En el lado de la derecha:

$$\nabla\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ F_1 & F_2 & F_3\end{vmatrix}=\left(\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z}\right)\vec{i}+\left(\frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x}\right)\vec{j}+\left(\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right)\vec{k}$$

$$\vec{G}\cdot(\nabla\times\vec{F})=\frac{\partial F_3}{\partial y}G_1-\frac{\partial F_2}{\partial z}G_1+\frac{\partial F_1}{\partial z}G_2-\frac{\partial F_3}{\partial x}G_2+\frac{\partial F_2}{\partial x}G_3-\frac{\partial F_1}{\partial y}G_3$$

Análogamente

$$-\vec{F}\cdot(\nabla\times\vec{G})=-F_1\frac{\partial G_3}{\partial y}+F_1\frac{\partial G_2}{\partial z}-F_2\frac{\partial G_1}{\partial z}+F_2\frac{\partial G_3}{\partial x}-F_3\frac{\partial G_2}{\partial x}+F_3\frac{\partial G_1}{\partial y}$$

Si sumamos estas dos cantidades

$$\vec{G}\cdot(\nabla\times\vec{F})-\vec{F}(\nabla\times\vec{G})=\underbrace{\frac{\partial F_3}{\partial y}G_1}_{5}-\underbrace{\frac{\partial F_2}{\partial z}G_1}_{11}+\underbrace{\frac{\partial F_1}{\partial z}G_2}_{9}-\underbrace{\frac{\partial F_3}{\partial x}G_2}_{3}+\underbrace{\frac{\partial F_2}{\partial x}G_3}_{1}-\underbrace{\frac{\partial F_1}{\partial y}G_3}_{7}$$

$$-\underbrace{F_1\frac{\partial G_3}{\partial y}}_{8}+\underbrace{F_1\frac{\partial G_2}{\partial z}}_{10}-\underbrace{F_2\frac{\partial G_1}{\partial z}}_{12}+\underbrace{F_2\frac{\partial G_3}{\partial x}}_{2}-\underbrace{F_3\frac{\partial G_2}{\partial x}}_{4}+\underbrace{F_3\frac{\partial G_1}{\partial y}}_{6}=\text{(reordenamos términos)}$$

$$=\frac{\partial F_2}{\partial x}G_3+F_2\frac{\partial G_3}{\partial x}-\frac{\partial F_3}{\partial x}G_2-F_3\frac{\partial G_2}{\partial x}+\frac{\partial F_3}{\partial y}G_1+F_3\frac{\partial G_1}{\partial y}$$

$$-\frac{\partial F_1}{\partial y}G_3-F_1\frac{\partial G_3}{\partial y}+\frac{\partial F_1}{\partial z}G_2+F_1\frac{\partial G_2}{\partial z}-\frac{\partial F_2}{\partial z}G_1-F_2\frac{\partial G_1}{\partial z}=$$

$$=\nabla\cdot(\vec{F}\times\vec{G})$$