I. E. S. " SAN ISIDRO "

Asignatura.................... Fecha ....

Alumno/a .... Curso.............. Nº..........

Apellidos Nombre

Calificación

7.-

a) $\sum_{n=0}^{\infty} r^n \cdot sen(nx)$ $r>0$

Sabemos que $\sum_{n=0}^{\infty} z^n \underset{|z|<1}{=} \frac{1}{1-z}$. Tomando $z = r\cdot(\cos x + i\,sen x)$, $0 \leq r < 1$

se tiene que $\sum_{n=0}^{\infty} r^n(\cos x + i\,sen x)^n = \sum_{n=0}^{\infty} r^n \cdot \cos nx + i \sum_{n=0}^{\infty} r^n sen(nx)$

$$\sum_{n=0}^{\infty} r^n(\cos x + i\,sen x)^n = \frac{1}{1-r(\cos x + i\,sen x)}$$

$$\frac{1}{1-r(\cos x + i\,sen x)} = \frac{1}{1-r\cos x - i r\,sen x} = \frac{(1-r\cos x)+i r\,sen x}{(1-r\cos x)^2 + r^2 sen^2 x} =$$

$$= \frac{1-r\cos x + i r\,sen x}{1-2r\cos x + r^2\cos^2 x + r^2 sen^2 x} = \frac{1-r\cos x}{1-2r\cos x + r^2} + i\,\frac{r\,sen x}{1-2r\cos x+r^2}$$

$$\Longrightarrow \sum_{n=0}^{\infty} r^n \cdot \cos nx = \frac{1-r\cos x}{1-2r\cos x+r^2}$$

$$\sum_{n=0}^{\infty} r^n\,sen(nx) = \frac{r\,sen x}{1-2r\cos x+r^2}$$