ACTIVITY 2:PROPERTIES AND APPLICATIONS OF THE 2D FOURIER TRANSFORM (PART 2 OF 2)

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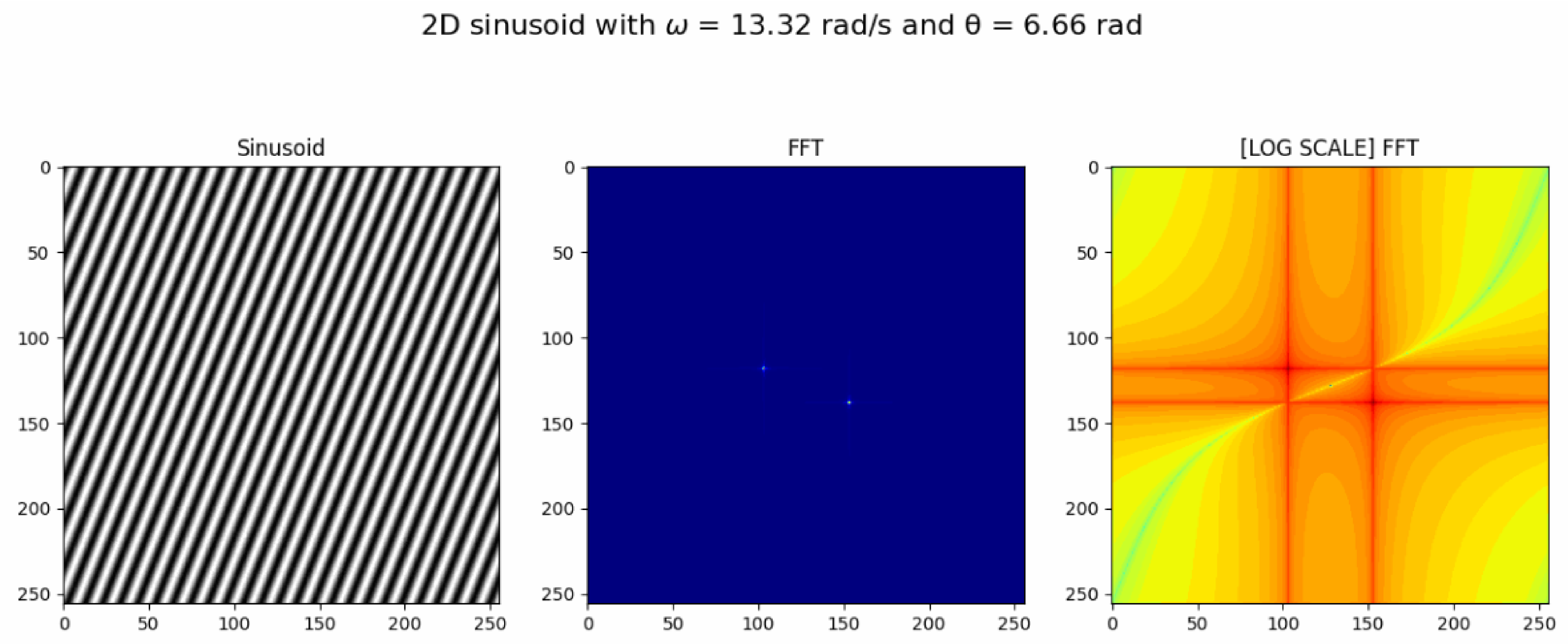
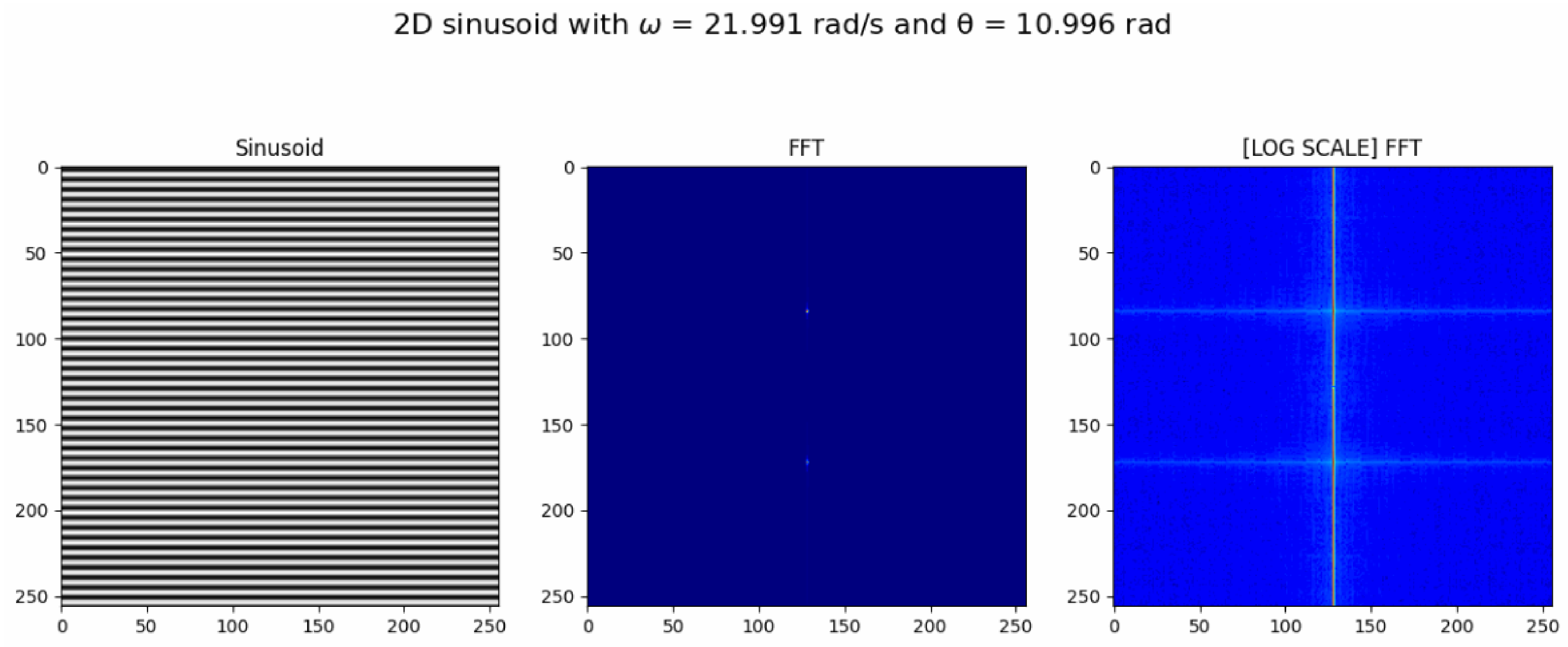
App Physics 157 WFY-FX-1

2.2.1 Rotation Property of the FT

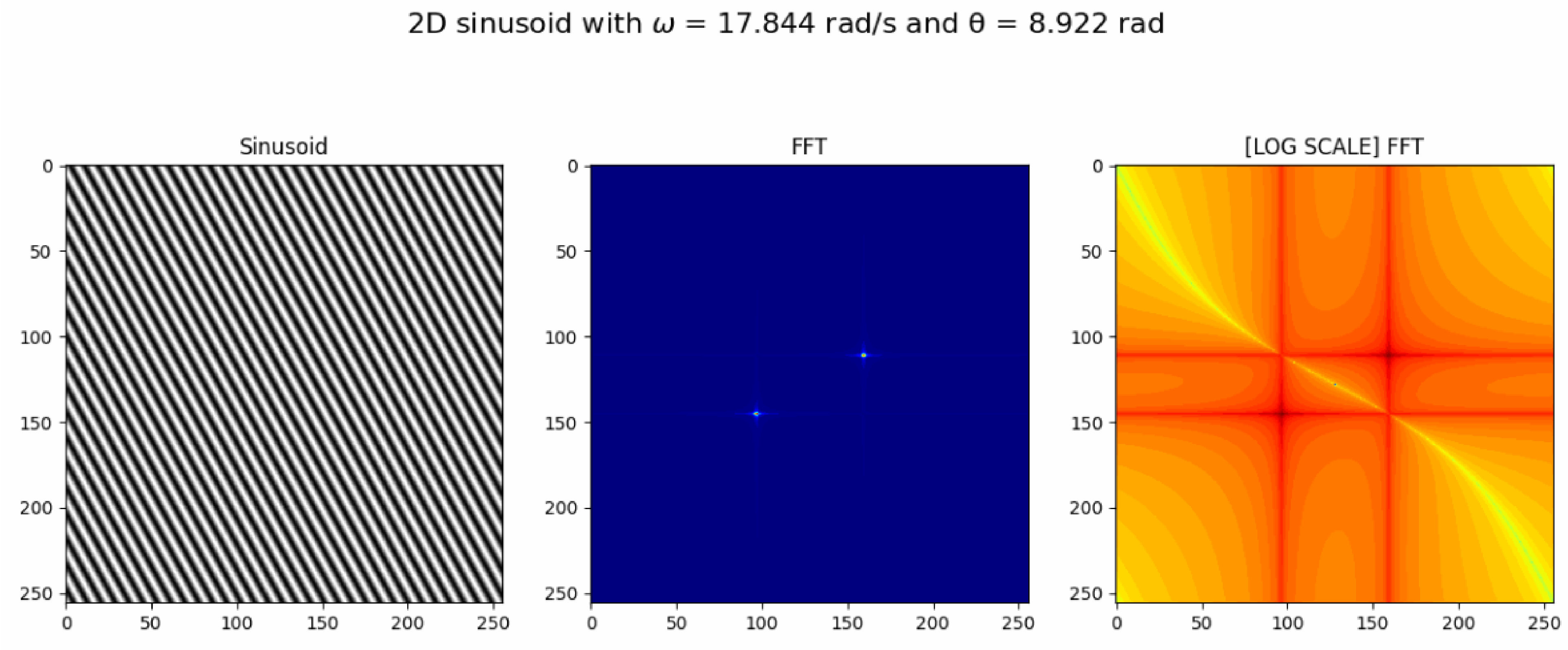
I have already done this in the previous activity, so I decided to make an animation by varying the frequency and rotation angle of the sinusoid. Unfortunately, I can't save GIFs in PDF format, so I'll upload the animations in the submission bin. The following are some screenshots of the animation.



Looking at the results, the FFT of the sinusoid are two peaks that are symmetric around the origin (i.e. the center of the image). As the sinusoid rotates, the peaks also rotate in the same direction. The separation distance between the two peaks are also proportional to the frequency of the sinusoid. As the frequency increases, the separation distance increases. And as the frequency decreases, the separation distance decreases.

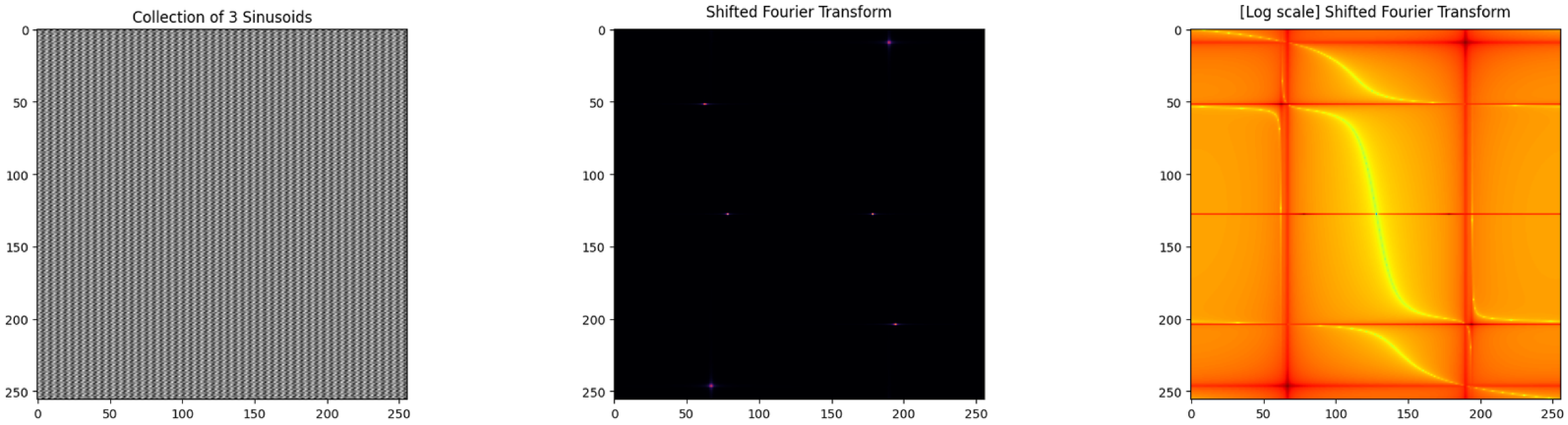


My results are consistent with my last activity's results. But it can now be seen better because of the animation. The results are correct because two peaks are to be expected in the sinusoid's FFT. This is because a peak corresponds to the dominating frequency from the sinusoid [1]. But due to the FFT being symmetrical, each peak has a reflection about the origin. Thus resulting in two peaks,



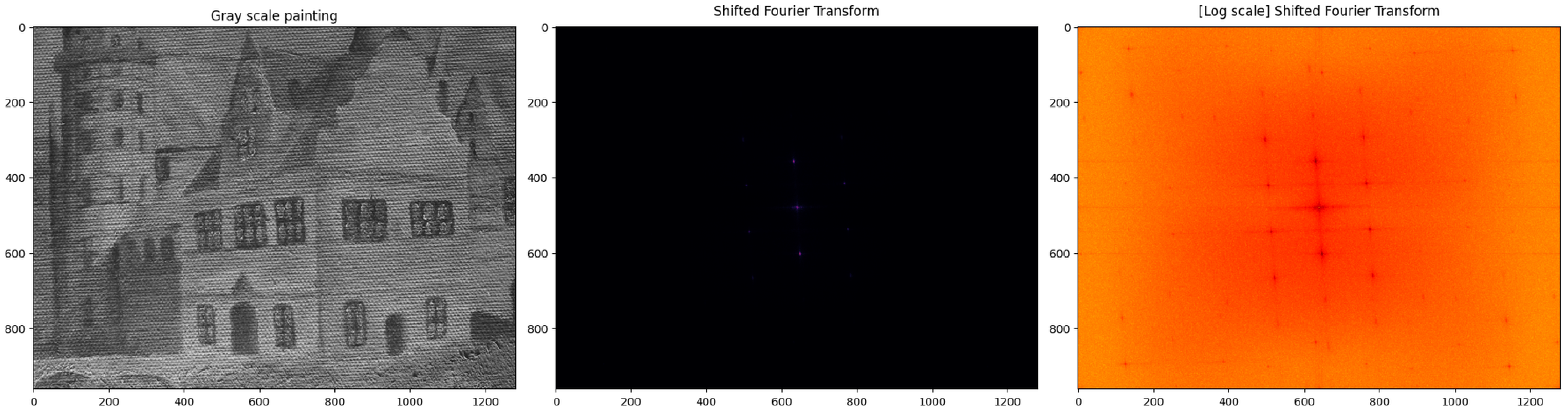
Superimposing multiple sinusoids

I created 3 sinusoids and added them all together. The sinusoids had the following paramters: 50π rad/s (angular frequency) and 0 (angle of rotation), 100π rad/s and 4π rad, 150π rad/s and 20π rad. I then got the FFT and it showed three pairs of peaks that are symmetrical to the origin. These peaks represent the frequencies of each individual sinusoid. This is the expected result because the Fourier Transform is a linear transformation [2]. So the FFT of the superposition of sinusoids is a superposition of the FFTs of the individual sinusoids.



