Si calculamos ahora dichas esperanzas

$$E[X_{(1)}] = \int_\theta^{4\theta} x\cdot f_{X_{(1)}}(x)\,dx = \int_\theta^{4\theta} x\cdot n\left(1-\frac{x-\theta}{3\theta}\right)^{n-1}\cdot\frac{1}{3\theta}\,dx =$$

$$= \frac{n}{3\theta}\int_\theta^{4\theta} x\cdot\left(\frac{4\theta-x}{3\theta}\right)^{n-1}dx \underset{\uparrow}{=} \frac{n}{3\theta}\int_1^0 (4\theta-3\theta y)\,y^{n-1}(-3\theta)\,dy =$$

$y=\frac{4\theta-x}{3\theta}$ $\quad x=4\theta\Rightarrow y=0$

$dy=-\frac{dx}{3\theta}$ $\quad x=\theta\Rightarrow y=1$

$x=4\theta-3\theta y$

$$= n\int_0^1 4\theta y^{n-1}\,dy - n\int_0^1 3\theta y^n\,dy = 4\theta - 3\theta\frac{n}{n+1} = \frac{4\theta n+4\theta-3\theta n}{n+1} =$$

$$= \frac{\theta n+4\theta}{n+1} = \theta\,\frac{n+4}{n+1}$$

$$E[X_{(n)}] = \int_\theta^{4\theta} x\, f_{X_{(n)}}(x)\,dx = \int_\theta^{4\theta} x\cdot n\left(\frac{x-\theta}{3\theta}\right)^{n-1}\frac{1}{3\theta}\,dx \underset{\uparrow}{=}$$

$y=\frac{x-\theta}{3\theta}$ $\quad x=4\theta\Rightarrow y=1$

$dy=\frac{dx}{3\theta}$ $\quad x=\theta\Rightarrow y=0$

$x=\theta+3\theta y$

$$= n\int_0^1(\theta+3\theta y)\,y^{n-1}\,dy = n\theta\left(\int_0^1 y^{n-1}dy + 3\int_0^1 y^n\,dy\right) = n\theta\left(\frac{1}{n}+\frac{3}{n+1}\right) =$$

$$= n\theta\left(\frac{n+1+3n}{n(n+1)}\right) = \theta\,\frac{4n+1}{n+1}$$

Basta entonces tomar como función $g(T)=g(X_{(1)},X_{(n)}) = \frac{X_{(1)}}{n+4} - \frac{X_{(n)}}{4n+1}$.

Esta función es claramente no nula y se esperanza es

$$E_\theta[g(T)] = E_\theta\left[\frac{X_{(1)}}{n+4}-\frac{X_{(n)}}{4n+1}\right] = \frac{1}{n+4}E[X_{(1)}] - \frac{1}{4n+1}E[X_{(n)}] =$$

$$= \frac{1}{n+4}\cdot\theta\,\frac{n+4}{n+1} - \frac{1}{4n+1}\cdot\theta\cdot\frac{4n+1}{n+1} = 0.$$