$\boxed{\text{b)}}$ $f(z) = \dfrac{z+1}{(z^2+4z-5)}$ en $z_0 = 0$

holomorfa en $D(0,1)$

$$\frac{z+1}{z^2+4z-5} = \frac{z+1}{(z-1)(z+5)} = \frac{A}{z-1} + \frac{B}{z+5} = \frac{1/3}{z-1} + \frac{2/3}{z+5}$$

$$\Updownarrow$$

$$z+1 = A(z+5) + B(z-1)$$

Para $z = -5 \Rightarrow -4 = -6B \Rightarrow B = \frac{2}{3}$

$z = 1 \Rightarrow 2 = 6A \Rightarrow A = 1/3$

Veamos que $f^{n)}(z) = \frac{1}{3} n!(z-1)^{-n-1}(-1)^n + \frac{2}{3} n!(z+5)^{-n-1} \cdot (-1)^n$

$n = 1$

$$f'(z) = \frac{1}{3}(z-1)^{-2} \cdot (-1) + \frac{2}{3}(z+5)^{-2} \cdot (-1)$$

Supuesto para n.

$$f^{n+1)}(z) = \frac{\partial}{\partial z}\left(\frac{n!}{3}(-1)^n \cdot (z-1)^{-n-1} + \frac{2n!}{3}(-1)^n \cdot (z+5)^{-n-1} \right) =$$

$$= \frac{n!}{3}(-1)^n \cdot (-n-1)(z-1)^{-n-1-1} + \frac{2n!}{3}(-1)^n(-n-1)(z+5)^{-n-1-1} =$$

$$= \frac{(n+1)!}{3}(-1)^{n+1}(z-1)^{-(n+1)-1} + \frac{2 \cdot (n+1)!}{3}(-1)^{n+1}(z-1)^{-(n+1)-1}$$

$$\Rightarrow f^{n)}(0) = \frac{n!}{3}(-1)^{-n-1}(-1)^n + \frac{2}{3} n!(5)^{-n-1} \cdot (-1)^n =$$

$$= \frac{n!}{3} \cdot \frac{(-1)^n}{(-1)^n \cdot (-1)} + \frac{2n!}{3} \cdot \frac{(-1)^n}{5^{n+1}} = -\frac{n!}{3} + \frac{2n!}{15} \cdot \left(-\frac{1}{5}\right)^n$$

$$\Rightarrow f(z) = \frac{z+1}{z^2+4z-5} = \sum_{n=0}^{\infty} \frac{f^{n)}(0)}{n!} \cdot (z-0)^n = \sum_{n=0}^{\infty} \frac{1}{n!}\left(-\frac{n!}{3} + \frac{2n!}{15}\left(-\frac{1}{5}\right)^n\right) z^n =$$

I. E. S. " SAN ISIDRO "

Asignatura.......... Fecha ....

Alumno/a .... Curso.......... Nº..........

Apellidos Nombre

Calificación

$$= \sum_{n=0}^{\infty} -\frac{z^n}{3} + \frac{2}{15}\left(-\frac{z}{5}\right)^n = -\frac{1}{3}\sum_{n=0}^{\infty} z^n + \frac{2}{15}\sum_{n=0}^{\infty}\left(-\frac{z}{5}\right)^n \quad \forall z \in D(0,1)$$

c) $f(z) = \frac{z^2}{(z+2)^2}$ $\qquad z_0 = 0$

Holomorfa en $D(0,2)$.

$$f(z) = \frac{z^2}{(z+2)^2} = \frac{z^2+4z+4-4z-4}{z^2+4z+4} = 1 - \frac{4z+4}{(z+2)^2} = 1 - 4\cdot\frac{z+1}{(z+2)^2} =$$

$$= 1 - 4\left(\frac{1}{z+2} - \frac{1}{(z+2)^2}\right) = 1 - \frac{4}{z+2} + \frac{4}{(z+2)^2}$$

Veamos que $f^{n)}(z) = -4\cdot(-1)^n (z+2)^{-n-1} n! + 4(-1)^n (z+2)^{-n-2}(n+1)!$

$n=1$

$$f'(z) = -4(z+2)^{-2}\cdot(-1) + 4(z+2)^{-3}\cdot(-2)$$

Supuesto para n.

$$f^{n+1)}(z) = \frac{\partial}{\partial z}\left(-4(-1)^n (z+2)^{-n-1} n! + 4(-1)^n (z+2)^{-n-2}(n+1)!\right) =$$

$$= -4(-1)^n n!\cdot(-n-1)\cdot(z+2)^{-n-1-1} + 4(-1)^n (n+1)!\,(-n-2)\cdot(z+2)^{-n-2-1} =$$

$$= -4(-1)^{n+1}(n+1)!\,(z+2)^{-(n+1)-1} + 4(-1)^{n+1}(n+2)!\,(z+2)^{-(n+1)-2}$$

$$\Rightarrow f^{n)}(0) = -4\cdot(-1)^n (2)^{-n-1} n! + 4(-1)^n (2)^{-n-2}(n+1)! =$$

$$= 4(-1)^n n!\left(-1 + 2^{-1}\cdot(n+1)\right) = \frac{4(-1)^n n!}{2^{n+1}}\cdot\left(\frac{n+1}{2} - 1\right) =$$

$$= \frac{4(-1)^n n!}{2^{n+1}}\left(\frac{n+1-2}{2}\right) = \frac{(-1)^n n!\,(n-1)}{2^n}$$