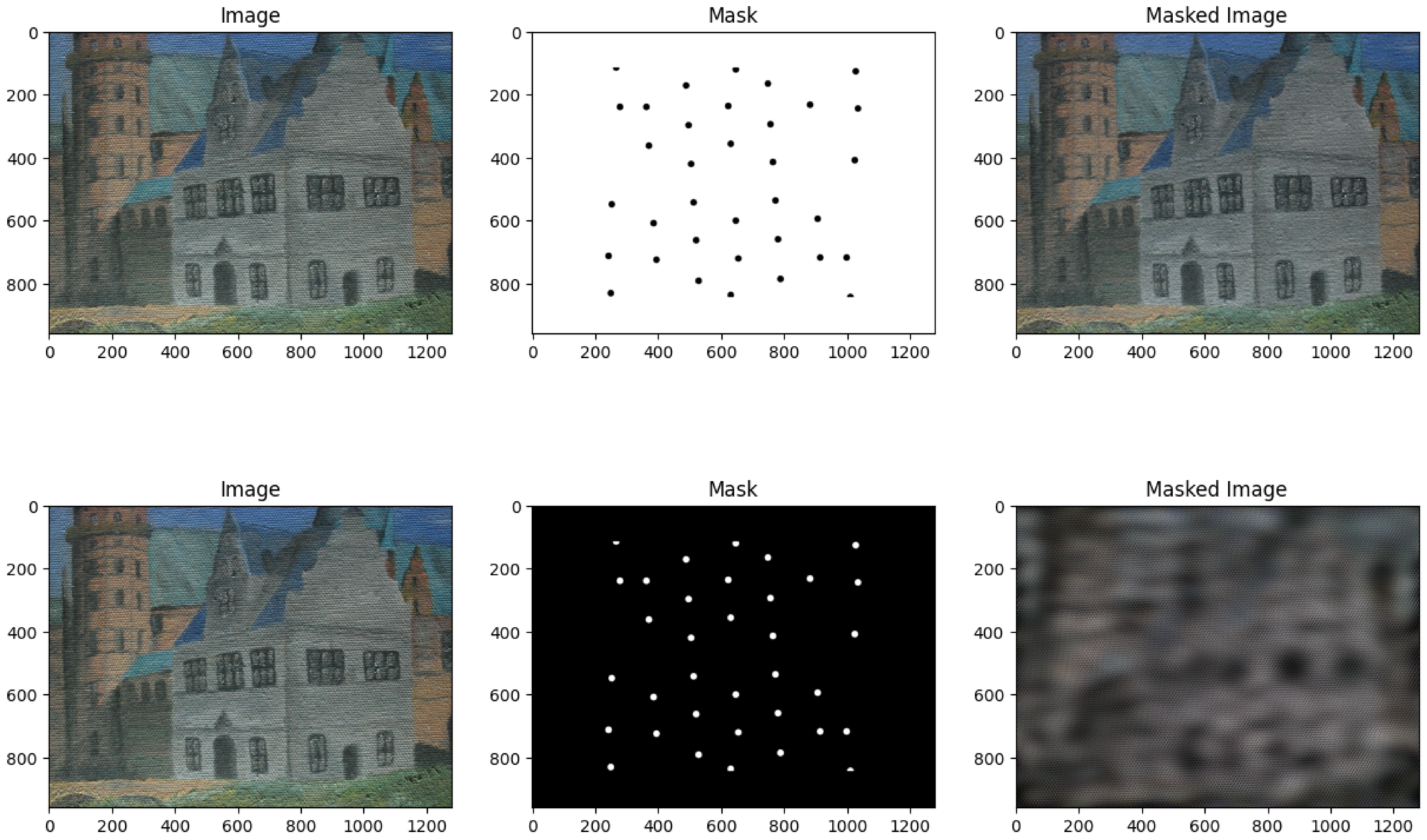
2.2.2 Application: Canvas Weave Modeling and Removal

For this part of the activity, I removed the canvas weave of the Daria painting. I did this by creating an automated mask function. The function first converts the image to gray scale, mean centers it, and applies FFT and FFTshift. The mask is then created by using the *maximum_filter* of scipy to get the locations of the peaks. Since most of the image's information is stored at the center of the FFT as a peak, I removed the peak at the center by ignoring it in my filter. I then convolve the peaks with a Gaussian function to enlarge them. With the mask created, I then multiplied it to the FFT of each of the color channels of the image. The results are as follows:



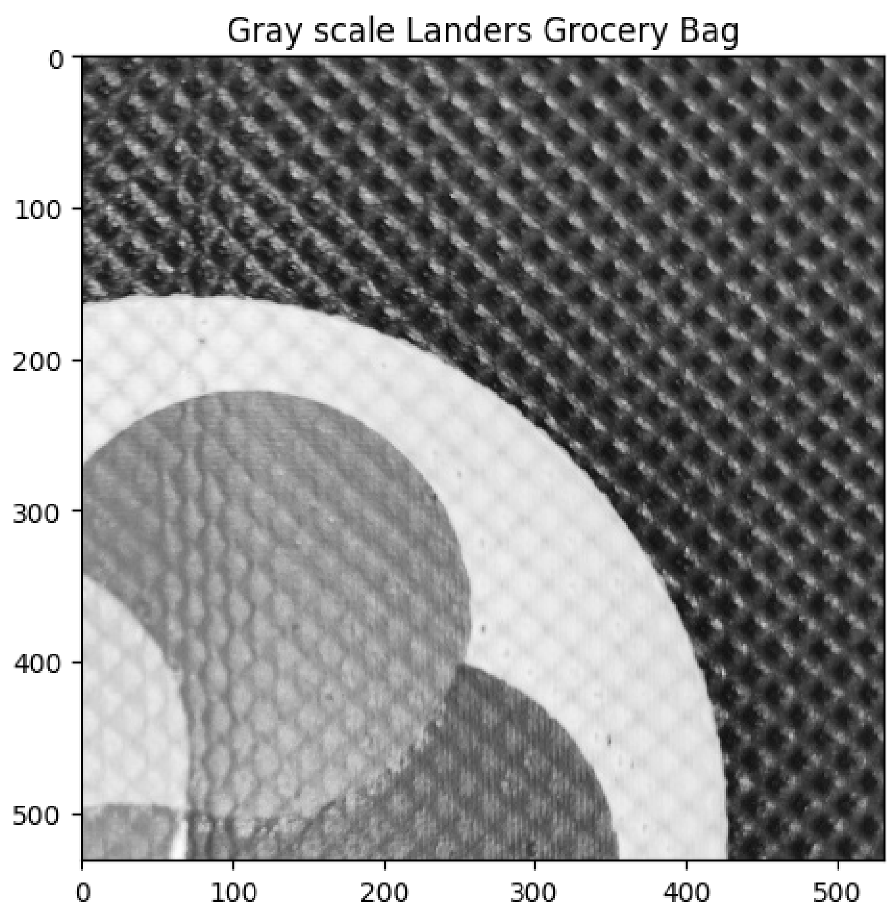
By applying the mask, I was able to remove the canvas pattern from the painting. And by inverting the mask, I was able to extract the canvas weave from the painting.

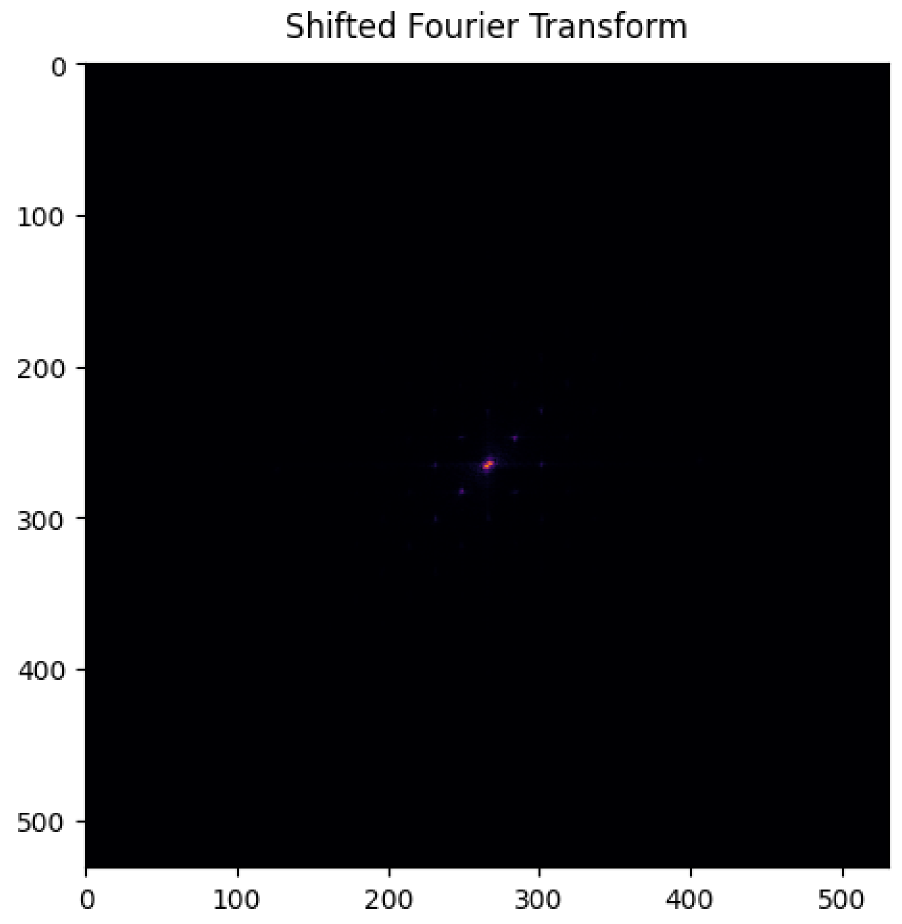
I believe that I got the correct results, as the brush strokes in the masked panting (upper row) is now more visible. I also got the canvas' repeating pattern by inverting the mask. The masked image (bottom row) shows a little bit of detail from the original painting. This is due to the dots in the mask being a bit too large. But through testing different parameters, I found that this was the most optimal mask that I could make.

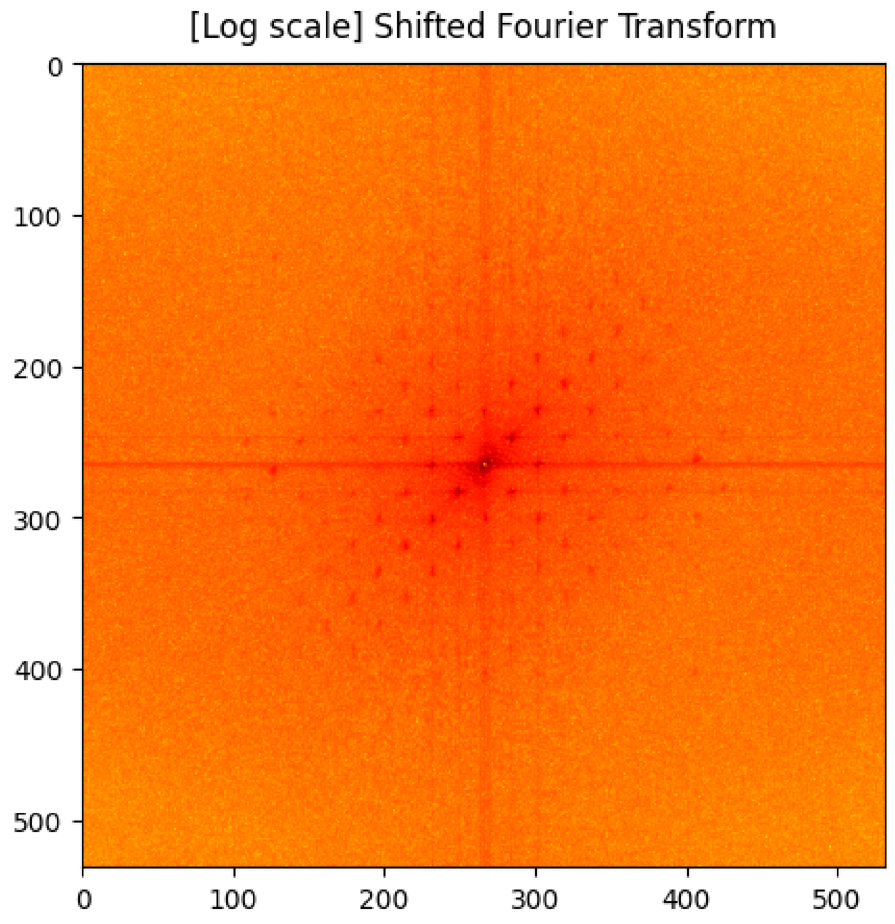


Extra Challenge 1

With an automated mask function, I can basically create masks for almost all images with repeating patterns. So I tried to removed the repeating pattern from a Landers grocery bag. The following are my results:

