Nos ha quedado la normal interior pero preferimos trabajar con la normal exterior así que reescribimos

$$\tilde{D}_1 = (1,\sqrt{3}) \times (0,2\pi) \quad \text{con} \quad \tilde{\Phi}_1 : \tilde{D}_1 \longrightarrow \mathbb{R}^3$$
$$(r,\theta) \rightarrow (r\cos\theta,\ r\,\mathrm{sen}\,\theta,\ 3-r^2)$$

y $\dfrac{\partial \tilde{\Phi}_1}{\partial r} \times \dfrac{\partial \tilde{\Phi}_1}{\partial \theta} = - \dfrac{\partial \Phi_1}{\partial \theta} \times \dfrac{\partial \Phi_1}{\partial r} = (2r^2\cos\theta,\ 2r^2\,\mathrm{sen}\,\theta,\ r).$

Sustituyendo

$$\iint_{S_1} \nabla f \cdot d\vec{S} = \iint_{\tilde{D}} (\nabla f \circ \tilde{\Phi}_1) \cdot \left(\frac{\partial \tilde{\Phi}_1}{\partial r} \times \frac{\partial \tilde{\Phi}_1}{\partial \theta} \right) dr\, d\theta =$$

$$= \iint_{\tilde{D}_1} \nabla f(r\cos\theta,\ r\,\mathrm{sen}\,\theta,\ 3-r^2) \cdot (2r^2\cos\theta,\ 2r^2\,\mathrm{sen}\,\theta,\ r)\, dr\, d\theta =$$

$$= \iint_{\tilde{D}_1} (2r\cos\theta + 2r\,\mathrm{sen}\,\theta - 3,\ 2r\cos\theta,\ 6-2r^2) \cdot (2r^2\cos\theta,\ 2r^2\,\mathrm{sen}\,\theta,\ r)\, dr\, d\theta =$$

$$= \iint_{\tilde{D}_1} (4r^3\cos^2\theta + 4r^3\,\mathrm{sen}\,\theta\cos\theta - 6r^2\cos\theta + 4r^3\,\mathrm{sen}\,\theta\cos\theta + 6r - 2r^3)\, dr\, d\theta =$$

$$= \int_1^{\sqrt{3}} \int_0^{2\pi} \cancel{2r^3} + 2r^3\cos 2\theta + 4r^3 \cdot 2\,\mathrm{sen}\,\theta\cos\theta - 6r^2\cos\theta + 6r - \cancel{2r^3})\, d\theta\, dr =$$

$$= \int_1^{\sqrt{3}} \left(\cancel{r^3\,\mathrm{sen}\,2\theta\Big]_0^{2\pi}}^{\,0} + 4r^3\, \cancel{\mathrm{sen}^2\theta\Big]_0^{2\pi}}^{\,0} + 6r^2\, \cancel{\mathrm{sen}\,\theta\Big]_0^{2\pi}}^{\,0} + 12\pi r \right) dr =$$

$$= 6\pi\ r^2\Big]_1^{\sqrt{3}} = 6\pi(3-1) = 12\pi$$