Si $\theta_0 \le \theta \le T \Rightarrow |\theta-\theta_0| + |\theta - T| = 2\theta - (T+\theta_0) = 2\theta - T - \theta_0 \underset{\substack{\Updownarrow \\ 2\theta \le 2T \\ OK}}{\le} T - \theta_0 = (T-\theta_0) I_{[\theta_0,\infty)}(\theta) + (\theta_0 - T) I_{(-\infty,\theta_0)}(\theta)$

Si $\theta \ge T \Rightarrow |\theta-\theta_0| + |\theta-T| = T - \theta_0 \le T - \theta_0 = (\theta_0 - T) I_{(-\infty,\theta_0]}(\theta) + (T-\theta_0) I_{[\theta_0,\infty)}(\theta)$

Integrando a ambos lados de la desigualdad.

$$\int_{\Theta} (|\theta-\theta_0| - |\theta - T|)\, dF(\theta|x_1 \dots x_n) \le \int_{-\infty}^{\theta_0} (\theta_0 - T)\, dF(\theta|x_1 \dots x_n) + \int_{\theta_0}^{\infty} (T-\theta_0)\, dF(\theta|x_1 \dots x_n) =$$

$$= (\theta_0 - T)\, P\{\theta \le \theta_0 | x_1 \dots x_n\} + (T-\theta_0)\, P\{\theta \ge \theta_0 | x_1 \dots x_n\} =$$

$$= (\theta_0 - T) \cdot F(\theta_0|x_1 \dots x_n) + (T-\theta_0)(1 - F(\theta_0|x_1 \dots x_n)) \underset{\substack{\uparrow \\ \theta_0 \text{ es la mediana}}}{=}$$

$$= (\theta_0 - T)\frac{1}{2} + (T-\theta_0)\left(1 - \frac{1}{2}\right) = 0$$

Por tanto $\displaystyle\int_{\Theta} |\theta-\theta_0|\, dF(\theta|x_1 \dots x_n) \le \int_{\Theta} |\theta - T|\, dF(\theta|x_1 \dots x_n)$

Análogamente, si $T < \theta_0$

Entonces $|\theta-\theta_0| - |\theta-T| = \begin{cases} \theta_0 - T & \text{si } \theta \le T \\ T+\theta_0 - 2\theta & \text{si } T \le \theta \le \theta_0 \\ T - \theta_0 & \text{si } \theta_0 \le \theta \end{cases}$

Podemos acotar de nuevo

$$|\theta-\theta_0| - |\theta - T| \le (\theta_0 - T) I_{(-\infty,\theta_0)}(\theta) + (T-\theta_0) I_{(\theta_0,\infty)}(\theta)$$

Si $\theta \le T \Rightarrow |\theta-\theta_0| - |\theta-T| = \theta_0 - T \le \theta_0 - T = (\theta_0 - T) I_{(-\infty,\theta_0)}(\theta) + (T-\theta_0) I_{(\theta_0,\infty)}(\theta)$

Si $T \le \theta \le \theta_0 \Rightarrow |\theta-\theta_0| - |\theta-T| = T+\theta_0 - 2\theta \underset{\substack{\Updownarrow \\ 2T \le 2\theta}}{\le} \theta_0 - T = (\theta_0-T) I_{(-\infty,\theta_0)}(\theta) + (T-\theta_0) I_{(\theta_0,\infty)}(\theta)$

Si $\theta_0 \le \theta \Rightarrow |\theta-\theta_0| - |\theta-T| = T - \theta_0 \le T - \theta_0 = (\theta_0 - T) I_{(-\infty,\theta_0)}(\theta) + (T-\theta_0) I_{(\theta_0,\infty)}(\theta)$