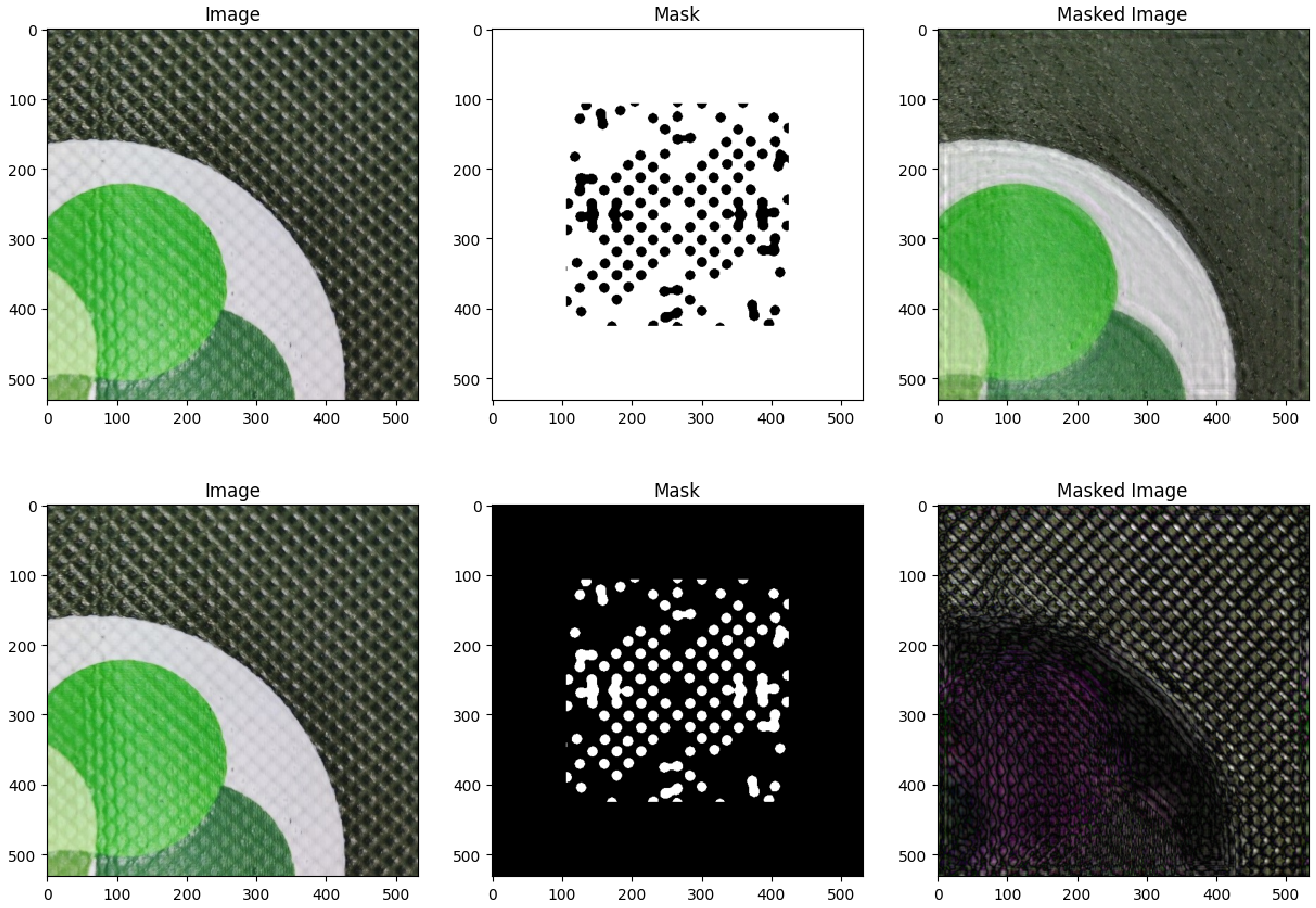






By applying the mask, I was able to smoothen the image by removing the grovcery bag's weave. And when I inverted the mask, I was able to extract the bag's weave pattern.

Ideally, the grocery bag should only have four peaks that correspond to the two perpendicular sinusoid-like patterns that make up the weave. But from the mask, it can be seen that there are several pairs of symmetrical peaks. This can be explained by the grocery bag's condition. The bag is used, so it has undergone stretching, which resulted in a distorted weave pattern. But overall, my program was able to separate the wave pattern from the bag's design.

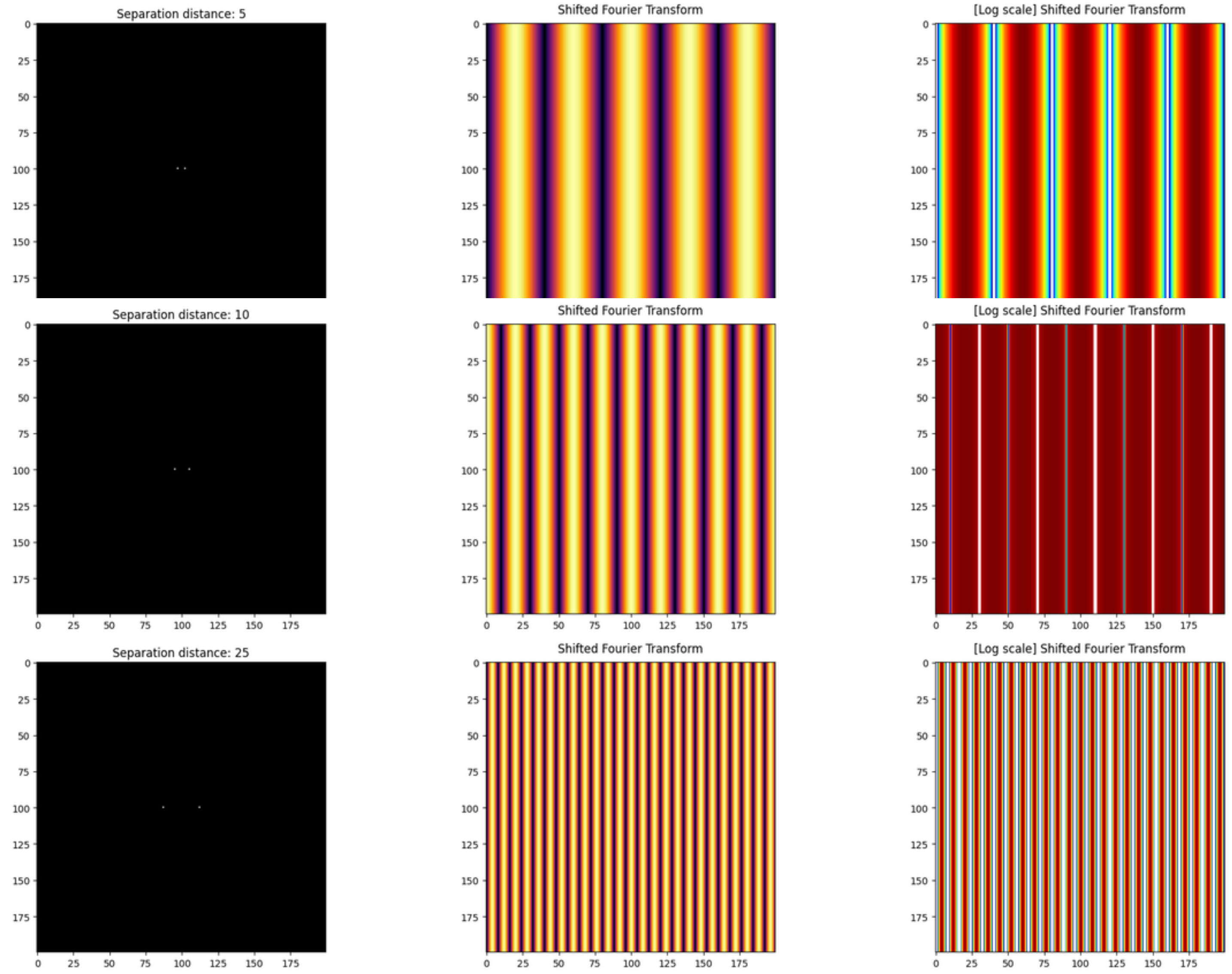


2.2.3 Convolution Theorem Redux

For this part of the activity, I simply followed the steps given by the Applied Physics 157 Laboratory Manual. First, I created two symmetrical dots on the horizontal axis and varied their separation distance. The following are my results:

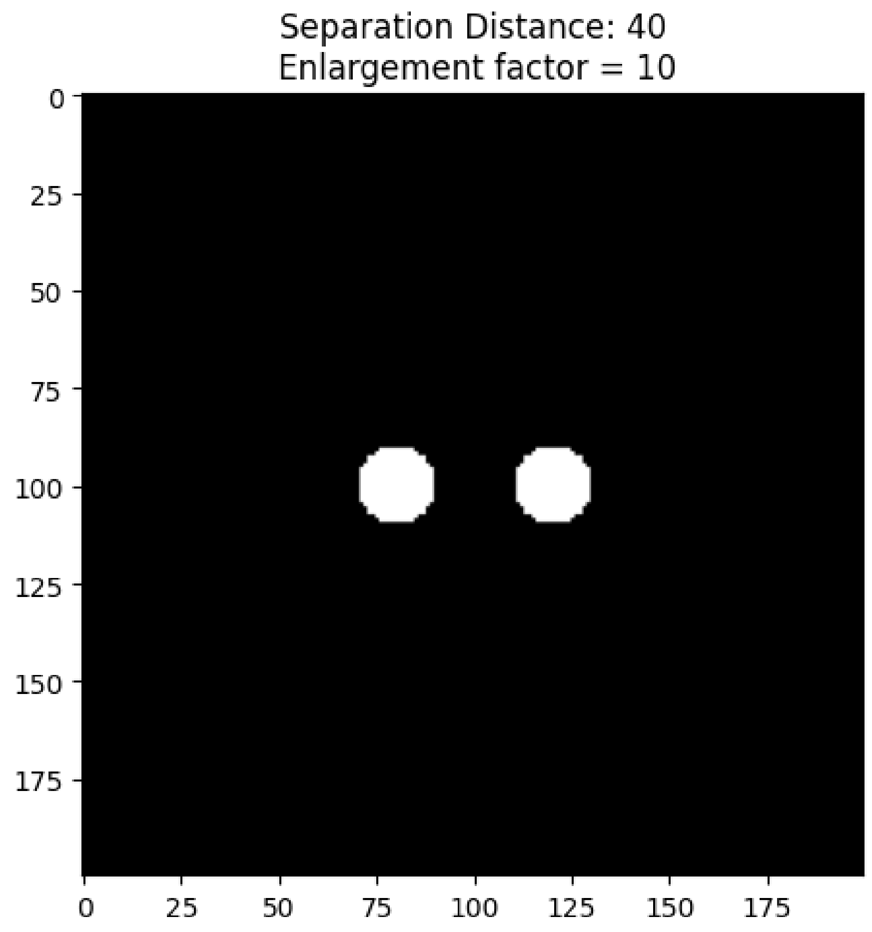
Looking at the results, it can be seen that the FFT of the two dots is a sinusoid. And as the separation distance increases, the frequency of the sinusoid also increases.

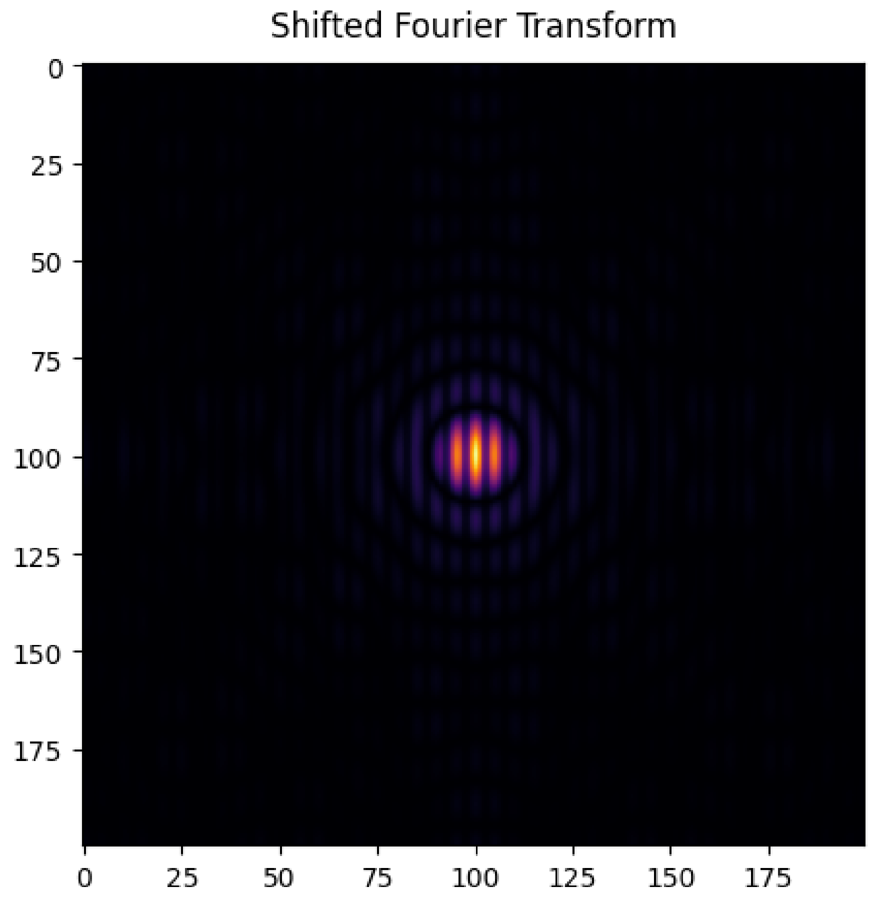
This is to be expected. Recall that in the last activity, an image can be recovered by applying an FFT on it twice. But the recovered image is also vertically flipped. This is basically what happened when we applied an FFT on the two dots. It is equivalent to applying an FFT on a sinusoid twice because the FFT of a sinusoid is two symmetrical dots. The recovered sinusoid (i.e. the FFT of the two dots) is verically flipped, but it can't be observed due to the sinusoid's symmetry across ther horizontal axis.

