Ejercicio 1:

Algoritmo de la sección áurea

Ext. Inf.	Ext. Sup.	$\frac{I_k}{C}$	a	f(a)	b	f(b)
0	2	1'236	0'764	-4'318	1'236	$\boxed{-3'348}$
0	1'236	0'764	0'472	-4'367	0'764	$\boxed{-4'318}$
0	0'764	0'472	0'252	$\boxed{-4'236}$	0'472	-4'367
0'252	0'764	0'292	0'472	$\boxed{-4'367}$	0'544	-4'383
0'472	0'764	0'180	0'544	-4'383	0'652	$\boxed{-4'375}$

⇒ Solución: [0'472, 0'652]

Algoritmo de Fibonacci.

$\frac{I_0}{F_k}$	F_k	I_k	Ext. Inf.	Ext. Sup.	a	f(a)	b	f(b)
2	1	2	0	2	0'75	-4'328	1'25	$\boxed{-3'30}$
1	2	1'25	0	1'25	0'5	-4'375	0'75	$\boxed{-4'328}$
2/3	3	0'75	0	0'75	0'25	$\boxed{-4'234}$	0'5	-4'375
2/5	5	0'5	0'25	0'75	0'49	$\boxed{-4'372}$	0'51	-4'377
1/4	8	0'25						

⇒ Solución: [0'5, 0'75]

Algoritmo de la bisección.

$f'(x) = 3x^2 - 1.$

Ext. Inf.	Ext. Sup.	c	f'(c).
0	2	1	2
0	1	0'5	-0'25
0'5	1	0'75	11/16

Algoritmo de Newton.

$f''(x) = 6x.$

Iteración 1: $x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} = 1 - \frac{2}{6} = \frac{2}{3}.$

Iteración 2: $x_2 = x_1 - \frac{f'(x_1)}{f''(x_1)} = \frac{2}{3} - \frac{1/3}{4} = \frac{2}{3} - \frac{1}{12} = \frac{7}{12}.$

Iteración 3: $x_3 = x_2 - \frac{f'(x_2)}{f''(x_2)} = \frac{7}{12} - \frac{1/48}{7/2} = \frac{7}{12} - \frac{1}{168} = \frac{97}{168}.$

Solución: $x_3 = \frac{97}{168}.$

Ejercicio 2:

Algoritmo de las coordenadas cíclicas.

Iteración 1: Paso 1: $\underset{d}{\text{Min}}\ f(d) = 6d + 18 + d^2 \equiv f(d, 3)$

$f'(d) = 6 + 2d \Rightarrow f'(d) = 0 \Leftrightarrow d = -3 \Rightarrow \boxed{X_0^1 = (-3, 3)}$

Paso 2: $\underset{d}{\text{Min}}\ f(d) = -6(3+d) + 2(3+d)^2 + 9 \equiv f(-3, 3+d)$

$f'(d) = -6 + 4(3+d) = 6 + 4d$

$f'(d) = 0 \Leftrightarrow d = -1'5 \Rightarrow \boxed{X_0^2 = (-3, 1'5)}$

Iteración 2: Paso 1: $\underset{d}{\text{Min}}\ f(d) = 3(-3+d) + 2 * 1'5^2 + (d-3)^2$

$f'(d) = 3 + 2(d-3) = 2d - 3$

$f'(d) = 0 \Leftrightarrow d = 1'5 \Rightarrow \boxed{X_1^1 = (-1'5, 1'5)}$

Paso 2: $\underset{d}{\text{Min}}\ f(d) = -3(1'5+d) + 2(1'5+d)^2 + 1'5^2$

$f'(d) = -3 + 4(1'5+d) = 3 + 4d.$

$f'(d) = 0 \Leftrightarrow d = \frac{-3}{4} \Rightarrow \boxed{X_1^2 = (-1'5, 0'75).}$

Algoritmo de Hooke y Jeeves.

Iteración 1: Paso 1.1. $\rightarrow$ $\boxed{X_0^1 = (-3,3)}$

Paso 1.2. $\rightarrow$ $\boxed{X_0^2 = (-3, 1'5)}$ $\Big\} \Rightarrow \vec{d} = (-3, 1'5) - (0,3) = (-3, -1'5)$

Obtenido en el método de coordenadas cíclicas.

Paso 2.: $\underset{\lambda \geq 0}{\text{Min}} \; \underbrace{2(-3\lambda)(3-1'5\lambda) + 2(3-1'5\lambda)^2 + (-3\lambda)^2}_{\equiv \, f(-3\lambda,\, 3-1'5\lambda).}$

$$f'(\lambda) = -18 + 9\lambda + 9\lambda - 6(3-1'5\lambda) + 18\lambda$$
$$= -36 + 45\lambda.$$

$$f'(\lambda) = 0 \Leftrightarrow \lambda = \frac{36}{45} = \frac{4}{5} \Rightarrow \vec{x}_1^{\,t} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \frac{4}{5}\begin{pmatrix} -3 \\ -1'5 \end{pmatrix} = \begin{pmatrix} -12/5 \\ 9/5 \end{pmatrix}$$

