



I. E. S. " SAN ISIDRO "

Calificación

Asignatura..... Fecha

Alumno/a..... Curso..... Nº.....

Apellidos

Nombre

$$\Rightarrow f(z) = \operatorname{tg} z \sim z + \frac{2}{3!} z^3 = z + \frac{z^3}{3}$$

$$d) f(z) = \sqrt{z^2 - 1} = (z^2 - 1)^{1/2} \quad f(0) = i$$

$$f'(z) = \frac{1}{2} (z^2 - 1)^{-1/2} \cdot 2z = z (z^2 - 1)^{-1/2} ; f'(0) = 0$$

$$f''(z) = (z^2 - 1)^{-1/2} + z \cdot (z^2 - 1)^{-3/2} \cdot (-1/2) \cdot 2z =$$

$$= (z^2 - 1)^{-1/2} - z^2 (z^2 - 1)^{-3/2} \quad f''(0) = i$$

$$f'''(z) = (z^2 - 1)^{-3/2} \cdot (-1/2) \cdot 2z - 2z (z^2 - 1)^{-3/2} - z^2 \cdot (z^2 - 1)^{-5/2} \cdot (-3/2) \cdot 2z =$$

$$= -z (z^2 - 1)^{-3/2} - 2z (z^2 - 1)^{-3/2} + 3z^3 (z^2 - 1)^{-5/2} = -3z (z^2 - 1)^{-3/2} + 3z^3 (z^2 - 1)^{-5/2}$$

$$f(z) = \sqrt{z^2 - 1} \sim i + \frac{i}{2!} z^2 = i + \frac{i}{2} z^2$$

$$e) f(z) = e^{\frac{1}{1-z}} = e^{(1-z)^{-1}} \quad f(0) = e$$

$$f'(z) = e^{(1-z)^{-1}} \cdot (-1) \cdot (1-z)^{-2} \cdot (-1) = e^{(1-z)^{-1}} \cdot (1-z)^{-2} ; f'(0) = e$$

$$f''(z) = e^{(1-z)^{-1}} \cdot (1-z)^{-2} \cdot (1-z)^{-2} + e^{(1-z)^{-1}} \cdot (1-z)^{-3} \cdot (-2) \cdot (-1) =$$

$$= e^{(1-z)^{-1}} (1-z)^{-4} + 2e^{(1-z)^{-1}} (1-z)^{-3} ; f''(0) = 3e$$

$$f'''(z) = e^{(1-z)^{-1}} (1-z)^{-2} (1-z)^{-4} + e^{(1-z)^{-1}} \cdot (1-z)^{-5} \cdot (-4) \cdot (-1)$$

$$+ 2e^{(1-z)^{-1}} (1-z)^{-2} \cdot (1-z)^{-3} + 2e^{(1-z)^{-1}} (1-z)^{-4} \cdot (-3) \cdot (-1) =$$

$$= e^{(1-z)^{-1}} (1-z)^{-6} + 4e^{(1-z)^{-1}} (1-z)^{-5} + 2e^{(1-z)^{-1}} (1-z)^{-5} + 6e^{(1-z)^{-1}} (1-z)^{-4}$$

$$f'''(0) = 13e$$

$$\Rightarrow f(z) = e^{\frac{1}{1-z}} \sim e + ez + \frac{3e}{2!} z^2 + \frac{13e}{3!} z^3 = e + ez + \frac{3e}{2} z^2 + \frac{13e}{6} z^3$$