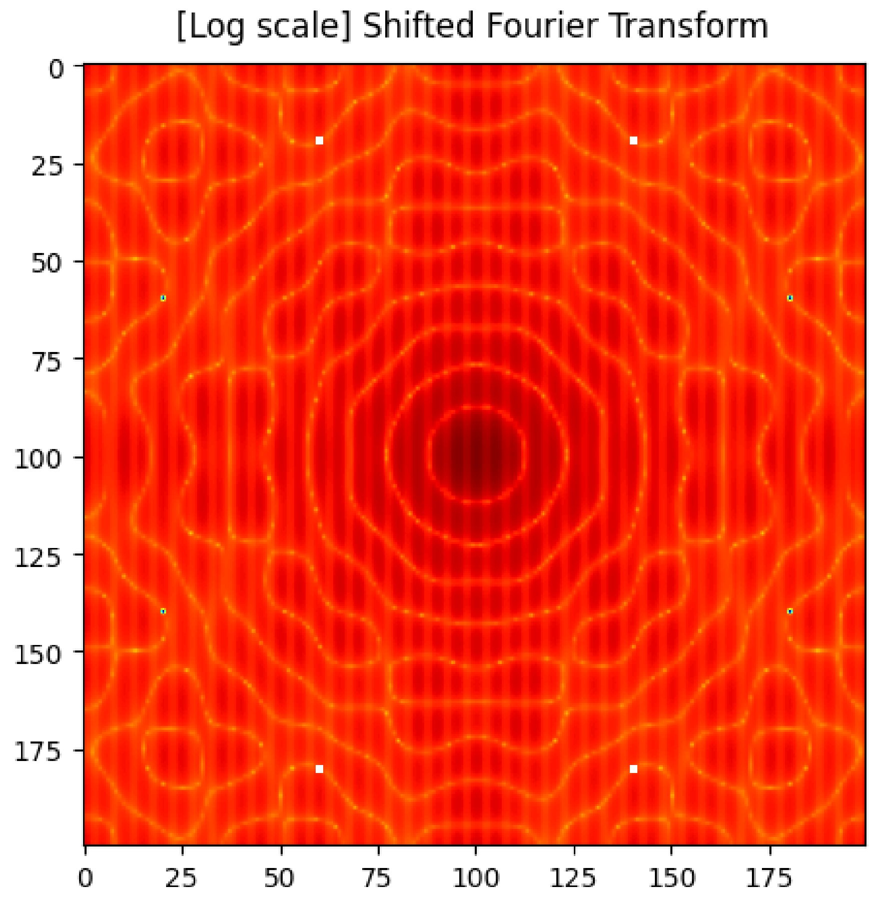


Replacing the dots with circles

I then replaced the dots with circles. The results are as follows:



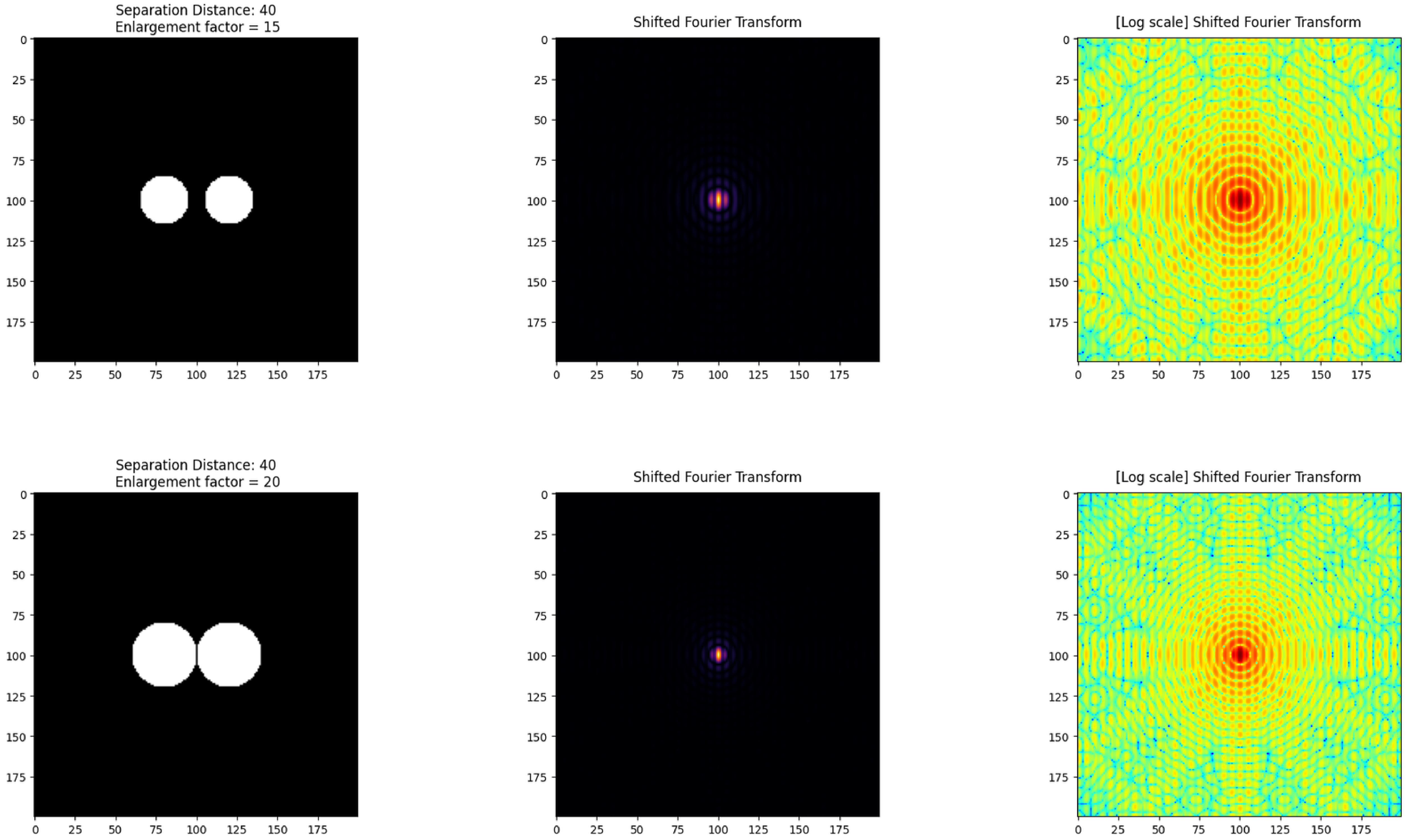




I set the separation distance to 40 and replaced the dots with circles. The enlargement factor that's indicated in the graphs refer to the radius of each circle. The results show that the FFT of the two circles is similar to the FFT of a single circluar aperture. The FFT also exhibits anamorphic properties. As the radii of the two circles increases, the radius in the FFT decreases

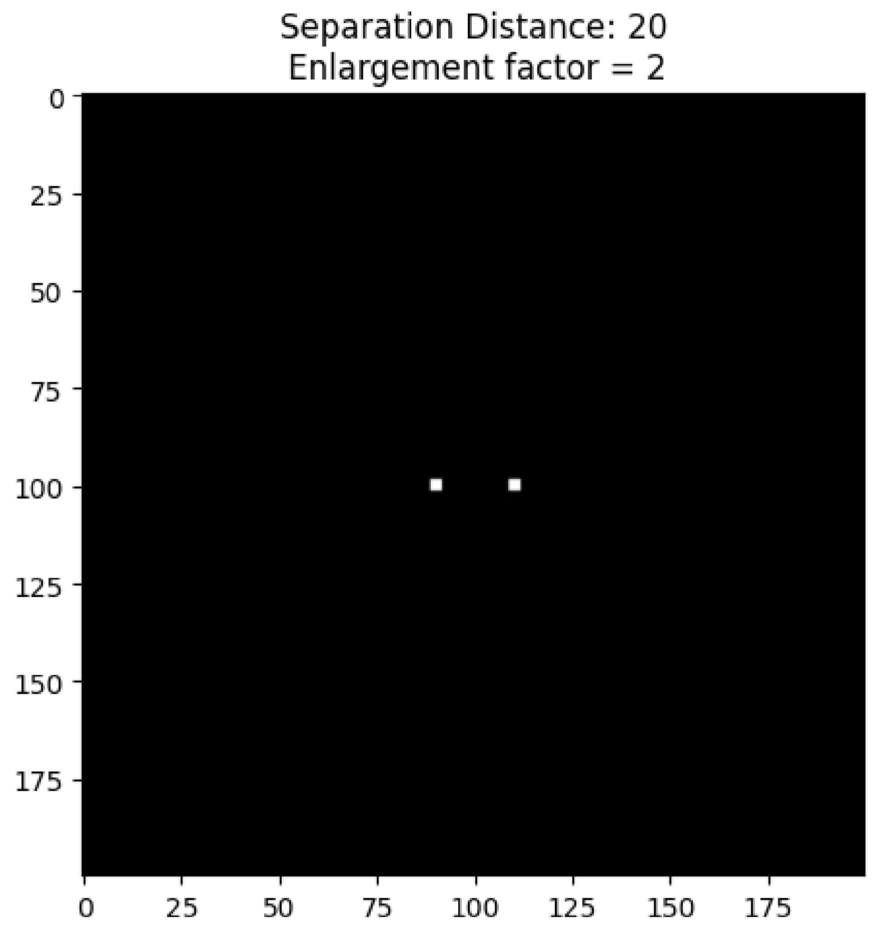
However, the main difference in the FFTs between a single circular aperture and a double circular aperture is that an interference pattern can be observed. There are maxima which are the bright parts, and the are minima which are the vertical black lines in between the maxima.

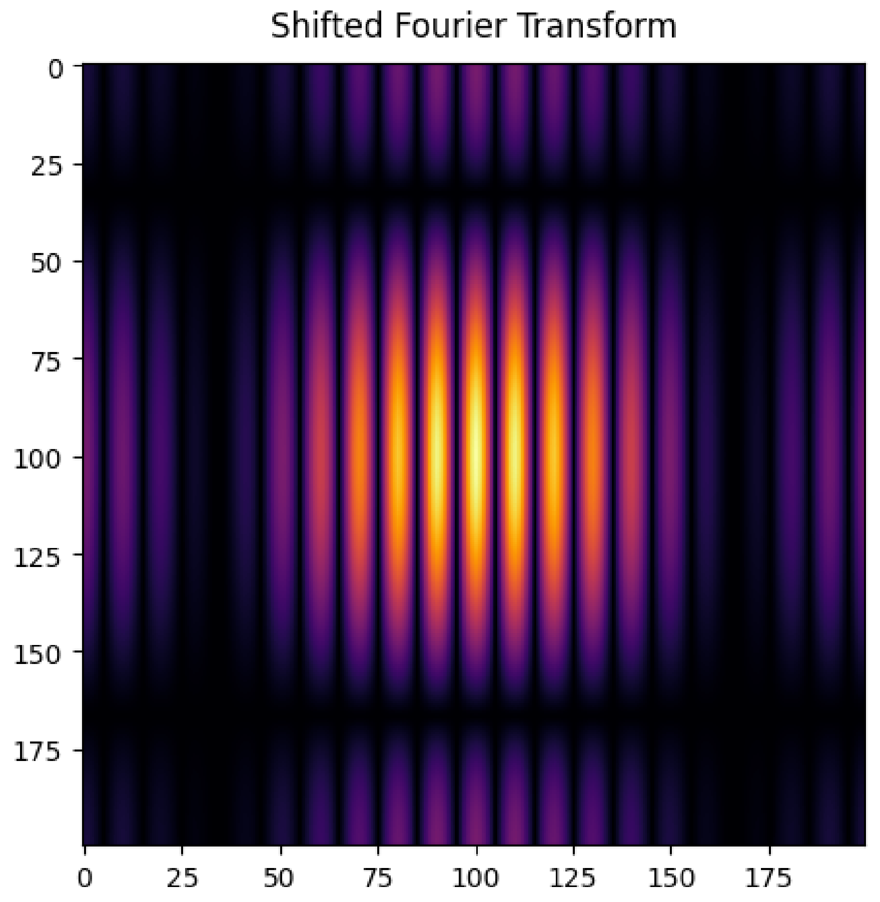
The physical interpretation for this is that if light were to pass through the two circular appertures, their FFT would be the interference pattern. This is because the Fourier transform of the two apertures converts it from the coordinate-space to a momentum density and a momentum distribution function which shows the interference pattern. [3]



Replacing the dots with squares

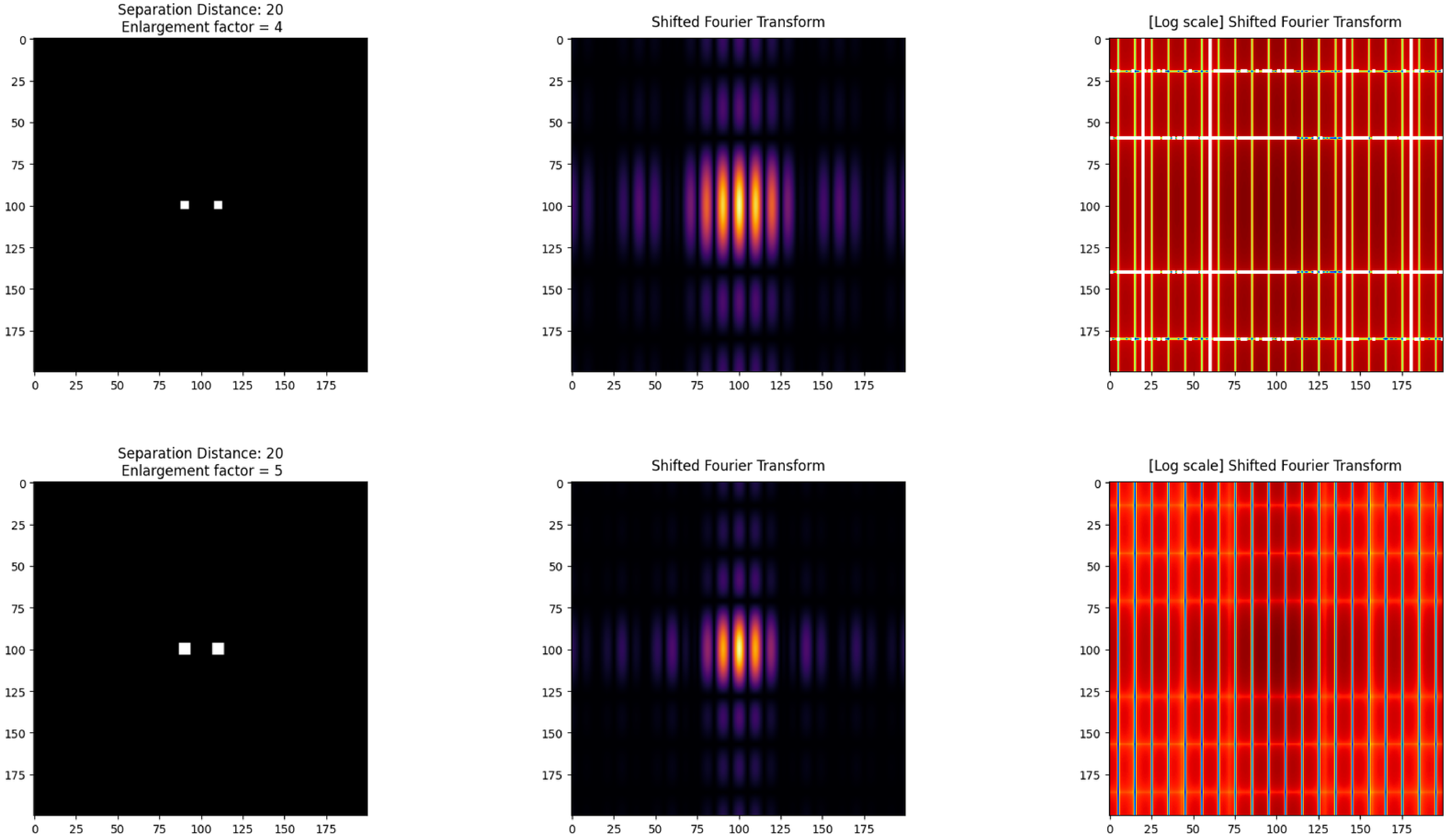
For this part of the activity, I replaced the dots with squares. The results are as follows:





