 \underset{\text{Simetría}}{=}$$

$$= \frac{2\,\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)} \int_{\pi/2}^{\pi/2+\beta} \operatorname{sen}^{n-2}\phi_1 \, d\phi_1$$

Juan Carlos Llamas Núñez DNI- 11867802-D