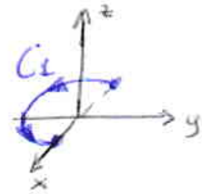
Entonces $\partial S_1 = C_1 + C_3$ y $\partial S_2 = C_2 + C_3^-$ donde

C_1, C_2, C_3 son las siguientes curvas orientadas simples:

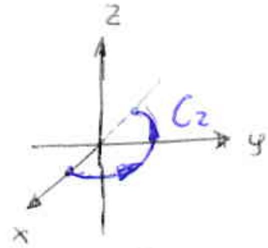
$$\gamma_1 : [\pi, 2\pi] \longrightarrow \mathbb{R}^3$$
$$t \longrightarrow (\cos t, \operatorname{sen} t, 0)$$

$\gamma_1([\pi, 2\pi]) = C_1$



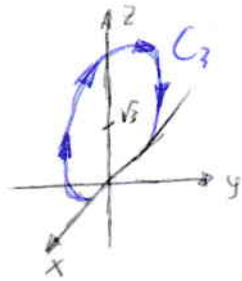
$$\gamma_2 : [0, \pi] \longrightarrow \mathbb{R}^3$$
$$t \longrightarrow (\cos t, \operatorname{sen} t, 0)$$

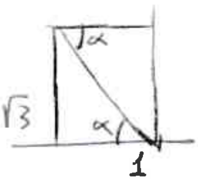
$\gamma_2([0, \pi]) = C_2$

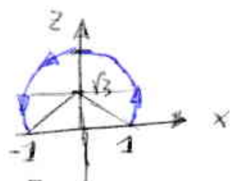


$$\gamma_3 : [-\tfrac{\pi}{3}, \tfrac{4\pi}{3}] \longrightarrow \mathbb{R}^3$$
$$t \longrightarrow (2\cos t, 0, 2\operatorname{sen} t) + \sqrt{3})$$

$\gamma_3([-\tfrac{\pi}{3}, \tfrac{4\pi}{3}]) = C_3$







$$tg\,\alpha = \frac{\sqrt{3}}{1} = \frac{\sqrt{3}/2}{1/2} = \frac{\operatorname{sen}\frac{\pi}{3}}{\cos\pi/3}$$

De esta forma, aplicando el teorema de Stokes a cada una de las superficies

$$\boxed{\iint_S \operatorname{rot}(\vec{F})\cdot d\vec{S}} = \iint_{S_1} \operatorname{rot}(\vec{F})\cdot d\vec{S} + \iint_{S_2} \operatorname{rot}(\vec{F})\cdot d\vec{S} =$$

$$= \iint_{C_1} \vec{F}\cdot d\vec{s} + \iint_{C_3} \vec{F}\cdot d\vec{s} + \iint_{C_2} \vec{F}\cdot d\vec{s} + \iint_{C_3^-} \vec{F}\cdot d\vec{s} = \iint_{C_1} \vec{F}\cdot d\vec{s} + \iint_{C_2} \vec{F}\cdot d\vec{s} =$$

$$= \int_0^{2\pi} \vec{F}(\cos t, \operatorname{sen} t, 0) \bullet (-\operatorname{sen} t, \cos t, 0)\; dt =$$

$$= \int_0^{2\pi} (\operatorname{sen} t, -\cos t, 1)\cdot(-\operatorname{sen} t, \cos t, 0)\, dt = -\int_0^{2\pi} (\operatorname{sen}^2 t + \cos^2 t)\, dt = \boxed{-2\pi}$$