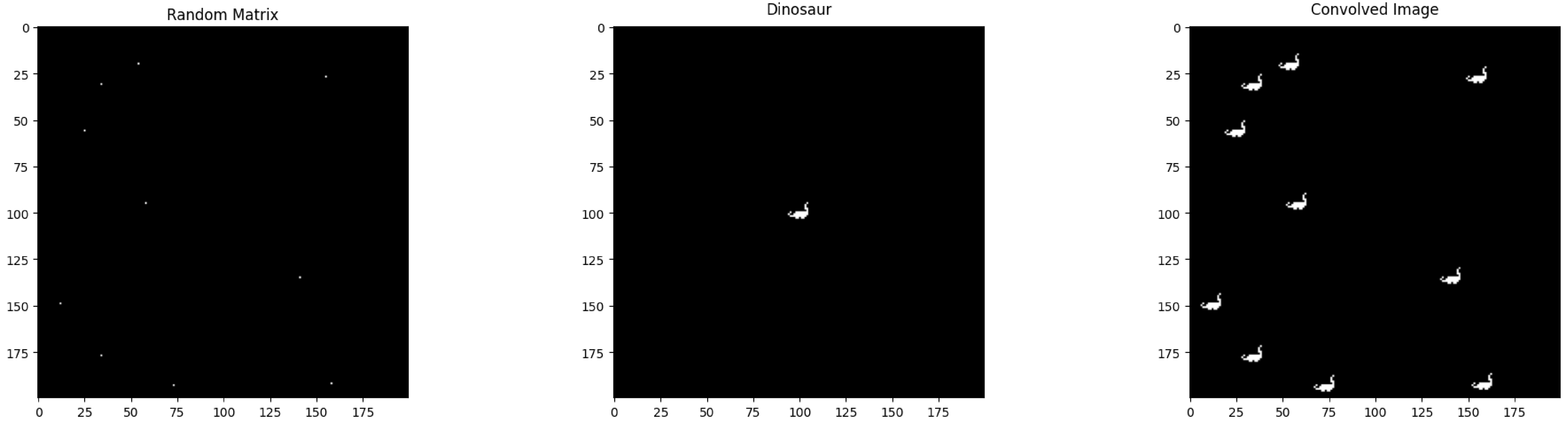
I kept the separation distance between the two points at 20 and then replaced them with squares. The enlargement factor in the graphs refer to the side length of the squares. The FFT exhibits anamorphic propoerties. That is, when the side length of the squares incerases, the side length of the central square in the FFT decreases.

I have done this in the previous activity, The physical representation of this is that the two squares act as a double slit. If light was shined on the slits, then they create an interference pattern which is equal to the FFT of the double slit [3].



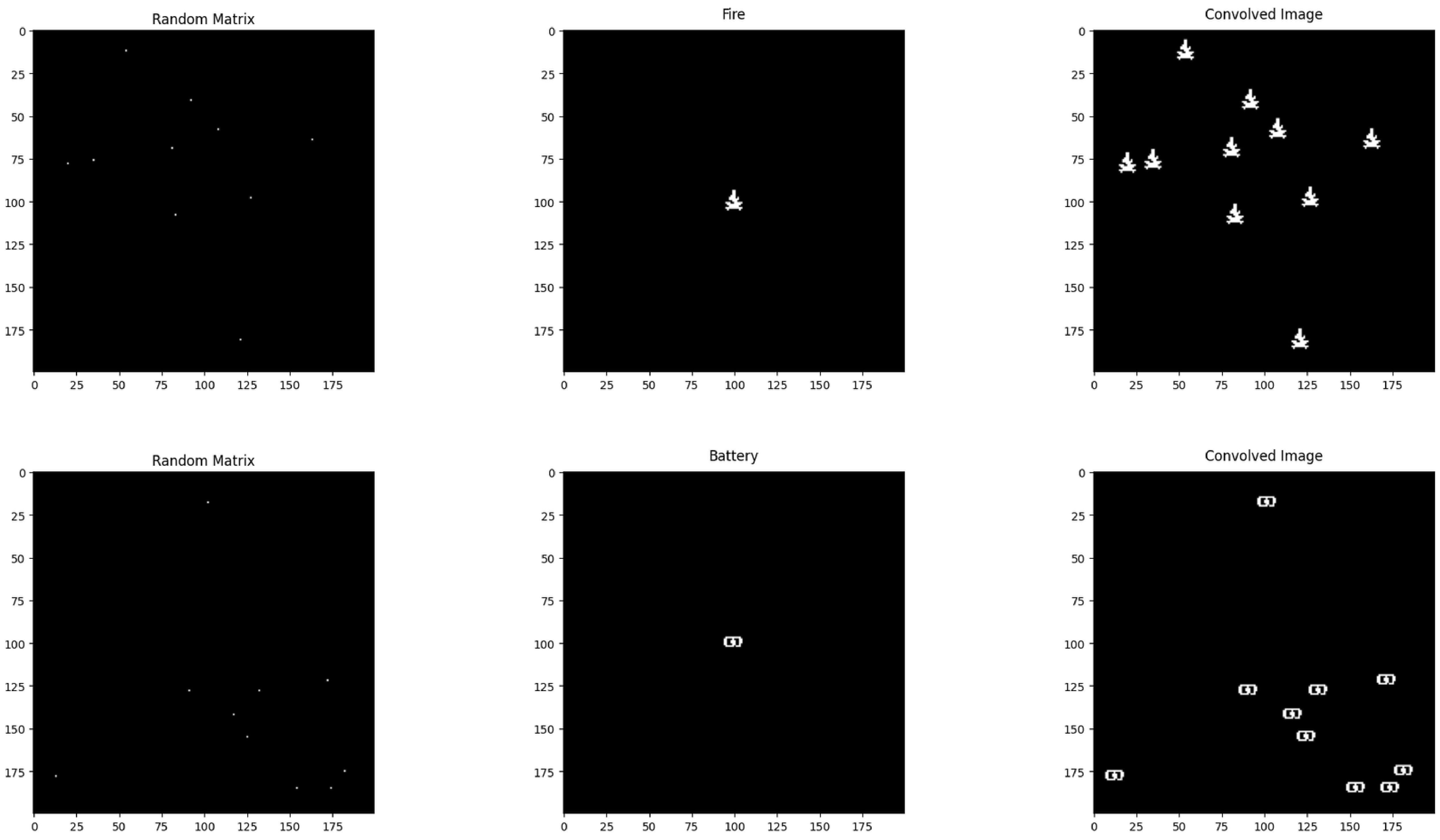
Convolution

I made a matrix with 10 1's at random locations. I then convolved it with a pattern. The results are as follows:



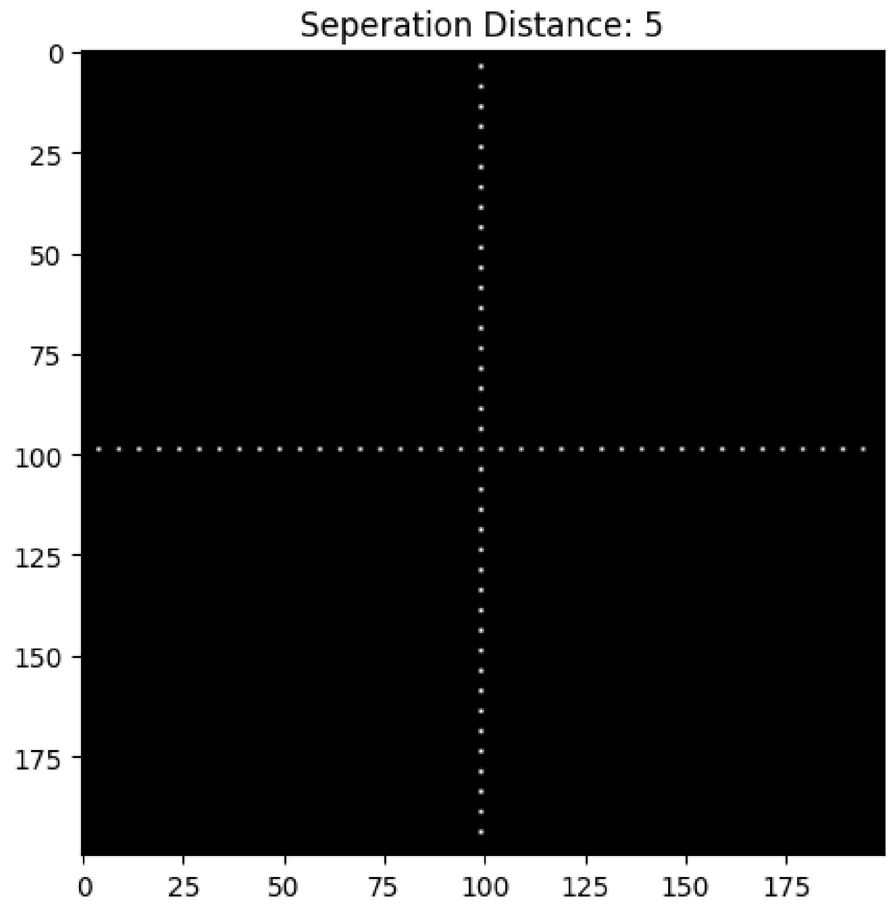
I convolved the matrix with 10 1's at random locations with different patterns. The patterns that I used were a dinosaur, a campfire, and a battery. The patterns were created by using Microsoft Powerpoint's icon feature and I binarized them before the convolution.

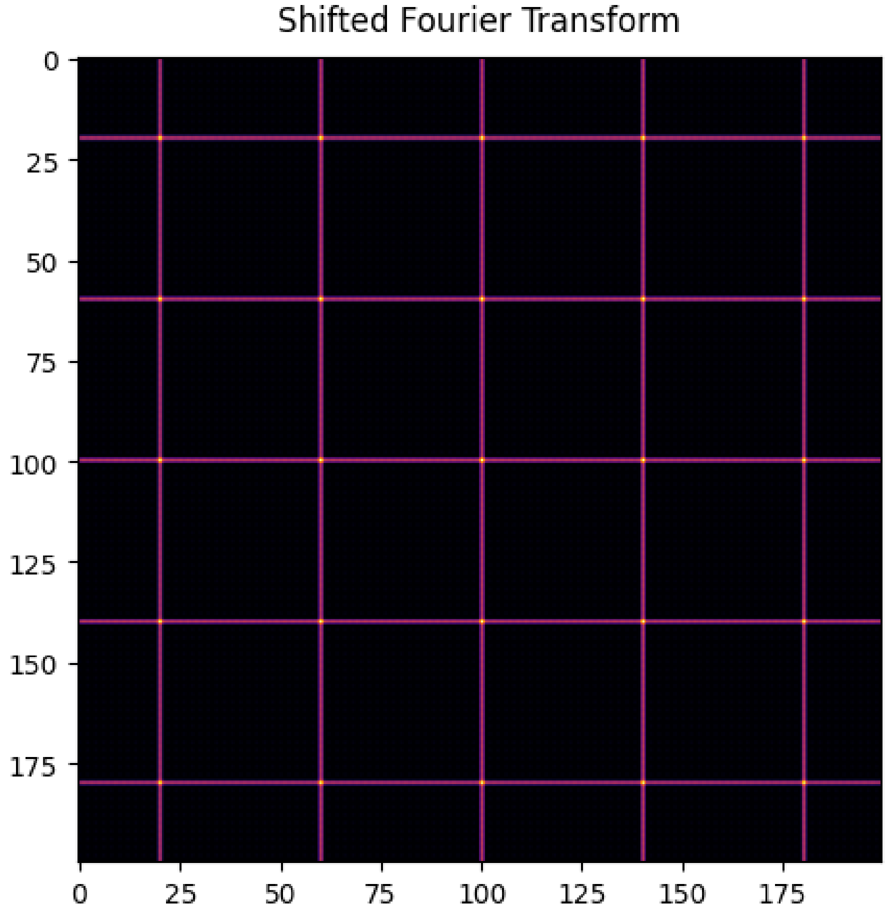
Looking at the results, it can be seen that after convolution, the pattern appeared at the location of the 1's in the random matrix. This is to be expected because this is how convolutions is defined to be. One function "blends" over to the other function [4]. And since one of the functions is similar to a collection of dirac deltas, then the other function will be copied onto the dirac deltas.

