

$$\Rightarrow f(z) = \frac{z^6}{\left(\frac{z}{2}\right)^2 - \left(\frac{\operatorname{sen} z}{2}\right)^2} = \frac{z^6}{\left(\frac{z}{2}\right)^2 - \left(\frac{zh(z)}{2}\right)^2} = \frac{z^6}{\left(\frac{z}{2}\right)^2 (1 - h^2(z))} = \frac{4z^4}{1 - h^2(z)}$$

Sabemos que $h(z) = \begin{cases} 1 & \text{si } z=0 \\ \frac{\operatorname{sen} z}{z} & \text{si } z \neq 0 \end{cases}$

Sea $g(z) = 1 - h^2(z) = \begin{cases} 0 & \text{si } z=0 \\ 1 - \frac{\operatorname{sen}^2 z}{z^2} & \text{si } z \neq 0 \end{cases}$

Es claro que g tiene un cero en 0 y $g(z) \in \mathcal{H}(\mathbb{C})$. Veamos la multiplicidad de dicho 0 .

$$g'(z) \stackrel{z \neq 0}{=} - \frac{2\operatorname{sen} z \cos z \cdot z^2 - \operatorname{sen}^2 z \cdot 2z}{z^4} = \frac{2\operatorname{sen}^2 z - z \operatorname{sen} 2z}{z^3}$$

$$g'(0) = \lim_{z \rightarrow 0} \frac{1 - \frac{\operatorname{sen}^2 z}{z^2} - 0}{z - 0} = \lim_{z \rightarrow 0} \frac{z^2 - \operatorname{sen}^2 z}{z^3} = \lim_{z \rightarrow 0} \frac{(z + \operatorname{sen} z)(z - \operatorname{sen} z)}{z^3}$$

$$= 2 \cdot \lim_{z \rightarrow 0} \frac{z - \operatorname{sen} z}{z^2} \stackrel{*1}{=} 0$$

$$\Rightarrow g'(z) = \begin{cases} \frac{2\operatorname{sen}^2 z - z \operatorname{sen} 2z}{z^3} & \text{si } z \neq 0 \\ 0 & \text{si } z = 0 \end{cases}$$

$$g''(0) = \lim_{z \rightarrow 0} \frac{\frac{2\operatorname{sen}^2 z - z \operatorname{sen} 2z}{z^3} - 0}{z - 0} = \lim_{z \rightarrow 0} \frac{2\operatorname{sen}^2 z - z \operatorname{sen} 2z}{z^4} \stackrel{*2}{=} \frac{2}{3}$$

Por tanto g tiene un cero en 0 de multiplicidad 2 es decir

$\exists g_2 \in \mathcal{H}(\mathbb{C})$ tal que $g_2(0) \neq 0$ y $g(z) = z^2 \cdot g_2(z)$.