f) $f(z) = \log(1+e^{-s})$ cte. del. principal.

$f(z) = \log(1+e^{-5z})$ $\qquad f(0) = \log 2 \overset{\downarrow}{=} \log 2$

$f'(z) = \frac{1}{1+e^{-5z}} \cdot e^{-5z} \cdot (-5)$ $\qquad f'(0) = -5/2$

$\qquad \| $

$-5 \cdot \frac{e^{-5z}}{1+e^{-5z}}$

$$f''(z) = -5 \cdot \frac{e^{-5z}(-5)(1+e^{-5z}) - e^{-5z} \cdot e^{-5z} \cdot (-5)}{(1+e^{-5z})^2} = -5 \frac{-5e^{-5z} \cancel{-5e^{-10z}} \cancel{+5e^{-10z}}}{(1+e^{-5z})^2} =$$

$$= 25 \frac{e^{-5z}}{(1+e^{-5z})^2} \quad ; \quad f''(0) = 25/2$$

$$f'''(z) = 25 \cdot \frac{e^{-5z}(-5)(1+e^{-5z})^2 - e^{-5z} \cdot 2(1+e^{-5z}) \cdot e^{-5z} \cdot (-5)}{(1+e^{-5z})^4} =$$

$$\not= \quad f'''(0) = 25 \cdot \frac{-5 \cdot 2^2 + 10 \cdot 2}{2^4} = 0$$

$$f(z) = \log(1+e^{-5z}) \sim \log 2 - 5/2\, z + \frac{25}{2 \cdot 2!} z^2 = \log 2 - \frac{5}{2} z + \frac{25}{4} z^2$$

g) $f(z) = \log(1+\cos z)$ $\qquad f(0) = \log 2$

$f'(z) = \frac{-\operatorname{sen} z}{1+\cos z}$ $\qquad f'(0) = 0$

$$f''(z) = \frac{-\cos z(1+\cos z) + \operatorname{sen} z(-\operatorname{sen} z)}{(1+\cos z)^2} = \frac{-\cos z - \cos^2 z - \operatorname{sen}^2 z}{(1+\cos z)^2} = -\frac{1+\cos z}{(1+\cos z)^2} =$$

$$= -(1+\cos z)^{-1} \qquad f''(0) = -\frac{1}{2}$$

$$f'''(z) = +(1+\cos z)^{-2} \cdot (-\operatorname{sen} z) \qquad f^{3)}(0) = 0$$

$$\Rightarrow f(z) = \log(1+\cos z) \sim \log 2 - \frac{1}{2 \cdot 2!} z^2 = \log 2 - \frac{z^2}{4}$$