$$\Rightarrow f(z) = \frac{z^2}{(z+2)^2} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(z-0)^n \overset{f(0)=0}{\downarrow}{=} \sum_{n=1}^{\infty} \frac{(-1)^n \cancel{n!}(n-1)}{2^n}\cdot\frac{1}{\cancel{n!}}\, z^n =$$

$$= \sum_{n=1}^{\infty} \left(\frac{-z}{2}\right)^n (n-1) \qquad \forall z \in D(0,2).$$

d) $f(z) = \dfrac{z^2}{(z-1)^2}$ $\qquad z_0 = -1.$

holomorfa en $D(-1,2)$

$$f(z) = \frac{z^2}{(z-1)^2} = \frac{z^2-2z+1+2z-1}{z^2-2z+1} = 1 + \frac{2z-1}{(z-1)^2} = 1 + \frac{2}{z-1} + \frac{1}{(z-1)^2}$$

Veamos que $f^{(n)}(z) = 2(-1)^n n!\,(z-1)^{-n-1} + (-1)^n (n+1)!\,(z-1)^{-n-2}$.

Para $n=1$

$$f'(z) = 2(-1)(z-1)^{-2} + (-2)\cdot(z-1)^{-3} = 2\cdot(-1)\cdot 1!\cdot(z-1)^{-1-1} + (-1)\cdot 2!\cdot(z-1)^{-1-2}$$

Supuesto para n

$$f^{(n+1)}(z) = \frac{\partial}{\partial z}\left(2(-1)^n n!\,(z-1)^{-n-1} + (-1)^n (n+1)!\,(z-1)^{-n-2}\right) =$$

$$= 2(-1)^n n!\,(-n-1)\,(z-1)^{-n-1-1} + (-1)^n (n+1)!\,(-n-2)\,(z-1)^{-n-2-1} =$$

$$= 2\,(-1)^{n+1}(n+1)!\,(z-1)^{-(n+1)-1} + (-1)^{n+1}(n+2)\,(z-1)^{-(n+1)-2}$$

$$\Rightarrow f^{(n)}(-1) = 2(-1)^n n!\,(-2)^{-n-1} + (-1)^n (n+1)!\,(-2)^{-n-2} =$$

$$= (-1)^n n!\,(-2)^{-n-1}\left(2 + \frac{n+1}{2}\right) = \frac{(-1)^n}{2^{n+1}\cdot(-1)^{n+1}}\, n!\left(\frac{n+5}{2}\right) =$$

$$= -\frac{n+5)\cdot n!}{2^{n+2}}$$