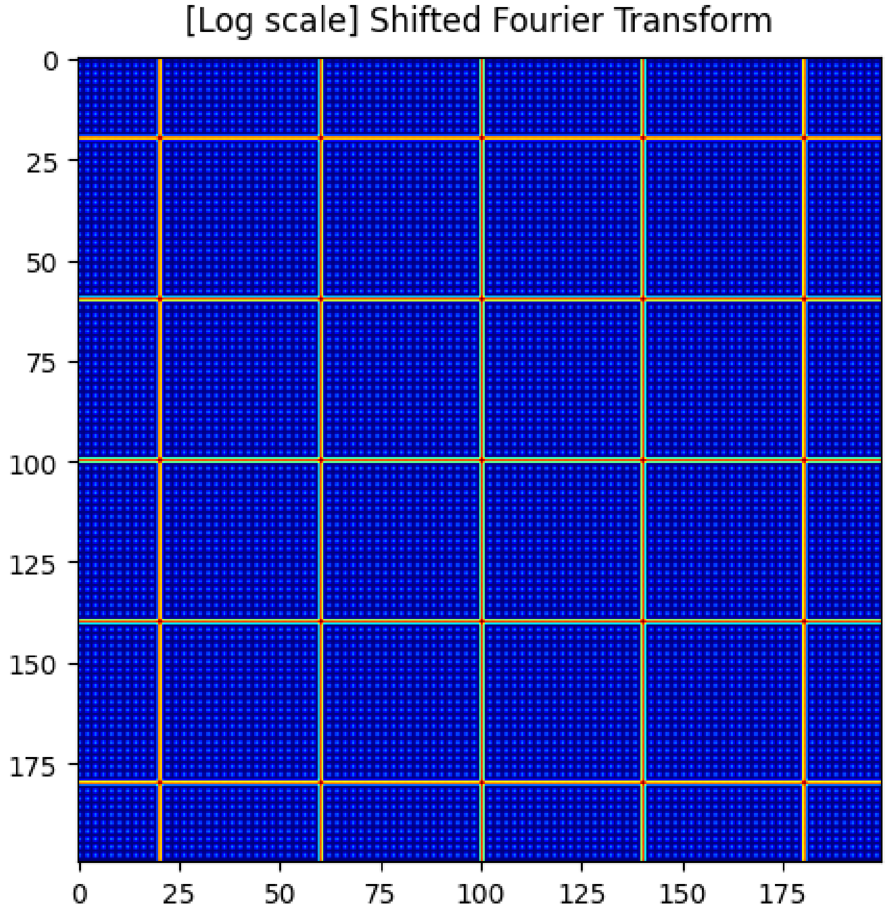


1s on the x and y axes.

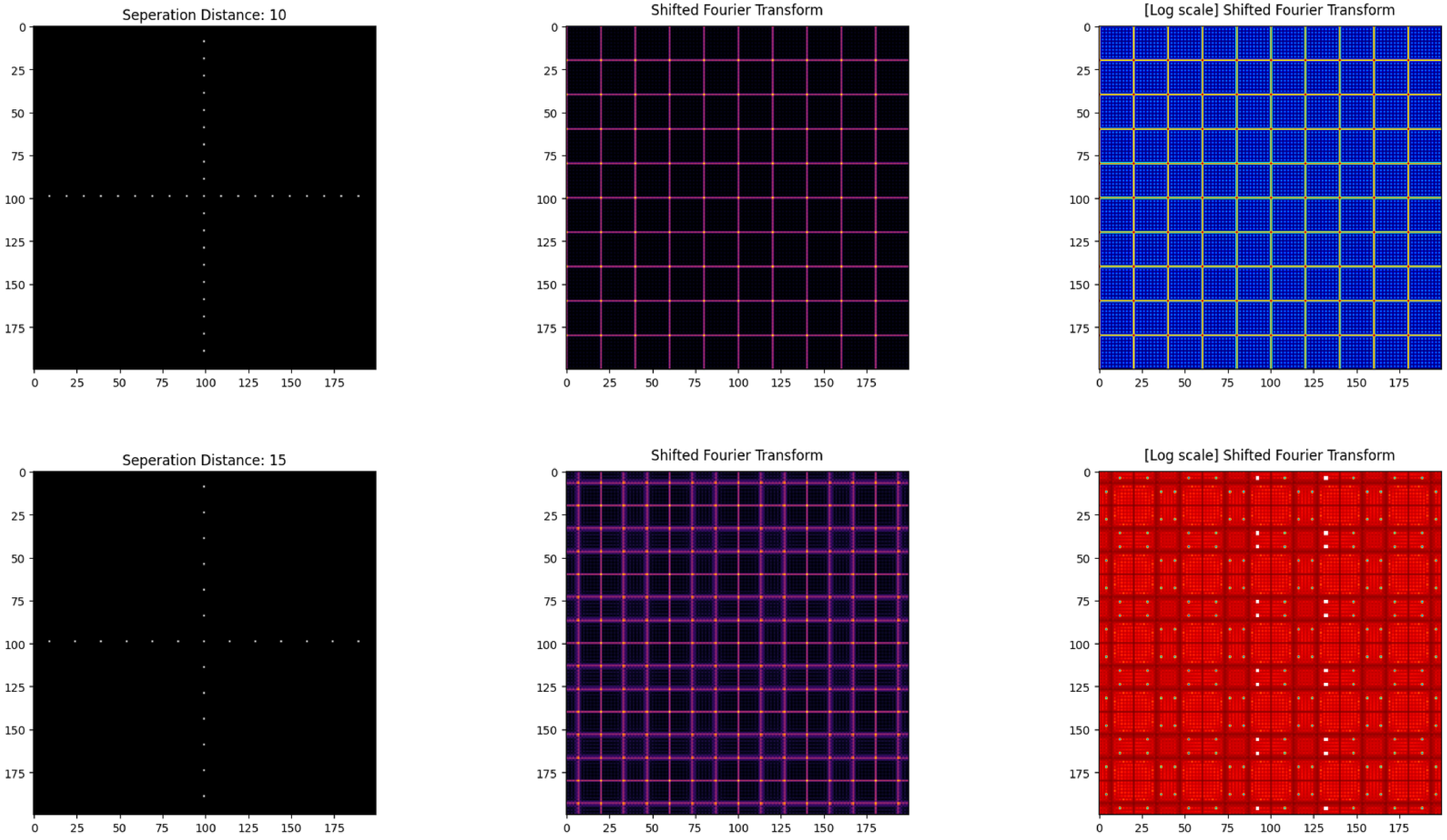
I generated 1's along the x and y axis of an image and got its FFT. The results are as follows:







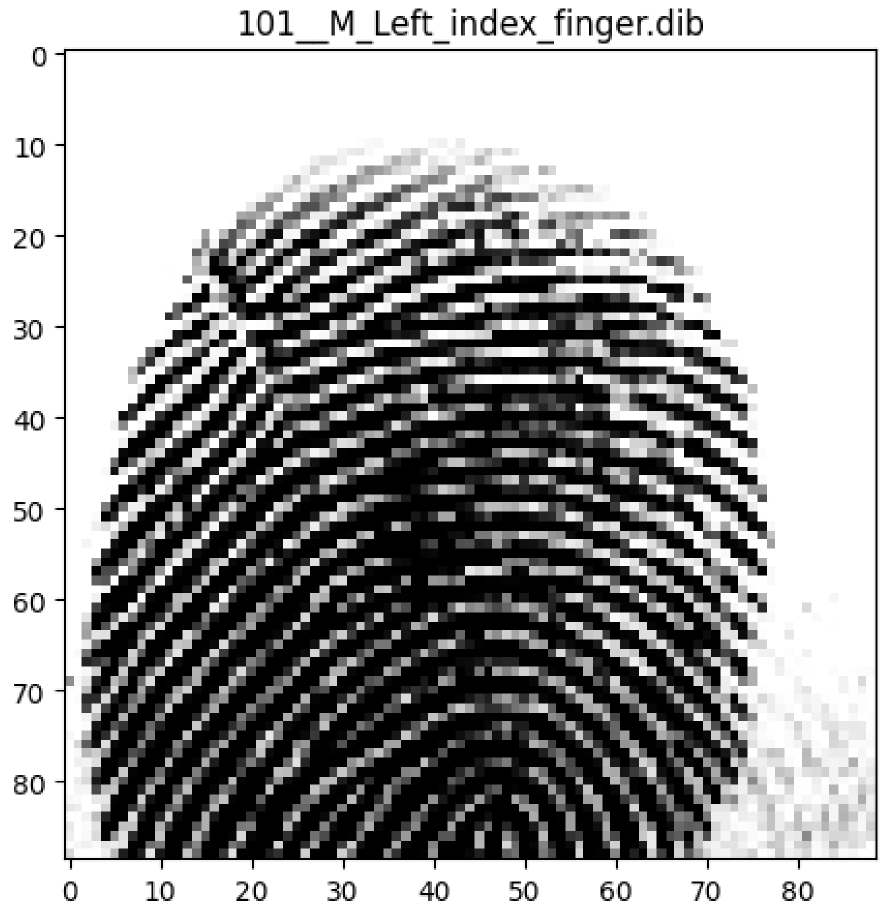
I varied the separation distance between the 1s and the values that I used were 5, 10, and 15. The FFT of these 1s form a grid-like pattern. It can also be observed that a smaller separation distance gives a wider grid while a larger separation distance gives a more condensed grid.

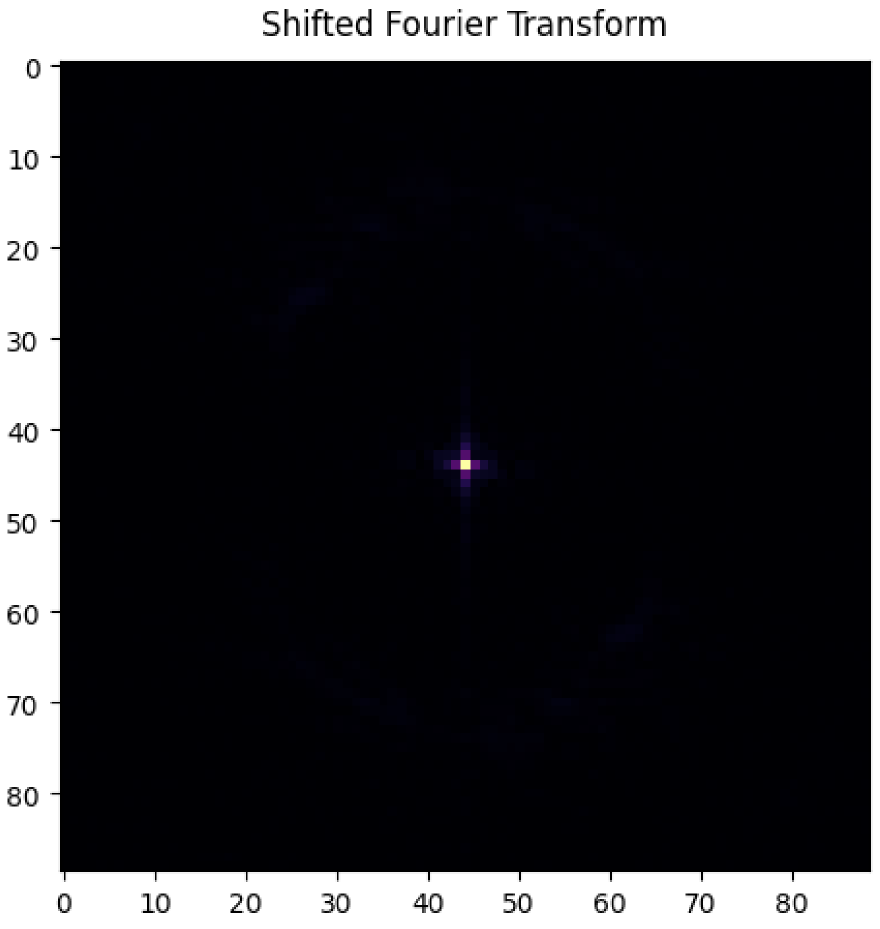


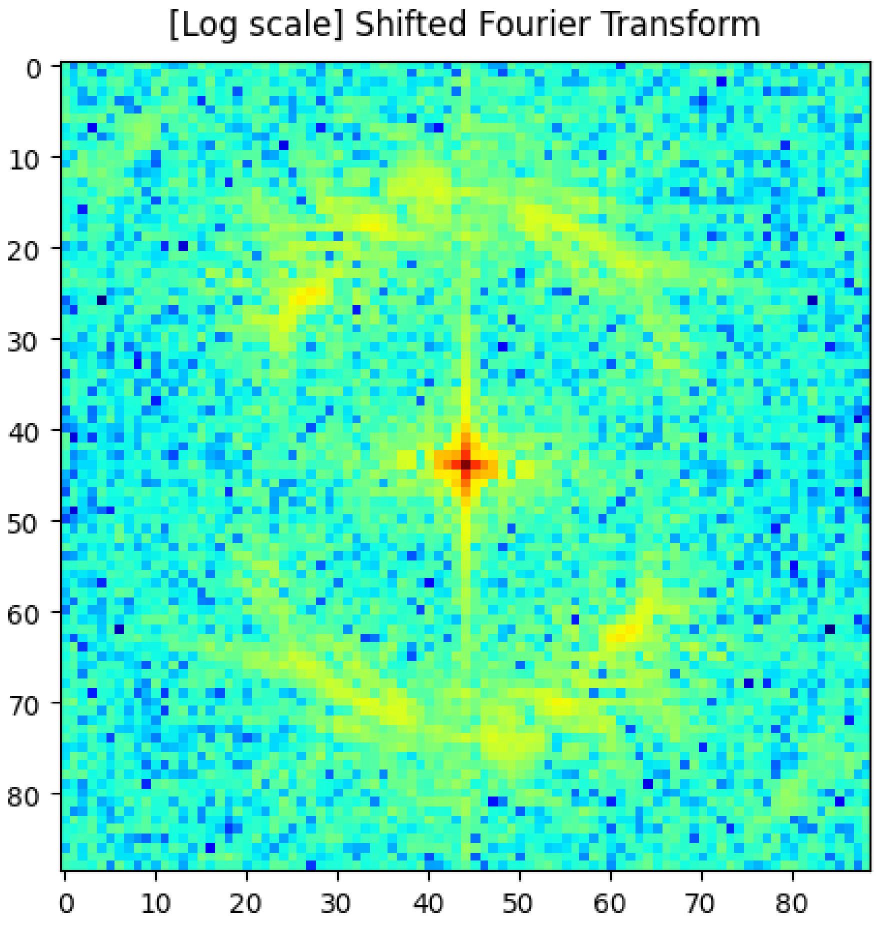
From the previous parts of this activity, we know that the FFT of two symmetrical dots is a sinusoid and vicec versa. And the FFT of a superposition of sinusoids is the superposition of the FFTs of each sinusoid. By knowing this, we can tell that the FFT of the dots is the superposition of multiple sinusoids with varying frequencies, each with an angle of rotation of either 0 (for the horizontal dots) or 180 degrees (for the vertical dots). A

