En resumen,

$$D = \{ (\theta, r) \in \mathbb{R}^2 \mid 0 < r < a\cos\theta \ , \ \underset{\theta\in(-\frac{\pi}{2},\frac{\pi}{2})}{\overset{\uparrow}{\theta \in (0, 2\pi)}} \}$$ es un conjunto abierto, con volumen bien definido y que verifica que $\Phi(D) = \hat{A}$

Por tanto tenemos una parametrización de $\hat{A}$.

Para calcular el área de $\hat{A}$ primero tenemos que calcular la norma de sus vectores normales.

$$\left.\begin{array}{l} \dfrac{\partial \Phi}{\partial \theta} = (-r\,\mathrm{sen}\,\theta,\ r\cos\theta,\ 0) \\[2ex] \dfrac{\partial \Phi}{\partial r} = (\cos\theta,\ \mathrm{sen}\,\theta,\ 1) \end{array}\right\} \Rightarrow \frac{\partial \Phi}{\partial \theta} \times \frac{\partial \Phi}{\partial r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -r\,\mathrm{sen}\,\theta & r\cos\theta & 0 \\ \cos\theta & \mathrm{sen}\,\theta & 1 \end{vmatrix} =$$

$$= r\cos\theta\ \vec{i} + r\,\mathrm{sen}\,\theta\ \vec{j} - (r\,\mathrm{sen}^2\theta + r\cos^2\theta)\vec{k} =$$

$$= (r\cos\theta,\ r\,\mathrm{sen}\,\theta,\ -r)$$

$$\left\| \frac{\partial \Phi}{\partial \theta} \times \frac{\partial \Phi}{\partial r} \right\| = \sqrt{r^2\cos^2\theta + r^2\mathrm{sen}^2\theta + r^2} = \sqrt{2r^2} = \sqrt{2}\,r$$

De esta forma

$$\boxed{\text{Área}(A)} = \text{Área}(\hat{A}) = \iint_{\hat{A}} 1 = \iint_D (1\circ\Phi)\cdot\left\| \frac{\partial \Phi}{\partial \theta} \times \frac{\partial \Phi}{\partial r} \right\| d\theta\, dr =$$

$$= \iint_D \sqrt{2}\, r \ d\theta\, dr \underset{\substack{\uparrow \\ \text{Corolario} \\ \text{Fubini}}}{\overset{*}{=}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\int_0^{a\cos\theta} \sqrt{2}\, r \ dr \right) d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sqrt{2}}{2} a^2 \cos^2\theta \ d\theta =$$

$$\underset{\substack{\uparrow \\ \cos^2\theta + \mathrm{sen}^2\theta = 1 \\ \cos^2\theta - \mathrm{sen}^2\theta = \cos 2\theta}}{=} \frac{\sqrt{2}}{2} a^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{\cos 2\theta}{2} \right) d\theta = \frac{\sqrt{2}}{2} a^2 \left(\frac{\pi}{2} + \left[\frac{\mathrm{sen}\, 2\theta}{4} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \right) = \boxed{\frac{\sqrt{2}}{4} a^2 \pi}$$