Modelización

jullamas

May 2021

1 Introduction

Objetivo:

$T(t)$ = temperatura en la Tierra

$$\frac{\partial T}{\partial t} \propto E_{in} - E_{out}$$

$$T' \propto E_{in} - E_{out}$$

$$S(x) = \frac{\text{Energía/tiempo}}{\text{Superficie a dist. } x}$$

$$S(r) = S_0 \approx 1361 \frac{W}{m^2}$$
$$r = 149.600.000 \ km$$

$$\text{Flujo total} = \pi R^2 S_0$$

$$Q = \frac{\text{Flujo atmos.}}{\text{Superficie}} = \frac{\pi R^2 S_0}{4\pi R^2} = \frac{S_0}{4}$$

$$\text{Albedo} = \alpha$$
$$\text{Co-albedo} = 1 - \alpha$$

$$E_{in} = (1 - \alpha)Q$$

$$T(t,\theta) \text{ o } T(t,y)$$
$$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$$
$$y = sin(\theta)$$

Ecuador

$$\theta = 0 \Rightarrow y = 0$$

Polo Norte

$$\theta = \pi/2 \Rightarrow y = 1$$

Polo Sur

$$\theta = -\pi/2 \Rightarrow y = -1$$

$$s(y) = \text{distribución radiación}$$
$$\text{solar por latitudes}$$
$$\int_0^1 s(y)\, dy = 1$$
$$\frac{\text{Flujo atmos.}}{\text{Superficie}}(y) = Qs(y)$$

$$\alpha(y) \neq cte$$

$$E_{in}(y) = (1 - \alpha(y))Qs(y)$$

$$T_c \approx \ -10°C$$
$$T(y_s) = T_c$$

$$\alpha(y) = \begin{cases} \alpha_1 = 0.32, & y < y_s \\ \alpha_2 = 0.62, & y > y_s \\ \alpha_0 = \frac{\alpha_1 + \alpha_2}{2} = 0.47, & y = y_s \end{cases}$$

$$E_{out} = \ ?$$

W. Sellers

Ley de Stefan-Boltzmann:

$I(T) = \sigma T^4$ con T en Kelvin y

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

Emisividad $= \epsilon$

$$0 < \epsilon < 1$$

$$E_{out} = I(T) = \epsilon \sigma T^4$$

M. Budyko

$$E_{out} = I(T) = A + BT$$

T en grados centígrados

$$f(t) = T^4, \quad f'(T_0) = 4T_0^3$$

$$y - f(T_0) = f'(t_0)(x - T_0) \quad \Longleftrightarrow \quad y = T_0^4 + 4T_0^3(x - T_0) = T_0^4\left(1 + \frac{4(x - T_0)}{T_0}\right)$$

$$f(T) = T^4 \approx T_0^4\left(1 + \frac{4(T - T_0)}{T_0}\right) \quad \Longrightarrow \quad I(T) = \epsilon\sigma T^4 \approx \epsilon\sigma T_0^4\left(1 + \frac{4(T - T_0)}{T_0}\right)$$

$$T_0 = 273K = 0°C, \quad A = \epsilon\sigma T_0^4, \quad B = \frac{4A}{T_0} = 4\epsilon\sigma T_0^3$$

$$I(T) \approx A + B(T - T_0) = A + BT$$

$$\frac{\partial T}{\partial t} \propto E_{in} - E_{out} + D(y)$$

$$D(y) = C(\bar{T} - T(y))$$

$$\bar{T} = \int_0^1 T(y)\, dy = \frac{1}{2}\int_{-1}^1 T(y)\, dy$$

$$\text{Si} \quad T(y) < \bar{T} \quad \Longrightarrow \quad D(y) > 0 \quad \Longrightarrow \quad T(y) \uparrow$$

$$\text{Si} \quad T(y) > \bar{T} \quad \Longrightarrow \quad D(y) < 0 \quad \Longrightarrow \quad T(y) \downarrow$$

$$R\frac{\partial T}{\partial t} = E_{in}(y) - E_{out} + D(y) =$$

$$= (1 - \alpha(y))Qs(y) - (A + BT) + C(\bar{T} - T(y))$$

$$R\bar{T}' = (1 - \bar{\alpha})Q - (A + B\bar{T})$$