2.2.4 Fingerprints : Ridge Enhancement

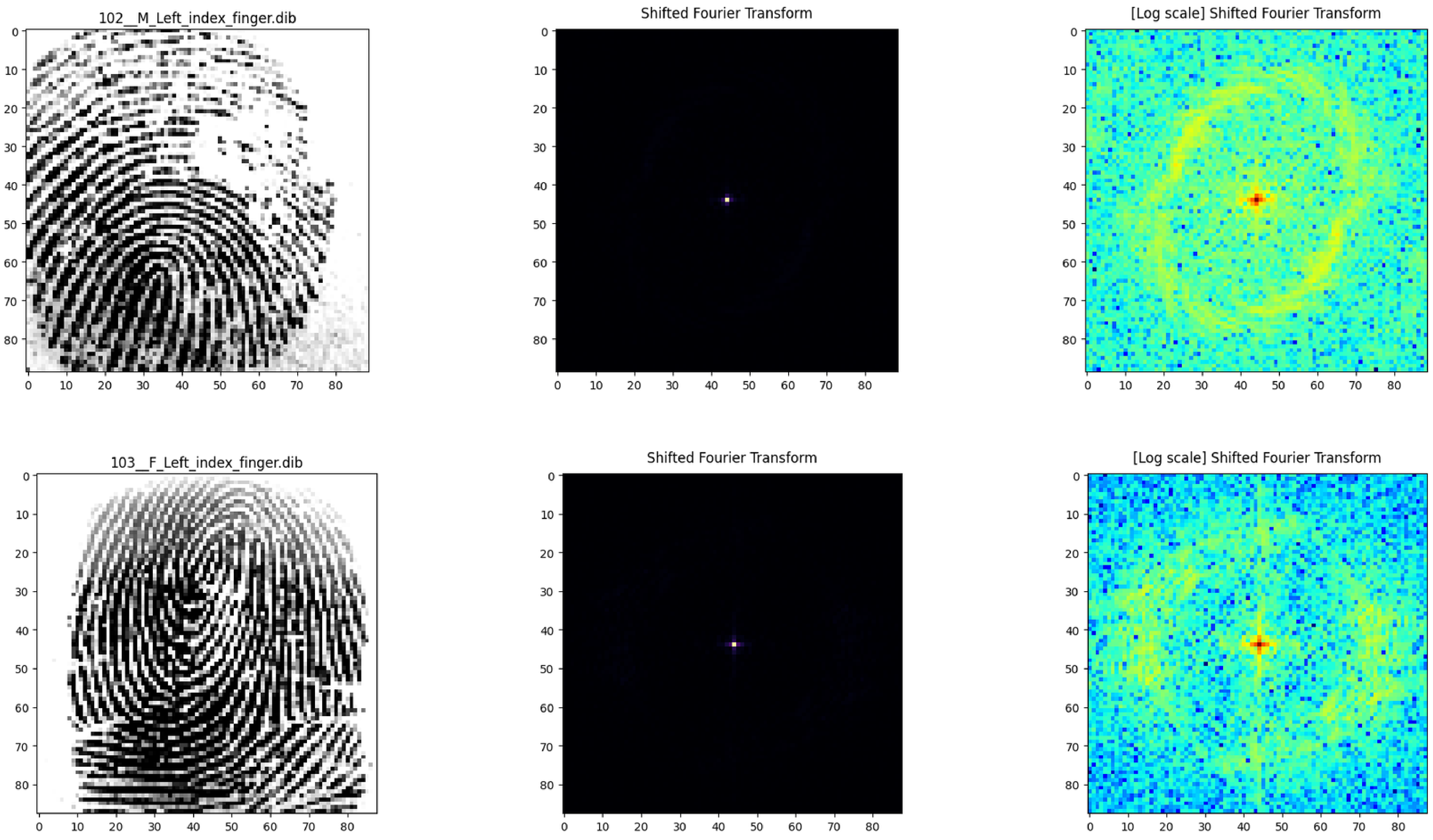
For this part of the activity, I used publicly available fingerprints. The fingerprints that I used were obtained from Sokoto Coventry Fingerprint Dataset (SOCOFing) which is readily available in kaggle (https://www.kaggle.com/datasets/ruizgara/socofing). The following are my results.



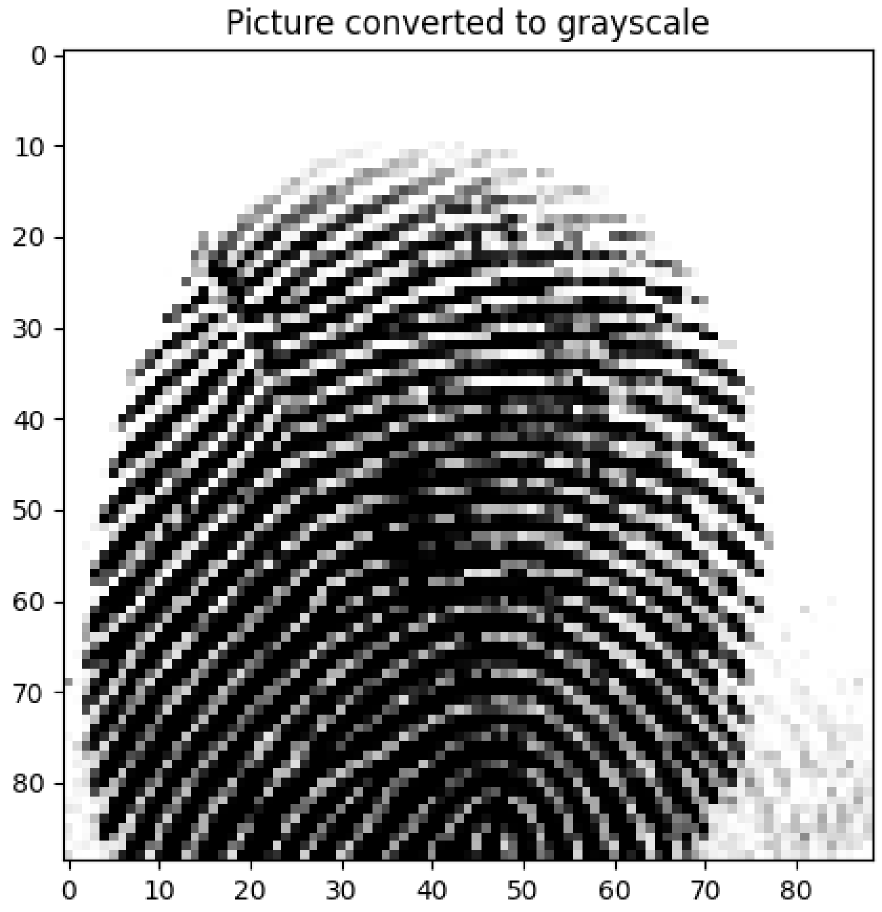


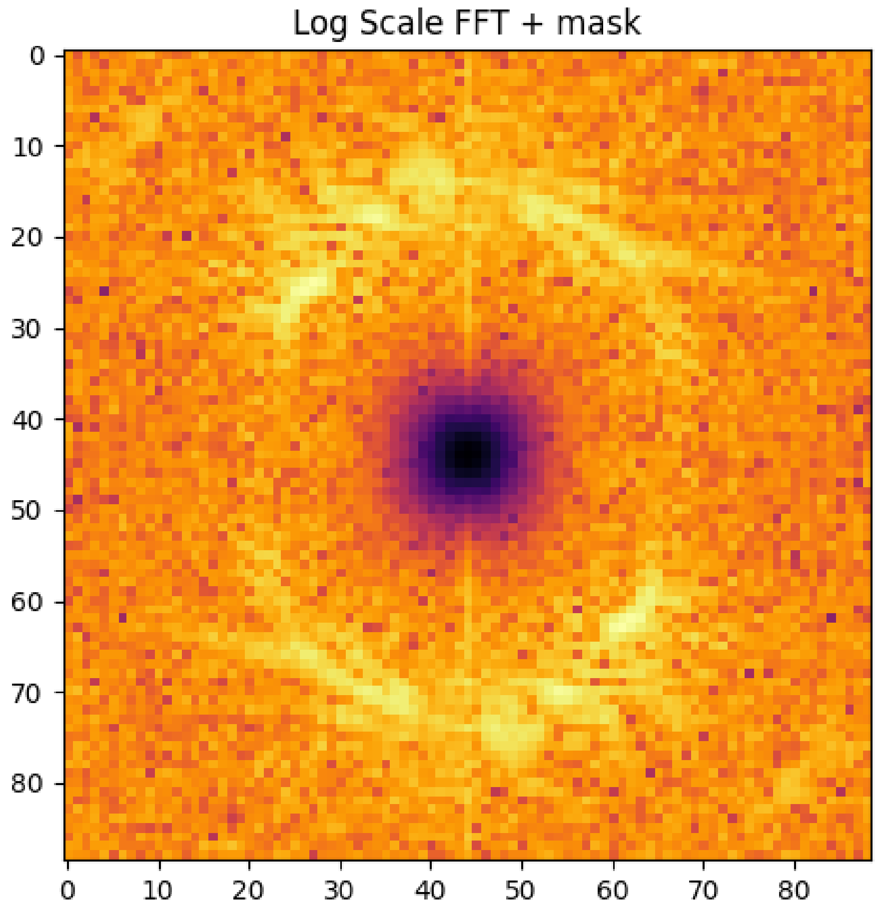


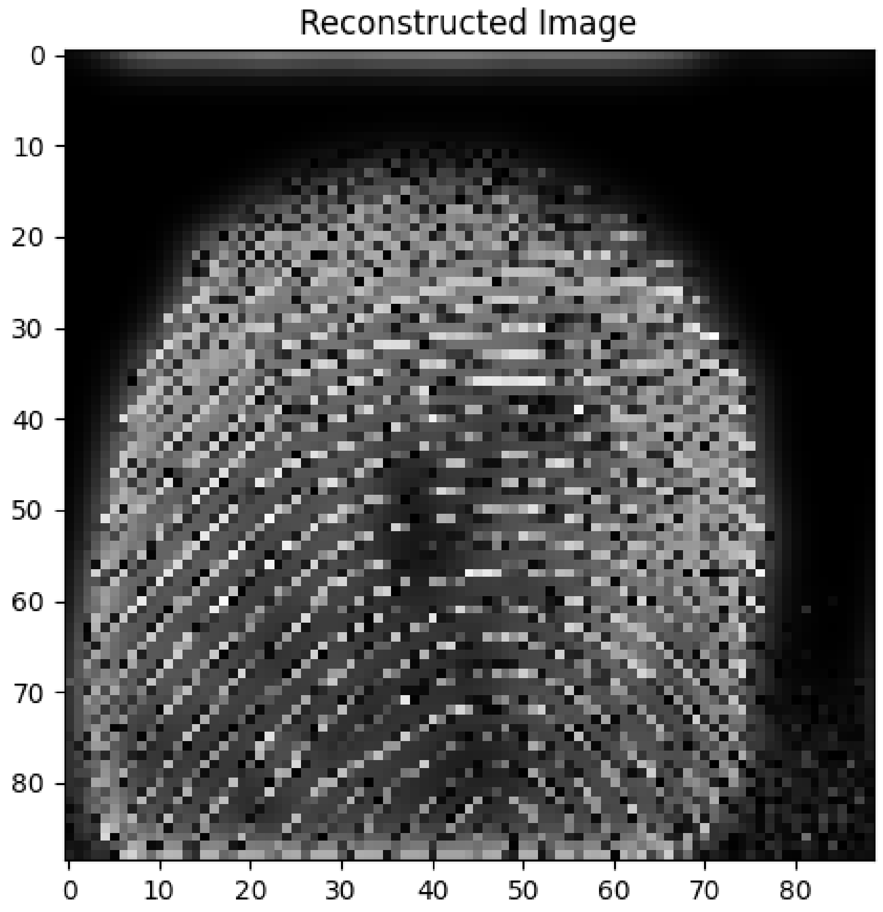
Looking at the log scale FFTs, it can be seen that there is a peak at the center of the FFT and a ring-like pattern around it. From the previous parts of the activity, we know that most of the image's information is stored at the center of the image and repeating patterns tend to appear as peaks that are relatively far from the center. Thus, we can concur that the ridge pattern of the finger print is contained in the ring while the smudges and other information are contained at the center of the FFT.



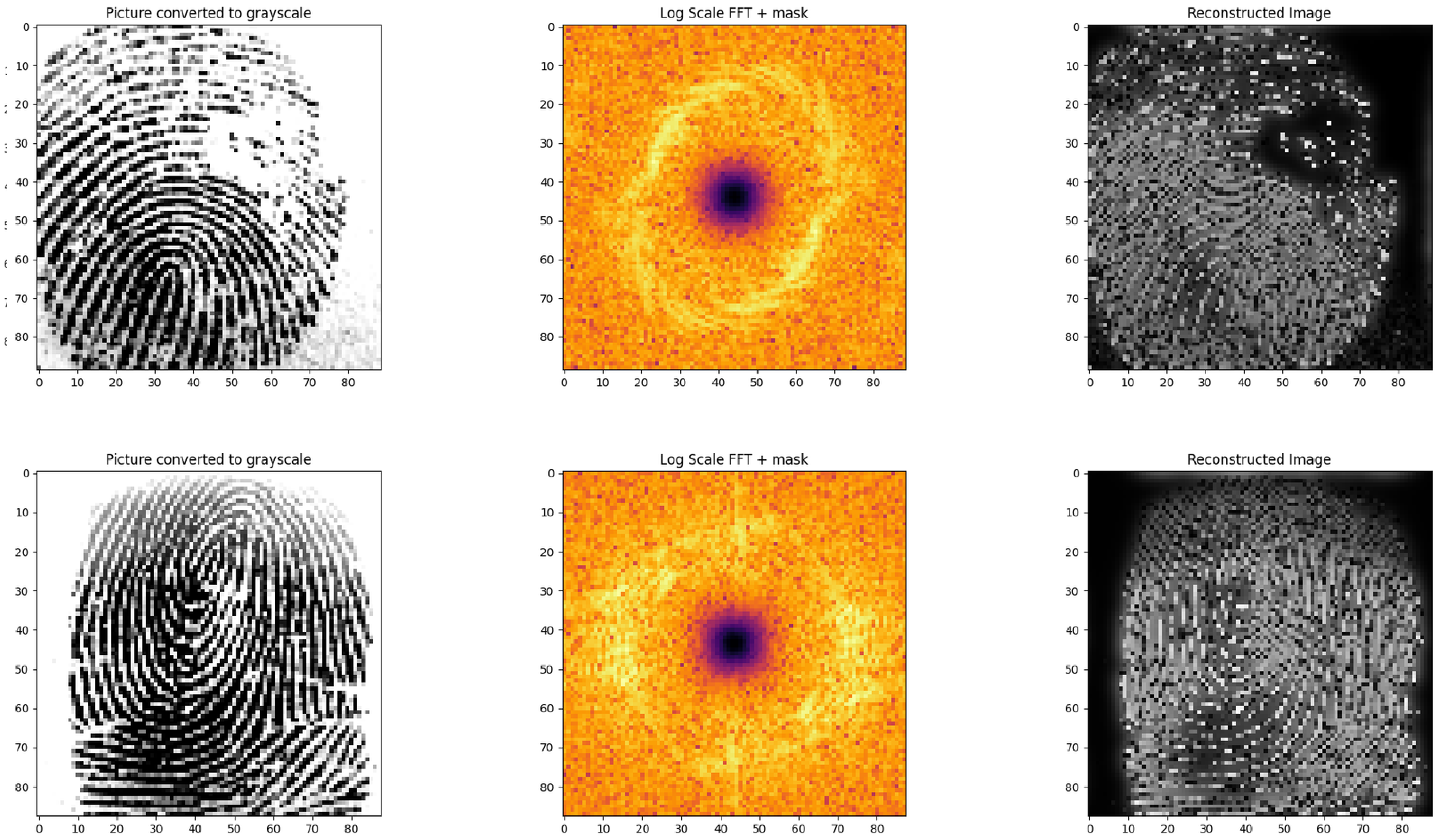
Since we now know that the ridge patterns of the fingerprint are stored in the higher frequencies, we can enhance them by blocking out the smudges. I did this by using a Gaussian mask and placed it at the center of the fingerprint's FFT. Also, I used a Gaussian mask because I did not want any Airy patterns after the image reconstruction.The following are my results.