Iteración 2: Paso 1.1 $\rightarrow$ $\underset{\lambda}{\text{Min}} \; f(-\frac{12}{5} + \lambda, \frac{9}{5}) =$

$$\underset{\lambda}{\text{Min}} \; \frac{18}{5}\left(\lambda - \frac{12}{5}\right) + 2\left(\frac{9}{5}\right)^2 + \left(\lambda - \frac{12}{5}\right)^2.$$

$$f'(\lambda) = \frac{18}{5} + 2\left(\lambda - \frac{12}{5}\right) = 2\lambda - \frac{6}{5}$$

$$f'(\lambda) = 0 \Leftrightarrow \lambda = \frac{3}{5} \Rightarrow x_1^1 = \left(\frac{-9}{5}, \frac{9}{5}\right)$$

Paso 1.2. $\rightarrow$ $\underset{\lambda}{\text{Min}} \; f(-\frac{9}{5}, \frac{9}{5} + \lambda) =$

$$\text{Min} \; -\frac{18}{5}\left(\frac{9}{5} + \lambda\right) + 2\left(\frac{9}{5} + \lambda\right)^2 + \left(\frac{9}{5}\right)^2$$

$$f'(\lambda) = \frac{-18}{5} + 4\left(\frac{9}{5} + \lambda\right) = \frac{18}{5} + 4\lambda$$

$$f'(\lambda) = 0 \Leftrightarrow \lambda = \frac{-9}{10} \Rightarrow x_1^2 = \left(\frac{-9}{5}, \frac{9}{5}\right) + \left(0, \frac{-9}{10}\right) = \left(\frac{-9}{5}, \frac{9}{10}\right).$$

$$\Rightarrow \vec{d} = \left(\frac{3}{5}, \frac{-9}{10}\right) \equiv (6, -9) \equiv (2, -3).$$

Paso 2: $\underset{\lambda \geq 0}{\text{Min}} \; f(-\frac{12}{5} + 2\lambda, \frac{9}{5} - 3\lambda) =$

$$\underset{\lambda \geq 0}{\text{Min}} \; 2\left(2\lambda - \frac{12}{5}\right)\left(\frac{9}{5} - 3\lambda\right) + 2\left(\frac{9}{5} - 3\lambda\right)^2 + \left(2\lambda - \frac{12}{5}\right)^2$$

$$f'(\lambda) = 4\left(\frac{9}{5} - 3\lambda\right) - 6\left(2\lambda - \frac{12}{5}\right) - 6\left(\frac{9}{5} - 3\lambda\right) + 4\left(2\lambda - \frac{12}{5}\right)$$

$$= +\frac{108}{5} - 24\lambda - \frac{54}{5} + 18\lambda + 8\lambda - \frac{48}{5}$$

$$= 2\lambda - \frac{4}{5}$$

$$f'(\lambda) = 0 \iff \lambda = \frac{2}{5}$$

$$\Rightarrow \overline{x}_2 = \left(\frac{-12}{5}, \frac{9}{5}\right) + \frac{2}{5}(2,-3) = \left(\frac{-8}{5}, \frac{3}{5}\right)$$

Algoritmo del gradiente.

$$\nabla f(x,y)^t = \begin{pmatrix} 2y + 2x \\ 2x + 4y \end{pmatrix}.$$

Iteración 1: $\nabla f(0,3)^t = \begin{pmatrix} 6 \\ 12 \end{pmatrix} \Rightarrow \vec{d} = -(1,2) = (-1,-2)$

$$\min_{\lambda \geq 0} f((0,3) + \lambda(-1,-2)) = \min_{\lambda \geq 0} f(-\lambda, 3-2\lambda) =$$

$$\min_{\lambda \geq 0} -2\lambda(3-2\lambda) + 2(3-2\lambda)^2 + \lambda^2$$

$$f'(\lambda) = -2(3-2\lambda) + 4\lambda - 8(3-2\lambda) + 2\lambda = -30 + 26\lambda$$

$$f'(\lambda) = 0 \iff \lambda = \frac{30}{26} = \frac{15}{13}. \Rightarrow \overline{x}_1 = (0,3) - \frac{15}{13}(1,2) = \left(\frac{-15}{13}, \frac{9}{13}\right)$$

Iteración 2: $\nabla f\left(\frac{-15}{13}, \frac{9}{13}\right) = \begin{pmatrix} -12/13 \\ 6/13 \end{pmatrix} \Rightarrow \vec{d} = -(-2,1) = (2,-1)$

$$\min_{\lambda \geq 0} f\left(\left(\frac{-15}{13}, \frac{9}{13}\right) + \lambda(2,-1)\right) = \min_{\lambda \geq 0} f\left(2\lambda - \frac{15}{13}, \frac{9}{13} - \lambda\right)$$

$$\min_{\lambda \geq 0} 2\left(2\lambda - \frac{15}{13}\right)\left(\frac{9}{13} - \lambda\right) + 2\left(\frac{9}{13} - \lambda\right)^2 + \left(2\lambda - \frac{15}{13}\right)^2$$

$$f'(\lambda) = 4\left(\frac{9}{13} - \lambda\right) - 2\left(2\lambda - \frac{15}{13}\right) - 4\left(\frac{9}{13} - \lambda\right) + 4\left(2\lambda - \frac{15}{13}\right)$$

$$= \frac{36}{13} - 4\lambda - 4\lambda + \frac{30}{13} - \frac{36}{13} + 4\lambda + 8\lambda - \frac{60}{13} = 4\lambda - \frac{30}{13}$$

$$f'(\lambda) = 0 \iff \lambda = \frac{15}{26}.$$

$$\Rightarrow \overline{x}_2 = \left(\frac{-15}{13}, \frac{9}{13}\right) + \frac{15}{26}(2,-1) = \left(0, \frac{3}{26}\right).$$