La región de rechazo es por tanto:

$$RC = \{(x_1 \dots x_n) \mid x_{(1)} \geq c\} \quad \text{con } c \text{ tal que}$$

$$\sup_{\theta \leq 0} E_\theta[\phi(X_1 \dots X_n)] = \alpha \qquad \text{siendo } \phi(X_1 \dots X_n) \text{ es test}$$

de hipótesis

$$\phi(x_1 \dots x_n) = \begin{cases} 1 & \text{si } x_{(1)} \geq c \\ 0 & \text{si } x_{(1)} < c \end{cases}$$

c) La función de potencia es:

$$\beta(\theta) = E_\theta[\phi(X_1 \dots X_n)] = P_\theta(X_{(1)} \geq c) = 1 - F_{X_{(1)}}(c)$$

Calculamos la distribución del mínimo para dar la solución concreta

$$F_{X_{(1)}}(y) = 1 - (1 - F_X(y))^n$$

$$f_{X_{(1)}}(y) = n(1 - F_X(y))^{n-1} f_X(y).$$

$$F_X(x) = \int_\theta^x \frac{1}{\lambda} e^{-\frac{1}{\lambda}(t-\theta)} dt = \frac{e^{\frac{\theta}{\lambda}}}{\lambda} \int_\theta^x e^{-\frac{t}{\lambda}} dt =$$

$$= \frac{e^{\theta/\lambda}}{\lambda} \left[ -\frac{e^{-t/\lambda}}{1/\lambda} \right]_\theta^x = e^{\theta/\lambda}\left(e^{-\theta/\lambda} - e^{-x/\lambda}\right) = 1 - e^{-\frac{x}{\lambda} + \frac{\theta}{\lambda}}$$

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