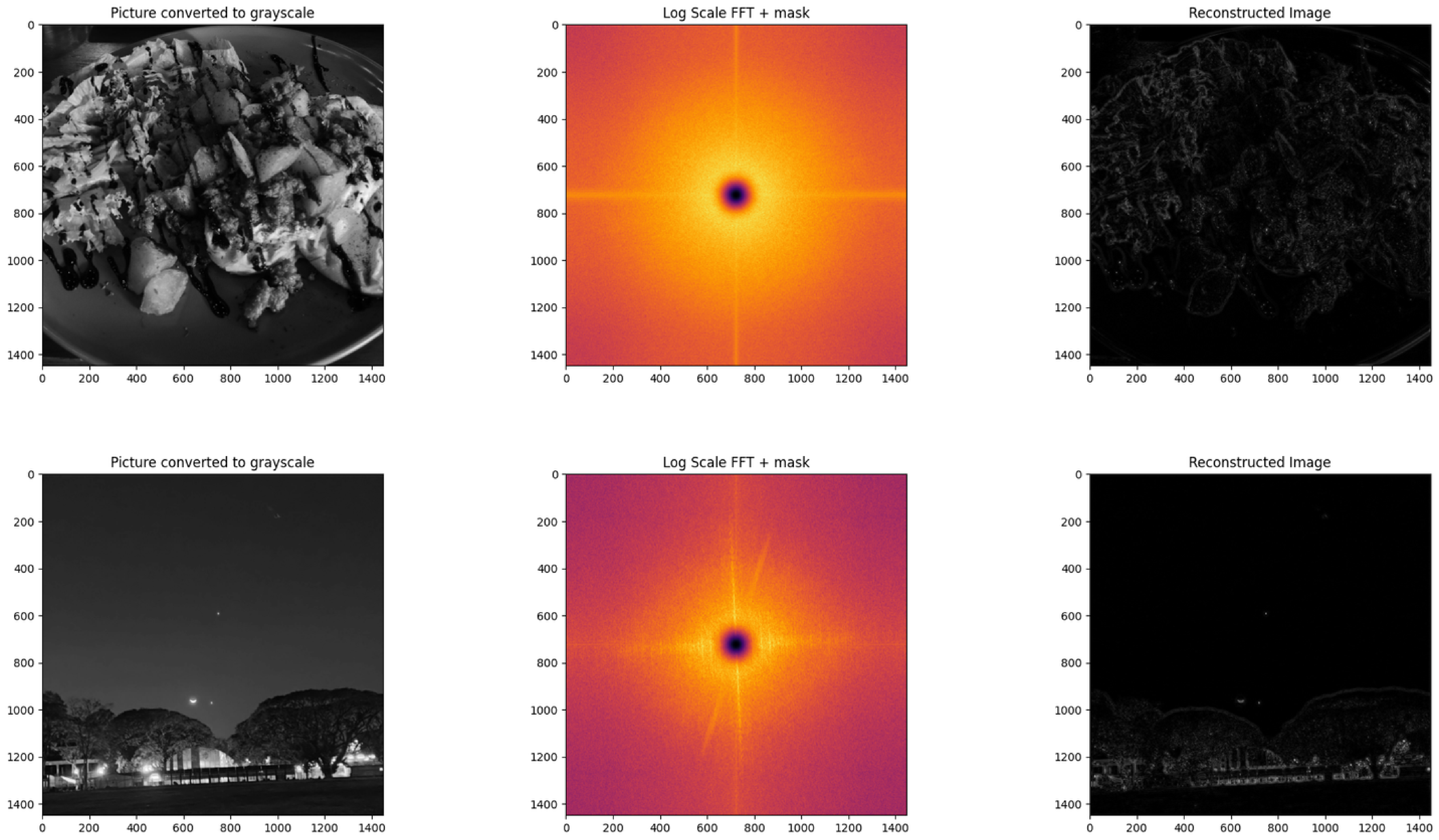




Looking at the outputs, it can be seen that the ridges of the fingerprints are enhanced. Most of the smudges were removed, which left us with a more defined fingerprint.

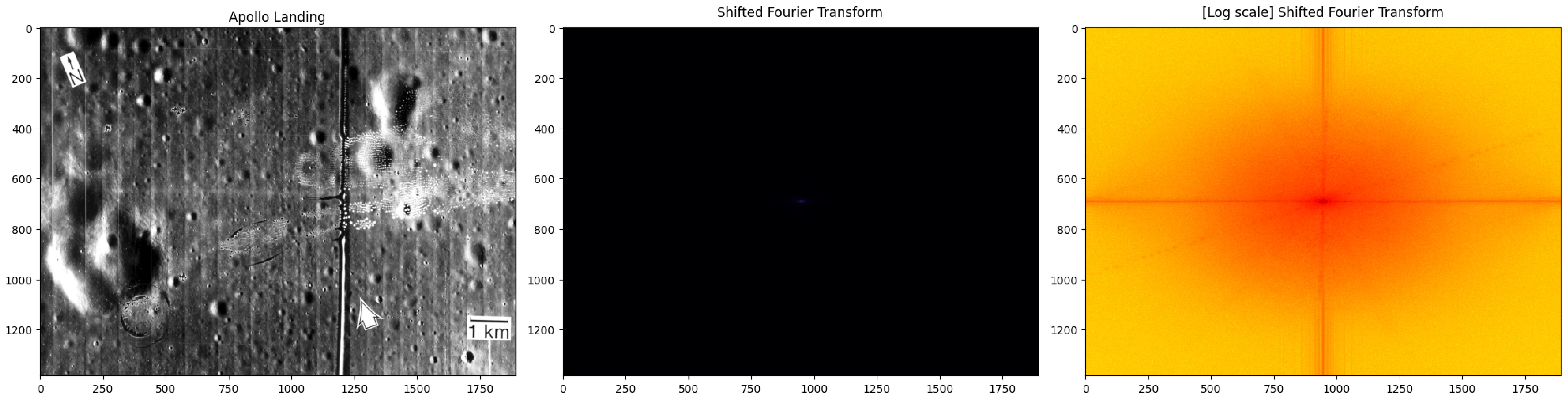


I have also tried masking a few personal images. Based on the results on the right, the mask removed most of the image's data and we are left with the edges of the objects.



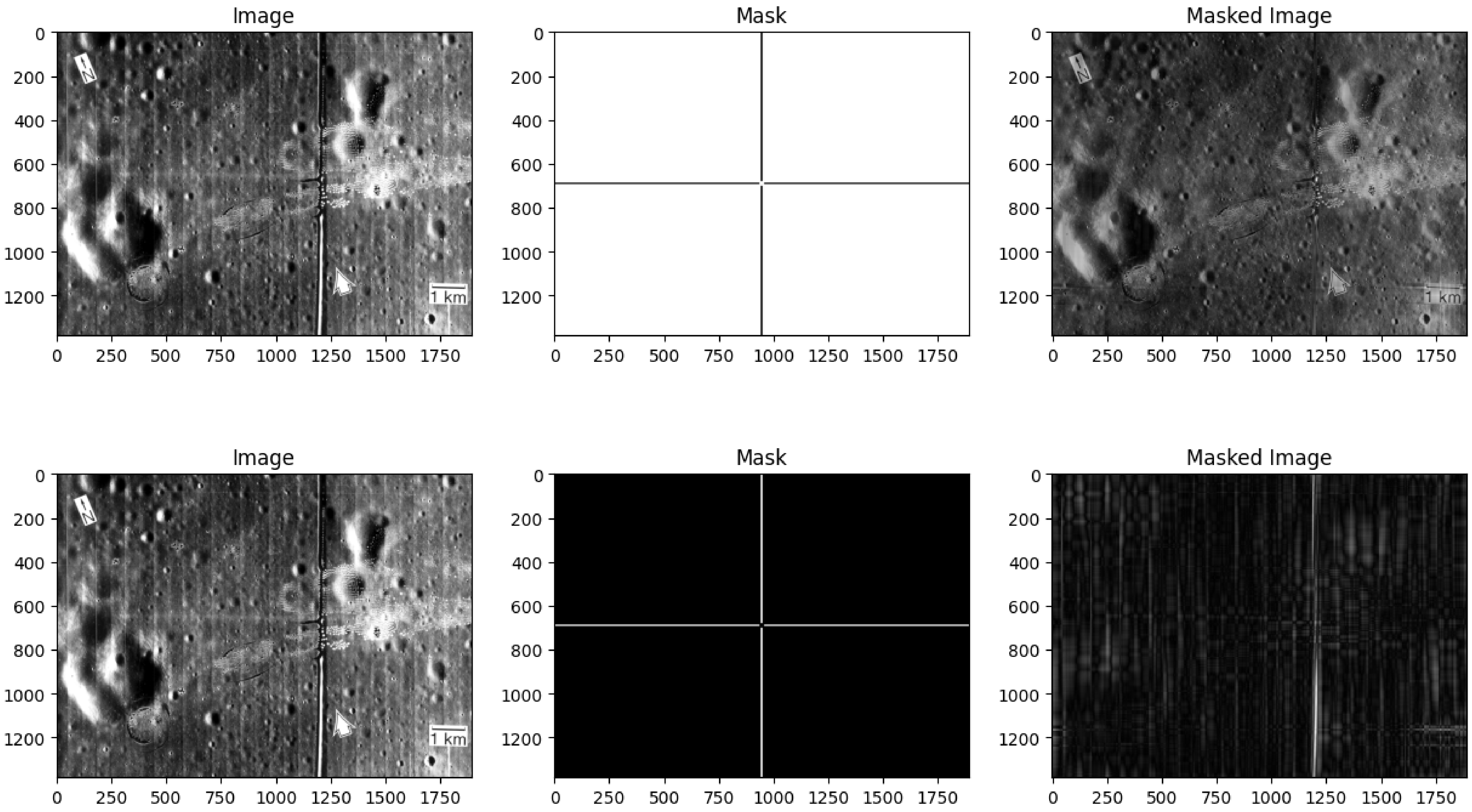
2.2.5 Lunar Landing Scanned Pictures : Line removal

For the final part of the activity, I removed the vertical lines from the Apollo landing image by filtering its Fourier Domain. My results are as follows.



Looking at the picture's FFT, we can see that there are peaks along the vertical and horizontal axes.

Recall that in a previous part of this activity, the FFT of a grid-like pattern is a series of dots along the horizontal and vertical axes . Thus, by masking the peaks along the vertical and horizontal axes of the image's FFT, we can remove the repeating vertical lines. By applying the mask, we can see that the masked image has a clearer picture of the Apollo landing. And by inverting the mask, we can observe the repeating vertical lines. With this, I have successfully masked the Apollo lunar landing picture.



Reflection

I really liked this activity because I was able to work with actual images. Seeing the output of my code really made me feel like I was doing actual work. For my results, I believe that I got all of them right as they made sense and lined up with the physics behind the processes. The only part where my code went wrong is with the Apollo lunar landing photo. My automated mask fails when the peaks are very close together, so I made a new masking algorithm instead and it is specifically for masking images with vertical or horizontal repeating lines.

I'd like to thank my instructors, Sir Rene Principe Jr. and Sir Kenneth Leo, for guiding me throughout the activity. I would also like to thank my professor, Ma'am Jing, for guiding me in my coding while my classmates and I worked in R202. I would also like to acknowledge my classmates: Abdel, Johnenn, Ja, Jonabel, Richmond, Lovely, Hans, Genesis, Jeruine, Rusher, and Ron for helping me complete this activity.

Self Grade

Technical Correctness	I understood the lesson and met all the objectives.	35
Quality of Presentation	The images I added to this report are of good quality and all the graphs are properly labelled. My code is also properly organized and labelled.	35
Self Reflection	I got the expected results, and acknowledged the contributions of my peers while doing this activity. I also properly cited online references.	30
Initiative	Apart from doing the required tasks, I also applied what I learned to sample images that I took and compiled. I also helped my classmates with their code and helped them by cross-referencing my results with their's.	10
	Total	110

References

[1] Cerna, Michael and Harvey, Audrey F. (2020 July) The Fundamentals of FFT-Based Signal Analysis and Measurement. National Instruments.

[2] Saini, Manish Kumar(2021, December 2). Linearity and Frequency Shifting Property of Fourier Transform. TutorialsPoint. Retrieved from: https://www.tutorialspoint.com/linearity-and-frequency-shifting-property-of-fourier-transform

[3] Rioux, F. (2023, January 11). 1.29: Single slit diffraction and the Fourier transform. Chemistry LibreTexts. Retrieved March 23, 2023, from https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Quantum_Tutorials_(Rioux)/01%3A_Quantum_Fundamentals/1.29%3A_Single_Slit_Diffraction_and_the_Fourier_Transform

[4] Weisstein, E. W. "Books about Convolution." http://www.ericweisstein.com/encyclopedias/books/Convolution.html.