$$\Pi(\theta|x_1 \ldots x_n) = \frac{\Pi(\theta) f(x_1 \ldots x_n|\theta)}{\int_0^\infty \Pi(\theta) f(x_1 \ldots x_n|\theta) d\theta} = \frac{\cancel{\frac{a^p}{\Gamma(p)}} \cdot e^{-a\theta} \theta^{p-1} \cdot \theta^n e^{-\theta \sum_{i=1}^n x_i}}{\int_0^\infty \cancel{\frac{a^p}{\Gamma(p)}} e^{-a\theta} \theta^{p-1} \theta^n e^{-\theta \sum_{i=1}^n x_i} d\theta} =$$

$$= \frac{e^{-\theta(a+\sum_{i=1}^n x_i)} \theta^{p+n-1}}{\frac{\Gamma(p+n)}{(a+\sum x_i)^{p+n}} \int_0^\infty \frac{(a+\sum x_i)^{p+n}}{\Gamma(p+n)} e^{-\theta(a+\sum_{i=1}^n x_i)} \theta^{p+n-1} d\theta} = \frac{(a+\sum_{i=1}^n x_i)^{p+n}}{\Gamma(p+n)} e^{-\theta(a+\sum_{i=1}^n x_i)} \theta^{p+n-1}$$

$\Rightarrow \Pi(\theta|x_1 \ldots x_n) \sim Gamma(a+\sum_{i=1}^n x_i, p+n)$

c) $(X_1 \ldots X_n)$ mas con $X \sim N(\theta, \frac{1}{r})$ con r conocido.

En clase de teoria se pudo ver que si $X \sim N(\theta, \sigma^2)$ con $\sigma^2 = Var(X)$ conocida y $E[X] = \theta$ a estimar y $\Pi(\theta) \sim N(\mu_0, \sigma_0^2)$ con μ_0, σ_0^2 conocidas entonces $\Pi(\theta|x_1 \ldots x_n) \sim N(\mu_1, \sigma_1^2)$ con

$$\mu_1 = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\bar{X}}{\sigma^2/n}}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2/n}} \quad y \quad \sigma_1^2 = \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2/n}}.$$

Para ello se vio que $\Pi(\theta|x_1 \ldots x_n) = \frac{\Pi(\theta) f(x_1 \ldots x_n|\theta)}{\int_{\mathbb{R}} \Pi(\theta) f(x_1 \ldots x_n|\theta) d\theta} = \ldots =$

$= \frac{1}{\sqrt{2\pi}\, \sigma_1} e^{-\frac{(\theta-\mu_1)^2}{2\sigma_1^2}}$. En este caso $\sigma^2 = \frac{1}{r}$

Por ello $\Pi(\theta|x_1 \ldots x_n) \sim N(\mu_1, \sigma_1^2)$ con

$$\mu_1 = \frac{\frac{\mu_0}{\sigma_0^2} + nr\bar{X}}{\frac{1}{\sigma_0^2} + nr} \quad y \quad \sigma_1^2 = \frac{1}{\frac{1}{\sigma_0^2} + nr}$$