

$$I_1 = \int_0^{2\pi} \left(\frac{a}{\sqrt{6}} \sin t - \frac{a}{\sqrt{2}} \cos t \right) \left(-\frac{a}{\sqrt{2}} \sin t + \frac{a}{\sqrt{6}} \cos t \right) dt =$$

$$= \int_0^{2\pi} \left(-\frac{a^2}{\sqrt{12}} \sin^2 t + \frac{a^2}{6} \sin t \cos t + \frac{a^2}{2} \sin t \cos t - \frac{a^2}{\sqrt{12}} \cos^2 t \right) dt$$

$$= -\frac{a^2}{\sqrt{12}} \int_0^{2\pi} (\sin^2 t + \cos^2 t) dt + \frac{2a^2}{3} \int_0^{2\pi} \sin t \cos t dt =$$

$$= -\frac{a^2 2\pi}{\sqrt{12}} + \frac{a^2}{3} \sin^2 t \Big|_0^{2\pi} = -\frac{a^2 \pi}{\sqrt{3}}$$

$$I_2 = \int_0^{2\pi} \left(-a\sqrt{\frac{2}{3}} \sin t \right) \left(\frac{a}{\sqrt{6}} \cos t + \frac{a}{\sqrt{2}} \sin t \right) dt =$$

$$= \int_0^{2\pi} \left(-a^2 \sqrt{\frac{2}{3 \cdot 6}} \sin t \cos t - a^2 \sqrt{\frac{2}{3 \cdot 2}} \sin^2 t \right) dt =$$

$$= -\frac{a^2}{6} \int_0^{2\pi} 2 \sin t \cos t dt - \frac{a^2}{\sqrt{3}} \int_0^{2\pi} \sin^2 t dt =$$

$$\sin^2 t + \cos^2 t = 1$$

$$\cos^2 t - \sin^2 t = \cos 2t$$

$$= -\frac{a^2}{6} \sin^2 t \Big|_0^{2\pi} - \frac{a^2}{2\sqrt{3}} \left(\int_0^{2\pi} dt - \int_0^{2\pi} \cos 2t dt \right) = -\frac{a^2}{2\sqrt{3}} \left(2\pi + \frac{\sin 2t}{2} \Big|_0^{2\pi} \right)$$

$$= -\frac{a^2 \pi}{\sqrt{3}}$$

$$I_3 = \int_0^{2\pi} \left(\frac{a}{\sqrt{2}} \cos t + \frac{a}{\sqrt{6}} \sin t \right) \left(-a\sqrt{\frac{2}{3}} \cos t \right) dt =$$

$$= \int_0^{2\pi} \left(-\frac{a^2}{\sqrt{3}} \cos^2 t - \frac{a^2 \sqrt{2}}{\sqrt{6 \cdot 3}} \sin t \cos t \right) dt = -\frac{a^2}{\sqrt{3}} \int_0^{2\pi} \cos^2 t dt - \frac{a^2}{6} \int_0^{2\pi} 2 \sin t \cos t dt =$$

$$= -\frac{a^2}{2\sqrt{3}} \left(\int_0^{2\pi} dt + \int_0^{2\pi} \cos 2t dt \right) - \frac{a^2}{6} \sin^2 t \Big|_0^{2\pi} = -\frac{a^2 \pi}{\sqrt{3}} - \frac{a^2}{2\sqrt{3}} \frac{\sin 2t}{2} \Big|_0^{2\pi} =$$

$$\sin^2 t + \cos^2 t = 1$$

$$\cos^2 t - \sin^2 t = \cos 2t$$

$$= -\frac{a^2 \pi}{\sqrt{3}}$$