

Calculamos  $K$ :

$$K = \frac{\lambda_1 \lambda_2}{\left( \frac{\lambda_2}{\lambda_1} \frac{w}{1-w} \lambda_1 + \lambda_2 \right)^2} \cdot \frac{\lambda_2}{\lambda_1} \cdot \frac{1}{(1-w)^2} = \frac{\lambda_2^2}{\left( \lambda_2 \left( \frac{w}{1-w} + 1 \right) \right)^2} \cdot \frac{1}{(1-w)^2} =$$

$$= \frac{1}{\left( \frac{w+1-w}{1-w} \right)^2} \cdot \frac{1}{(1-w)^2} = 1$$

Por tanto  $f_W(w) = \frac{1}{\text{Beta}(n_1, n_2)} \cdot w^{n_1-1} (1-w)^{n_2-1}$  con  $w \in (0, 1)$ ,  
es decir,  $W \sim \text{Beta}(n_1, n_2)$ .

Esta va a ser nuestra cantidad pivotal porque recordemos que

$$W = \frac{1}{1 + \frac{\lambda_2}{\lambda_1} w} = \frac{1}{1 + \frac{\lambda_2}{\lambda_1} A/B} = \frac{1}{1 + \frac{\lambda_2 B}{\lambda_1 A}} = \frac{1}{1 + \frac{\lambda_2 \sum_{i=1}^{n_2} Y_i}{\lambda_1 \sum_{i=1}^{n_1} X_i}} =$$

$$= \frac{1}{1 + \frac{\lambda_2 n_2 \bar{Y}}{\lambda_1 n_1 \bar{X}}} \quad \text{y esta distribución de esta cantidad no}$$

depende de  $\lambda_1$  ni  $\lambda_2$ .

Por tanto nos podemos construir nuestro intervalo de nivel de confianza  $1-\alpha$  como

$$P(a \leq W \leq b) = 1-\alpha$$

$$a \leq W \leq b \Leftrightarrow a \leq \frac{1}{1 + \frac{\lambda_2 n_2 \bar{Y}}{\lambda_1 n_1 \bar{X}}} \leq b \Leftrightarrow \frac{1}{a} \geq 1 + \frac{\lambda_2 n_2 \bar{Y}}{\lambda_1 n_1 \bar{X}} \geq \frac{1}{b} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{a} - 1 \geq \frac{\lambda_2 n_2 \bar{Y}}{\lambda_1 n_1 \bar{X}} \geq \frac{1}{b} - 1 \Leftrightarrow \frac{1-a}{a} \geq \frac{\lambda_2 n_2 \bar{Y}}{\lambda_1 n_1 \bar{X}} \geq \frac{1-b}{b} \Leftrightarrow$$

$$\Leftrightarrow \frac{a}{1-a} \leq \frac{\lambda_1 n_1 \bar{X}}{\lambda_2 n_2 \bar{Y}} \leq \frac{b}{1-b} \Leftrightarrow \frac{a}{1-a} \cdot \frac{n_2 \bar{Y}}{n_1 \bar{X}} \leq \frac{\lambda_1}{\lambda_2} \leq \frac{b}{1-b} \cdot \frac{n_2 \bar{Y}}{n_1 \bar{X}}$$