Para $z = e^{i\frac{3\pi}{4}} \Rightarrow e^{i\frac{3\pi}{4}} = A\left(e^{i\frac{3\pi}{4}} - e^{i\frac{7\pi}{4}}\right)$

$$\boxed{\begin{array}{l} e^{i\frac{3\pi}{4}} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \\ e^{i\frac{7\pi}{4}} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \end{array}}$$

$$\Updownarrow$$

$$e^{i\frac{3\pi}{4}} = -e^{i\frac{7\pi}{4}}$$

$$A = \frac{e^{i\frac{3\pi}{4}}}{e^{i\frac{3\pi}{4}} - e^{i\frac{7\pi}{4}}} = \frac{e^{i\frac{3\pi}{4}}}{2 e^{i\frac{3\pi}{4}}} = \frac{1}{2}$$

Para $z = e^{i\frac{7\pi}{4}} \Rightarrow e^{i\frac{7\pi}{4}} = B\left(e^{i\frac{7\pi}{4}} - e^{i\frac{3\pi}{4}}\right) \Leftrightarrow$

$$\Leftrightarrow B = \frac{e^{i\frac{7\pi}{4}}}{e^{i\frac{7\pi}{4}} - e^{i\frac{3\pi}{4}}} = \frac{e^{i7\pi/4}}{2 e^{i7\pi/4}} = \frac{1}{2}$$

$$\Rightarrow f(z) = \frac{z}{z^2+i} = \frac{1/2}{z - e^{\frac{3\pi}{4}i}} + \frac{1/2}{z - e^{\frac{7\pi}{4}i}} = \frac{1}{2}\left(z - e^{\frac{3\pi}{4}i}\right)^{-1} + \frac{1}{2}\left(z - e^{\frac{7\pi}{4}i}\right)^{-1}$$

$$\Rightarrow f'(z) = \frac{1}{2} \cdot (-1) \cdot \left(z - e^{\frac{3\pi}{4}i}\right)^{-2} + \frac{1}{2}(-1)\left(z - e^{\frac{7\pi}{4}i}\right)^{-2}$$

$$f''(z) = \frac{1}{2}(-1)\cdot(-2)\left(z - e^{\frac{3\pi}{4}i}\right)^{-3} + \frac{1}{2}(-1)(-2)\left(z - e^{\frac{7\pi}{4}i}\right)^{-3}$$

En general $f^{n)}(z) = \frac{1}{2}(-1)^n n! \left(z - e^{\frac{3\pi}{4}i}\right)^{-n-1} + \frac{1}{2}(-1)^n n! \left(z - e^{\frac{7\pi}{4}i}\right)^{-n-1}$

Los casos base ya están y el paso inductivo

$$f^{n+1)}(z) = \frac{\partial}{\partial z}\left(\frac{1}{2}(-1)^n n! \left(z - e^{\frac{3\pi}{4}i}\right)^{-(n+1)} + \frac{1}{2}(-1)^n n! \left(z - e^{\frac{7\pi}{4}i}\right)^{-(n+1)}\right) =$$

$$= \frac{1}{2}(-1)^n n! \left(z - e^{\frac{3\pi}{4}i}\right)^{-(n+1)-1} \cdot (-n-1) + \frac{1}{2}(-1)^n n! \left(z - e^{\frac{7\pi}{4}i}\right)^{-(n+1)-1} (-n-1) =$$

$$= \frac{1}{2}(-1)^{n+1}(n+1)! \left(z - e^{\frac{3\pi}{4}i}\right)^{-(n+1)-1} + \frac{1}{2}(-1)^{n+1}(n+1)! \left(z - e^{\frac{7\pi}{4}i}\right)^{-(n+1)-1}$$

$$f^{n)}(0) = \frac{1}{2}(-1)^n n! \left[\frac{1}{\left(-e^{\frac{3\pi}{4}i}\right)^{n+1}} + \frac{1}{\left(-e^{\frac{7\pi}{4}}\right)^{n+1}}\right] = \frac{(-1)^n n!}{2}\left[\frac{(-1)^{n+1}}{\left(e^{\frac{3\pi}{4}i}\right)^{n+1}} + \frac{1}{\left(e^{\frac{3\pi}{4}i}\right)^{n+1}}\right.$$