

f) $f(z) = \log(1+e^{-sz})$ cte. del. principal.

$$f(z) = \log(1+e^{-sz}) \quad f(0) = \log 2 = \log 2$$

$$f'(z) = \frac{1}{1+e^{-sz}} \cdot e^{-sz} \cdot (-s) \quad f'(0) = -s/2$$

$$\parallel -s \cdot \frac{e^{-sz}}{1+e^{-sz}}$$

$$f''(z) = -s \cdot \frac{e^{-sz} \cdot (-s)(1+e^{-sz}) - e^{-sz} \cdot e^{-sz} \cdot (-s)}{(1+e^{-sz})^2} = -s \frac{-se^{-sz} - se^{-2sz} + se^{-2sz}}{(1+e^{-sz})^2} =$$

$$= 2s \frac{e^{-sz}}{(1+e^{-sz})^2} ; f''(0) = 2s/2$$

$$f'''(z) = 2s \cdot \frac{e^{-sz}(-s)(1+e^{-sz})^2 - e^{-sz} \cdot 2(1+e^{-sz}) \cdot e^{-sz} \cdot (-s)}{(1+e^{-sz})^4} =$$

$$\frac{1}{2} f'''(0) = 2s \cdot \frac{-s \cdot 2^2 + 10 \cdot 2}{2^4} = 0$$

$$f(z) = \log(1+e^{-sz}) \sim \log 2 - \frac{s}{2}z + \frac{2s}{2 \cdot 2!}z^2 = \log 2 - \frac{s}{2}z + \frac{2s}{4}z^2$$

g) $f(z) = \log(1+\cos z)$ $f(0) = \log 2$

$$f'(z) = \frac{-\sin z}{1+\cos z} \quad f'(0) = 0$$

$$f''(z) = \frac{-\cos z(1+\cos z) + \sin z(-\sin z)}{(1+\cos z)^2} = \frac{-\cos z - \cos^2 z - \sin^2 z}{(1+\cos z)^2} = -\frac{1+\cos z}{(1+\cos z)^2} =$$

$$= -(1+\cos z)^{-1} \quad f''(0) = -\frac{1}{2}$$

$$f'''(z) = + (1+\cos z)^{-2} \cdot (-\sin z) \quad f'''(0) = 0$$

$$\Rightarrow f(z) = \log(1+\cos z) \sim \log 2 - \frac{1}{2 \cdot 2!}z^2 = \log 2 - \frac{z^2}{4}$$