$f''(0) = e^0 \cdot 1 + e^0 \cdot 0 - e^0 \cdot 1 - e^0 \cdot 0 = 0$

$f^{3)}(z) = e^{\operatorname{sen} z} \cos^3 z - 2e^{\operatorname{sen} z} \cos z \operatorname{sen} z - e^{\operatorname{sen} z} \cos z \operatorname{sen} z - e^{\operatorname{sen} z} \cos z$

$- e^{\operatorname{tg} z}(1+\operatorname{tg}^2 z)^3 - e^{\operatorname{tg} z} \cdot 2(1+\operatorname{tg}^2 z) \cdot 2\operatorname{tg} z \cdot (1+\operatorname{tg}^2 z)$

$- 2e^{\operatorname{tg} z}(1+\operatorname{tg}^2 z)^2 \cdot \operatorname{tg} z - 2e^{\operatorname{tg} z} \cdot (1+\operatorname{tg}^2 z + 3\operatorname{tg}^2 z(1+\operatorname{tg}^2 z))$

$f^{3)}(0) = 1 - 0 - 0 - 1 - 1 - 0 - 0 - 2 = -3 \neq 0.$

Por tanto la serie de Taylor para $f(z)$ centrada en $z_0 = 0$ es

$$f(z) = e^{\operatorname{sen} z} - e^{\operatorname{tg} z} = \sum_{n=3}^{\infty} \frac{f^n(0)}{n!} z^n \quad \text{con } f^3(0) \neq 0.$$

Por tanto la multiplicidad de 0 es 3.

c) $f(z) = 6\operatorname{sen}^3 z + z^3(z^6 - 6) = 6\operatorname{sen}^3 z + z^9 - 6z^3$ $\quad f(0) = 0$

$f'(z) = 18\operatorname{sen}^2 z \cos z + 9z^8 - 18z^2. \quad f'(0) = 0.$

$f''(z) = 18(2\operatorname{sen} z \cos^2 z + \operatorname{sen}^2 z \cdot (-\operatorname{sen} z)) + 72z^7 - 36z =$ $\quad f''(0) = 0$

$= 36\operatorname{sen} z \cos^2 z - 18\operatorname{sen}^3 z + 72z^7 - 36z.$

$f^{3)}(z) = 36(\cos^3 z - \operatorname{sen}^2 z \, 2\cos z) - 54\operatorname{sen}^2 z \cdot \cos z + 72 \cdot 7 \cdot z^6 - 36 =$

$= 36\cos^3 z - 36 - 72\operatorname{sen}^2 z \cos z - 54\operatorname{sen}^2 z \cos z + 504z^6 =$

$= 36\cos^3 z - 36 - 126\operatorname{sen}^2 z \cos z + 504z^6 \quad ; \quad f^{3)}(0) = 0.$

$f^{4)}(z) = -108\cos^2 z \operatorname{sen} z - 126(2\operatorname{sen} z \cos^2 z - \operatorname{sen}^3 z) + 3024z^5 =$

$= -108\cos^2 z \operatorname{sen} z - 252\cos^2 z \operatorname{sen} z + 126\operatorname{sen}^3 z + 3024z^5 =$

$= -360\cos^2 z \operatorname{sen} z + 126\operatorname{sen}^3 z + 3024z^5 \quad ; \quad f^{4)}(0) = 0$

$f^{5)}(z) = -360(2\cos z \cdot \operatorname{sen}^2 z(-1) + \cos^3 z) + 126\operatorname{sen}^2 z \cdot \cos z \cdot 3 + 3024 \cdot 5z^4.$

$f^{5)}(0) = -360 \neq 0.$