I. E. S. " SAN ISIDRO "

Asignatura .... Fecha ....

Alumno/a .... Curso .... Nº ....

Apellidos Nombre

Calificación

$$= \sum_{n=0}^{\infty} -\frac{z^n}{3} + \frac{2}{15}\left(-\frac{z}{5}\right)^n = -\frac{1}{3}\sum_{n=0}^{\infty} z^n + \frac{2}{15}\sum_{n=0}^{\infty}\left(-\frac{z}{5}\right)^n \quad \forall z \in D(0,1)$$

c) $f(z) = \frac{z^2}{(z+2)^2}$ $\quad z_0 = 0$

Holomorfa en $D(0,2)$.

$$f(z) = \frac{z^2}{(z+2)^2} = \frac{z^2+4z+4-4z-4}{z^2+4z+4} = 1 - \frac{4z+4}{(z+2)^2} = 1 - 4\cdot\frac{z+1}{(z+2)^2} =$$

$$= 1 - 4\left(\frac{1}{z+2} - \frac{1}{(z+2)^2}\right) = 1 - \frac{4}{z+2} + \frac{4}{(z+2)^2}$$

Veamos que $f^{n)}(z) = -4\cdot(-1)^n (z+2)^{-n-1} n! + 4(-1)^n (z+2)^{-n-2}(n+1)!$

$n=1$

$$f'(z) = -4(z+2)^{-2}\cdot(-1) + 4(z+2)^{-3}\cdot(-2)$$

Supuesto para n.

$$f^{n+1)}(z) = \frac{\partial}{\partial z}\left(-4(-1)^n(z+2)^{-n-1}n! + 4(-1)^n(z+2)^{-n-2}(n+1)!\right) =$$

$$= -4(-1)^n n!\cdot(-n-1)\cdot(z+2)^{-n-1-1} + 4(-1)^n(n+1)!(-n-2)\cdot(z+2)^{-n-2-1} =$$

$$= -4(-1)^{n+1}(n+1)!\,(z+2)^{-(n+1)-1} + 4(-1)^{n+1}(n+2)!\,(z+2)^{-(n+1)-2}$$

$$\Rightarrow f^{n)}(0) = -4\cdot(-1)^n(2)^{-n-1}n! + 4(-1)^n(2)^{-n-2}(n+1)! =$$

$$= 4(-1)^n n!\left(-1 + 2^{-1}\cdot(n+1)\right) = \frac{4(-1)^n n!}{2^{n+1}}\cdot\left(\frac{n+1}{2} - 1\right) =$$

$$= \frac{4(-1)^n n!}{2^{n+1}}\left(\frac{n+1-2}{2}\right) = \frac{(-1)^n n!\,(n-1)}{2^n}$$

$$f(0)=0$$

$$\Rightarrow f(z)=\frac{z^2}{(z+2)^2}=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}(z-0)^n \overset{\downarrow}{=} \sum_{n=1}^{\infty}\frac{(-1)^n \cancel{n!}(n-1)}{2^n}\cdot\frac{1}{\cancel{n!}}\cdot z^n =$$

$$=\sum_{n=1}^{\infty}\left(\frac{-z}{2}\right)^n(n-1) \quad \forall z\in D(0,2).$$

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c) $f(z)=\dfrac{z^2}{(z-1)^2}$ $\quad z_0=-1.$

holomorfa en $D(-1,2)$

$$f(z)=\frac{z^2}{(z-1)^2}=\frac{z^2-2z+1+2z-1}{z^2-2z+1}=1+\frac{2z-1}{(z-1)^2}=1+\frac{2}{z-1}+\frac{1}{(z-1)^2}$$

Veamos que $f^{(n)}(z)=2(-1)^n n!\,(z-1)^{-n-1}+(-1)^n(n+1)!\,(z-1)^{-n-2}$.

Para $n=1$

$$f'(z)=2(-1)(z-1)^{-2}+(-2)\cdot(z-1)^{-3}=2\cdot(-1)\cdot 1!\cdot(z-1)^{-1-1}+(-1)\cdot 2!\cdot(z-1)^{-1-2}$$

Supuesto para n

$$f^{(n+1)}(z)=\frac{\partial}{\partial z}\left(2(-1)^n n!\,(z-1)^{-n-1}+(-1)^n(n+1)!\,(z-1)^{-n-2}\right)=$$

$$=2(-1)^n n!\,(-n-1)\,(z-1)^{-n-1-1}+(-1)^n(n+1)!\,(-n-2)\,(z-1)^{-n-2-1}=$$

$$=2\,(-1)^{n+1}(n+1)!\,(z-1)^{-(n+1)-1}+(-1)^{n+1}(n+2)\,(z-1)^{-(n+1)-2}$$

$$\Rightarrow f^{(n)}(-1)=2(-1)^n n!\,(-2)^{-n-1}+(-1)^n(n+1)!\,(-2)^{-n-2}=$$

$$=(-1)^n n!\,(-2)^{-n-1}\left(2+\frac{n+1}{2}\right)=\frac{(-1)^n}{2^{n+1}\cdot(-1)^{n+1}}n!\left(\frac{n+5}{2}\right)=$$

$$=-\frac{n+5)\cdot n!}{2^{n+2}}$$