10.- Calcula los cuatro primeros términos del desarrollo en serie de Taylor en $z=0$ de la función:

a) $f(z) = \frac{1}{1+e^z} = (1+e^z)^{-1}$; $f(0) = 1/2$

$f'(z) = -(1+e^z)^{-2} \cdot e^z \qquad f'(0) = -\frac{1}{4}$

$f''(z) = -e^z(1+e^z)^{-2} + e^z \cdot 2(1+e^z)^{-3} \cdot e^z = 2e^{2z}(1+e^z)^{-3} - e^z(1+e^z)^{-2}$

$f''(0) = 0$

$f'''(z) = 2 \cdot e^{2z} \cdot 2(1+e^z)^{-3} - 2e^{2z} \cdot 3(1+e^z)^{-4} \cdot e^z - e^z(1+e^z)^{-2}$

$+ 2e^z(1+e^z)^{-3} \cdot e^z = 6e^{2z}(1+e^z)^{-3} - 6e^{3z}(1+e^z)^{-4} - e^z(1+e^z)^{-2}$

$f'''(0) = 1/8$

$\Rightarrow f(z) \sim \frac{1}{2} - \frac{1}{4}z + \frac{1}{8 \cdot 3!}z^3 = \frac{1}{2} - \frac{z}{4} + \frac{z^3}{48}$

b) $f(z) = e^z \cos z \qquad f(0) = 1$

$f'(z) = e^z \cos z - e^z \operatorname{sen} z \qquad f'(0) = 1$

$f''(z) = e^z \cos z - e^z \operatorname{sen} z - e^z \operatorname{sen} z - \cancel{e^z \cos z} = -2e^z \operatorname{sen} z$

$f''(0) = 0$

$f'''(z) = -2e^z \operatorname{sen} z - 2e^z \cos z$

$f^{(3)}(0) = -2$

$\Rightarrow f(z) \sim 1 + z - \frac{2}{3!}z^3 = 1 + z - \frac{z^3}{3}$

c) $f(z) = \operatorname{tg} z = \frac{\operatorname{sen} z}{\cos z} \qquad f(0) = 0$

$f'(z) = \frac{\cos^2 z + \operatorname{sen}^2 z}{(\cos z)^2} = \frac{1}{\cos^2 z} = 1 + \operatorname{tg}^2 z \qquad f'(0) = 1$

$f''(0) = 0 \qquad f'''(0) = 2$

$f''(z) = 2\operatorname{tg} z \cdot (1 + \operatorname{tg}^2 z) = 2\operatorname{tg} z + 2\operatorname{tg}^3 z$

$f'''(z) = 2(1+\operatorname{tg}^2 z) + 2 \cdot 3 \cdot \operatorname{tg}^2 z(1+\operatorname{tg}^2 z) = 2 + 2\operatorname{tg}^2 z + 6\operatorname{tg}^2 z + 6\operatorname{tg}^4 z = 2 + 8\operatorname{tg}^2 z + 6\operatorname{tg}^4 z$