Si integramos en ambos lados:

$$\int_0^{\pi} \cot g\left(\frac{x}{2}\right) \cdot sen(nx)\, dx = \int_0^{\pi} \left(1 + 2\sum_{k=1}^{n-1} \cos kx + \cos nx\right) dx =$$

$$= \int_0^{\pi} dx + 2\sum_{k=1}^{n-1} \int_0^{\pi} \cos kx\, dx + \int_0^{\pi} \cos nx\, dx = \pi$$ ya que

la integral de todos los cosenos es 0 porque $\int_0^{\pi} \cos kx\, dx = \left.\frac{sen\, kx}{k}\right]_0^{\pi} = 0$

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$$1 + 2\sum_{k=1}^{n} \cos kx = 1 + 2 \cdot \frac{1}{sen\frac{x}{2}} \sum_{k=1}^{n} sen\frac{x}{2} \cdot \cos kx =$$

$$= 1 + \frac{1}{sen\frac{x}{2}} \sum_{k=1}^{n} 2 sen\frac{x}{2} \cos kx =$$

$$= 1 + \frac{1}{sen\frac{x}{2}} \sum_{k=1}^{n} \left(sen\left(kx + \frac{x}{2}\right) - sen\left(kx - \frac{x}{2}\right)\right) \underset{\uparrow}{=}$$ Suma telescópica

$$= 1 + \frac{1}{sen\frac{x}{2}} \left(sen\left(nx + \frac{x}{2}\right) - sen\frac{x}{2}\right) = 1 + \frac{sen\left(nx + \frac{x}{2}\right)}{sen\frac{x}{2}} - 1 =$$

$$= \frac{sen\left(\frac{n+1}{2}x\right)}{sen\frac{x}{2}}$$

Bibliografía

$*_1$ Video youtube: Improper Integral of ln(sinx) from 0 to pi/2 : MIT Integration Bee (4)

$*_3$ math.stackexchange.com

$*_4$ math.stackexchange.com