*1 $\int_0^{\pi/2} \ln(\operatorname{sen} u)\,du = -\frac{\pi}{2}\ln 2$

Sea $I = \int_0^{\pi/2} \ln(\operatorname{sen} u)\,du = \int_0^{\pi/2} \ln\left(\cos\left(\frac{\pi}{2}-u\right)\right) du =$

$\operatorname{sen} u = \cos\left(\frac{\pi}{2}-u\right)$

$\frac{\pi}{2} - u = x$

$-du = dx$

$u = \pi/2 \Rightarrow x = 0$

$u = 0 \Rightarrow x = \pi/2$

$= \int_{\pi/2}^{0} -\ln(\cos x)\,dx = \int_0^{\pi/2} \ln(\cos x)\,dx$

$\Rightarrow 2I = I + I = \int_0^{\pi/2} \ln(\operatorname{sen} x)\,dx + \int_0^{\pi/2} \ln(\cos x)\,dx =$

$= \int_0^{\pi/2} \ln(\operatorname{sen} x \cos x)\,dx = \int_0^{\pi/2} \ln\left(\frac{1}{2}\operatorname{sen} 2x\right) dx =$

$= \int_0^{\pi/2} \ln\left(\frac{1}{2}\right) dx + \int_0^{\pi/2} \ln(\operatorname{sen} 2x)\,dx = \left[\begin{array}{l} 2x = u \\ 2dx = du \\ x = 0 \Rightarrow u = 0 \\ x = \pi/2 \Rightarrow u = \pi \end{array}\right]$

$= -\ln 2 \cdot \frac{\pi}{2} + \frac{1}{2}\int_0^{\pi} \ln(\operatorname{sen} u)\,du =$

I $\ln(\operatorname{sen} u)$ es simétrica respecto a $\pi/2$

$= -\ln 2 \frac{\pi}{2} + \frac{2}{2}\int_0^{\pi/2} \ln(\operatorname{sen} u)\,du = -\ln 2 \cdot \frac{\pi}{2} + I$

$\Rightarrow 2I = -\ln 2 \cdot \frac{\pi}{2} + I \iff \boxed{I = -\ln 2 \cdot \frac{\pi}{2}}$