$$= \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n} x_i^2}$$

Para maximizar esta función en σ^2 recurrimos a la función soporte

$$\ell(\sigma^2 | x_1 \dots x_n) = Ln(L(\sigma^2|x_1 \dots x_n)) = -\frac{n}{2} Ln(2\pi) - \frac{n}{2} Ln(\sigma^2) - \frac{\sum_{i=1}^{n} x_i^2}{2\sigma^2}$$

Ahora

$$\ell'(\sigma^2|x_1 \dots x_n) = -\frac{n}{2}\frac{1}{\sigma^2} + \frac{\sum_{i=1}^{n} x_i^2}{2(\sigma^2)^2} = \frac{-n\sigma^2 + \sum_{i=1}^{n} x_i^2}{2(\sigma^2)^2} = 0 \Leftrightarrow \sigma^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} \geq 0$$

Para comprobar que es un máximo volvemos a derivar:

$$\ell''(\sigma^2|x_1 \dots x_n) = \frac{1}{2}\frac{-n(\sigma^2)^2 - (-n\sigma^2 + \sum_{i=1}^{n} x_i^2) 2\sigma^2}{(\sigma^2)^4}$$

$$\ell''\left(\frac{\sum_{i=1}^{n} x_i^2}{n} \Big| x_1 \dots x_n\right) = \frac{1}{2}\frac{-n\left(\frac{\sum x_i^2}{n}\right)^2 - (\overbrace{\cancel{-\sum x_i^2 + \sum x_i^2}}^{0}) 2\left(\frac{\sum x_i^2}{n}\right)}{\left(\frac{\sum x_i^2}{n}\right)^4} =$$

$$= -\frac{n}{2}\frac{1}{\left(\frac{\sum x_i^2}{n}\right)^2} < 0$$

Efectivamente, es un máximo y

$$\sup_{\sigma^2>0}\{f(x_1 \dots x_n|\sigma^2)\} = L\left(\frac{\sum x_i^2}{n} \Big| x_1 \dots x_n\right) = \left(\frac{n}{2\pi\sum_{i=1}^{n} x_i^2}\right)^{n/2} e^{-\frac{\sum x_i^2}{2\frac{\sum x_i^2}{n}}} =$$

$$= \left(\frac{n}{2\pi\sum_{i=1}^{n} x_i^2}\right)^{n/2} e^{-n/2}$$