I. E. S. " SAN ISIDRO "

Asignatura .... Fecha ....

Alumno/a .... Curso .... Nº ....

Apellidos Nombre

Calificación

$$= \begin{cases} (n \text{ par}) = \dfrac{n!}{2}\left[\dfrac{-1}{(e^{\frac{3\pi}{4}i})^{n+1}} + \dfrac{1}{(e^{\frac{3\pi}{4}i})^{n+1}}\right] = 0 \\ (n \text{ impar}) \\ \quad = -\dfrac{n!}{2}\left[\dfrac{1}{(e^{\frac{3\pi}{4}i})^{n+1}} + \dfrac{1}{(e^{\frac{3\pi}{4}i})^{n+1}}\right] = -\dfrac{n!}{(e^{\frac{3\pi}{4}i})^{n+1}} \end{cases}$$

$$\Longrightarrow f(z) = \frac{z}{z^2+i} = \sum_{n=0}^{\infty} \frac{f^{n)}(0)}{n!}(z-0)^n = \sum_{\substack{n=0 \\ n \text{ impar}}}^{\infty} -\frac{\cancel{n!}}{(e^{\frac{3\pi}{4}i})^{n+1}} \cdot \frac{z^n}{\cancel{n!}} =$$

$$= -\frac{1}{e^{\frac{3\pi}{4}i}} \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(e^{\frac{3\pi}{4}i})^{2n+1}} \qquad \forall z \in D(0,1)$$

$$f^{n)}(1) = \frac{(-1)^n n!}{2}\left[\left(1-e^{\frac{3\pi}{4}}\right)^{-n-1} + \left(1+e^{\frac{3\pi}{4}}\right)^{-n-1}\right]$$

$$\Longrightarrow f(z) = \frac{z}{z^2+i} = \sum_{n=0}^{\infty} \frac{f^{n}(1)}{n!}(z-1)^n = \sum_{n=0}^{\infty} \frac{(-1)^n n!}{2\, n!}\left[\left(1-e^{\frac{3\pi}{4}}\right)^{-n-1} + \left(1+e^{3\pi/4}\right)^{-n-1}\right](z-1)^n$$

$$= \frac{1}{2}\sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{(1-e^{3\pi/4 i})^{n+1}} + \frac{1}{(1+e^{3\pi/4 i})^{n+1}}\right)(z-1)^n =$$

$$= \frac{1}{2(1-e^{\frac{3\pi i}{4}})} \sum_{n=0}^{\infty} \left(\frac{z-1}{e^{3/4 i}-1}\right)^n + \frac{1}{2(1+e^{3\pi/4 i})} \sum_{n=0}^{\infty} \left(\frac{1-z}{e^{3/4 i}+1}\right)^n \qquad \forall z \in D(1, \sqrt{2-\sqrt{2}})$$