I. E. S. " SAN ISIDRO "

Asignatura.................... Fecha ....

Alumno/a .... Curso.............. Nº..........

Apellidos Nombre

Calificación

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx \overset{g(x)\text{ es impar.}}{=} \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\,dx = \frac{2}{\pi}\int_0^{\pi} -\ln 2\cos nx\,dx + \frac{2}{\pi}\int_0^{\pi} h(x)\cdot\cos nx\,dx$$

Nótese que $\int_0^{\pi} \ln 2 \cdot \cos(nx)dx = \ln 2 \cdot \frac{\operatorname{sen}(nx)}{n}\Big]_0^{\pi} = 0$

$$\int_0^{\pi} h(x)\cos(nx)dx = -\int_0^{\pi} \ln(\operatorname{sen}(\tfrac{x}{2}))\cos(nx)\,dx =$$

$\ln(\operatorname{sen}\frac{x}{2}) = u$ $\quad \cos(nx)dx = dv$

$\frac{1}{2}\frac{\cos\frac{x}{2}}{\operatorname{sen}\frac{x}{2}}dx = du$ $\quad \frac{\operatorname{sen}(nx)}{n} = v$

$$= -\frac{\ln(\operatorname{sen}\frac{x}{2})\cdot \operatorname{sen}(nx)}{2n}\Big]_0^{\pi} + \frac{1}{2n}\int_0^{\pi} \operatorname{cotg}(\tfrac{x}{2})\cdot\operatorname{sen}(nx)\,dx =$$

$$= 0 + \frac{1}{2n}\lim_{x\to 0^+} \ln(\operatorname{sen}\tfrac{x}{2})\cdot\operatorname{sen}(nx) + \frac{1}{2n}\int_0^{\pi} \operatorname{cotg}(\tfrac{x}{2})\cdot \operatorname{sen}(nx)\,dx =$$

$$= 0 + \frac{\pi}{2n}$$

(Queda demostrar que $\ln(\operatorname{sen}\frac{x}{2})\cdot\operatorname{sen}(nx) \xrightarrow[x\to 0^+]{} 0$ *2

y $\int_0^{\pi}\operatorname{cotg}(\frac{x}{2})\cdot\operatorname{sen}(nx)\,dx = \pi$ *3)

$$\Rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = 0 + \frac{2}{\pi}\cdot\frac{\pi}{2n} = \frac{1}{n}$$

Por tanto $-\ln\left|2\operatorname{sen}\frac{x}{2}\right| = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + b_n\operatorname{sen}(nx) =$

$$= 0 + \sum_{n=1}^{\infty}\frac{1}{n}\cos(nx) \iff \sum_{n=1}^{\infty}\cos(nx) = -\ln\left|2\operatorname{sen}\frac{x}{2}\right| \quad 0<|x|\le\pi$$