$$\Pi(p|x_1 \dots x_n) = \frac{\Pi(p)\cdot f(x_1 \dots x_n|p)}{\int_0^1 \Pi(p)\, f(x_1 \dots x_n|p)\,dp} = \frac{\cancel{\frac{1}{Beta(\alpha_0,\beta_0)}}\, p^{\alpha_0-1}(1-p)^{\beta_0-1}\, p^{\sum x_i}(1-p)^{n^2-\sum x_i}\, \cancel{\prod_{i=1}^{n}\binom{n}{x_i}}}{\int_0^1 \cancel{\frac{1}{Beta(\alpha_0,\beta_0)}}\, p^{\alpha_0-1}(1-p)^{\beta_0-1}\, p^{\sum x_i}(1-p)^{n^2-\sum x_i}\, \cancel{\prod_{i=1}^{n}\binom{n}{x_i}}\,dp} =$$

$$= \frac{p^{\alpha_0+\sum_{i=1}^{n} x_i-1}\,(1-p)^{\beta_0+n^2-\sum x_i-1}}{Beta(\alpha_0+\sum x_i, \beta_0+n^2-\sum x_i)\int_0^1 \frac{1}{Beta(\alpha_0+\sum x_i, \beta_0+n^2-\sum x_i)}\, p^{\alpha_0+\sum_{i=1}^{n} x_i-1}\,(1-p)^{\beta_0+n^2-\sum_{i=1}^{n} x_i-1}\,dp} =$$

$$= \frac{1}{Beta(\alpha_0+\sum_{i=1}^{n} x_i, \beta_0+n^2-\sum_{i=1}^{n} x_i)}\, p^{\alpha_0+\sum_{i=1}^{n} x_i-1}\,(1-p)^{\beta_0+n^2-\sum_{i=1}^{n} x_i-1} \sim Beta(\alpha_0+\sum_{i=1}^{n} x_i, \beta_0+n^2-\sum_{i=1}^{n} x_i)$$

Efectivamente la distribución de la muestra es la familia conjugada natural de la distribución Beta.

Nótese que tanto $\alpha_0+\sum x_i$ con $\beta_0+n^2-\sum_{i=1}^{n} x_i$ son positivos ya que

$$x_i \leq n \quad \forall i=1 \dots n \implies \sum_{i=1}^{n} x_i \leq n^2 \implies \beta_0+n^2-\sum_{i=1}^{n} x_i > 0$$