02-04-CI

miércoles, 8 de abril de 2020 20:52

Ejercicio 1

(a)

$$\operatorname{rot}(\nabla f) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix} =$$

$$= \left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}\right)\vec{i} + \left(\frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z}\right)\vec{j} + \left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x}\right)\vec{k}$$

$= 0$ por tma Schwarz (asumimos f de clase C^2)

(b) $\vec{F}(x,y,z) = -\frac{MG}{r^2}\frac{\vec{r}(x,y,z)}{r}$, $r = r(x,y,z) = \|\vec{r}(x,y,z)\|$

$\vec{r}(x,y,z) = (x,y,z)$

$$f(x,y,z) = \frac{MG}{r(x,y,z)} = \frac{MG}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial f}{\partial x} = \frac{-MG}{r^2}\frac{x}{r}, \quad \frac{\partial f}{\partial y} = \frac{-MG}{r^2}\frac{y}{r}, \quad \frac{\partial f}{\partial z} = \frac{-MG}{r^2}\frac{z}{r}$$

$\Rightarrow \boxed{\nabla f = \vec{F}}$

(c) ¿$\operatorname{div}(\operatorname{rot}(\vec{F})) = 0$? $\vec{F} = (F_1, F_2, F_3)$

$$\operatorname{rot}(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\vec{i} +$$

$$+ \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\vec{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\vec{k}$$

$$\operatorname{div}(\operatorname{rot}(\vec{F})) = \frac{\partial^2 F_3}{\partial x \partial y} - \frac{\partial^2 F_2}{\partial x \partial z} + \frac{\partial^2 F_1}{\partial y \partial z} - \frac{\partial^2 F_3}{\partial y \partial x} +$$

$$+ \frac{\partial^2 F_2}{\partial z \partial x} - \frac{\partial^2 F_1}{\partial z \partial y} = 0$$ por tma Schwarz

(nuevamente asumimos que $\vec{F}$ es C^2)

(d) ¿$\operatorname{div}(\vec{F}) = 0$?

$$\vec{F} = \frac{-MG}{r^2}\frac{\vec{r}}{r}$$

$$\frac{\partial F_1}{\partial x} = -MG\left[(x^2+y^2+z^2)^{-3/2} - 3x^2(x^2+y^2+z^2)^{-5/2}\right]$$

$$\frac{\partial F_2}{\partial y} = -MG\left[(x^2+y^2+z^2)^{-3/2} - 3y^2(x^2+y^2+z^2)^{-5/2}\right]$$

$$+ \quad \frac{\partial F_3}{\partial z} = -MG\left[(x^2+y^2+z^2)^{-3/2} - 3z^2(x^2+y^2+z^2)^{-5/2}\right]$$

$$\operatorname{div}(\vec{F}) = -MG\left[3(x^2+y^2+z^2)^{-3/2} - 3(x^2+y^2+z^2)(x^2+y^2+z^2)^{-5/2}\right]$$

$$= 0 \;//$$

Ejercicio 2

Definimos $g(t) = V(\vec{c}(t))$

$$g'(t) = \nabla V(\vec{c}(t)) \cdot \vec{c}'(t) = -\|\nabla V(\vec{c}(t))\|^2 \leq 0$$

Regla de la cadena

$\| \leftarrow$ línea de flujo

$\vec{F}(c(t)) = -\nabla V(\vec{c}(t))$

Interpretación: las trayectorias buscan disminuir el potencial.

Ejercicio 3:

(a) $\operatorname{div}(\vec{F})(x,y,z) = ye^{xy} - xe^{xy} + ye^{yz}$

$= (y-x)e^{xy} + ye^{yz}$

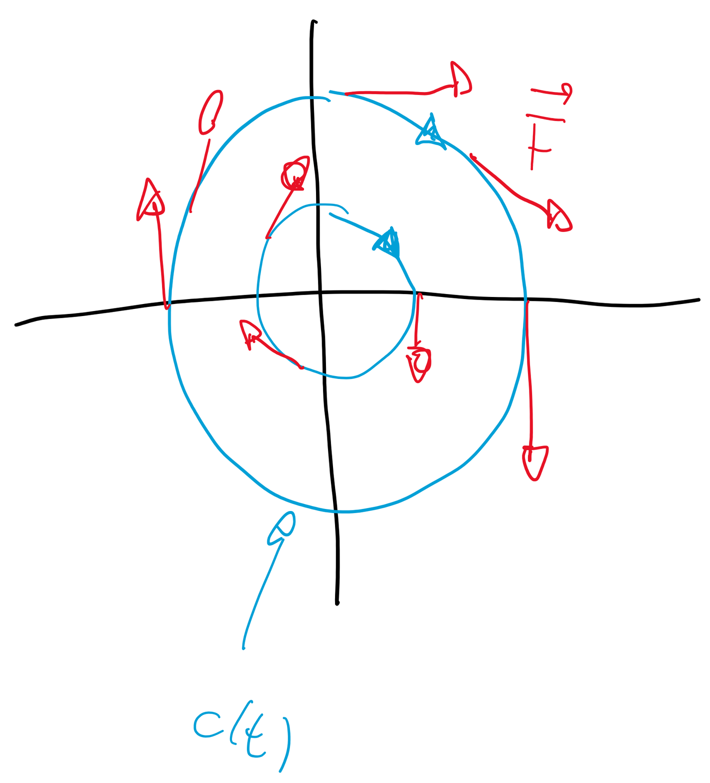
(b) $\operatorname{div}(\vec{F})(x,y,z) = 1 + 1 + 1 = 3$

Ejercicio 4:

(a) $\vec{c}(t) = \left(R\operatorname{sen}\left(\frac{1}{R^2}t\right), R\cos\left(\frac{1}{R^2}t\right), K\right) \leftarrow$ circunf. de radio R en el plano $z = K$

$$\vec{c}'(t) = \frac{1}{R^2}\left(R\cos\left(\frac{1}{R^2}t\right), -R\operatorname{sen}\left(\frac{1}{R^2}t\right), 0\right)$$

$$= \vec{F}(\vec{c}(t))$$



(b) $\operatorname{rot}(\vec{F}) = 0$