ACTIVITY 2. FOURIER TRANSFORM MODEL OF IMAGE FORMATION (PART 1 OF 2)

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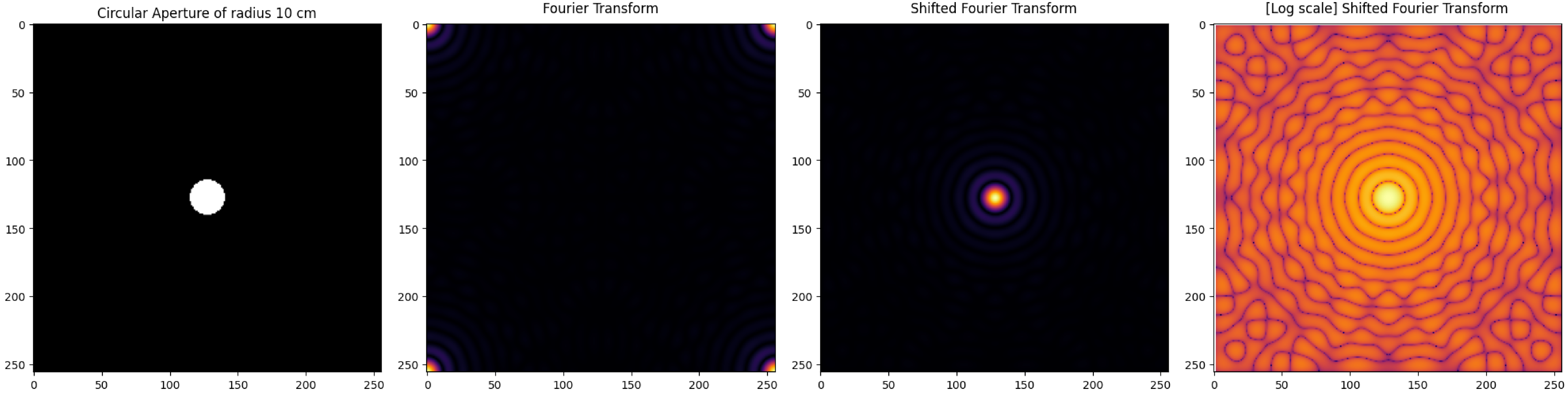
App Physics 157 WFY-FX-1

Activity 2.1 Familiarization with Discrete FT

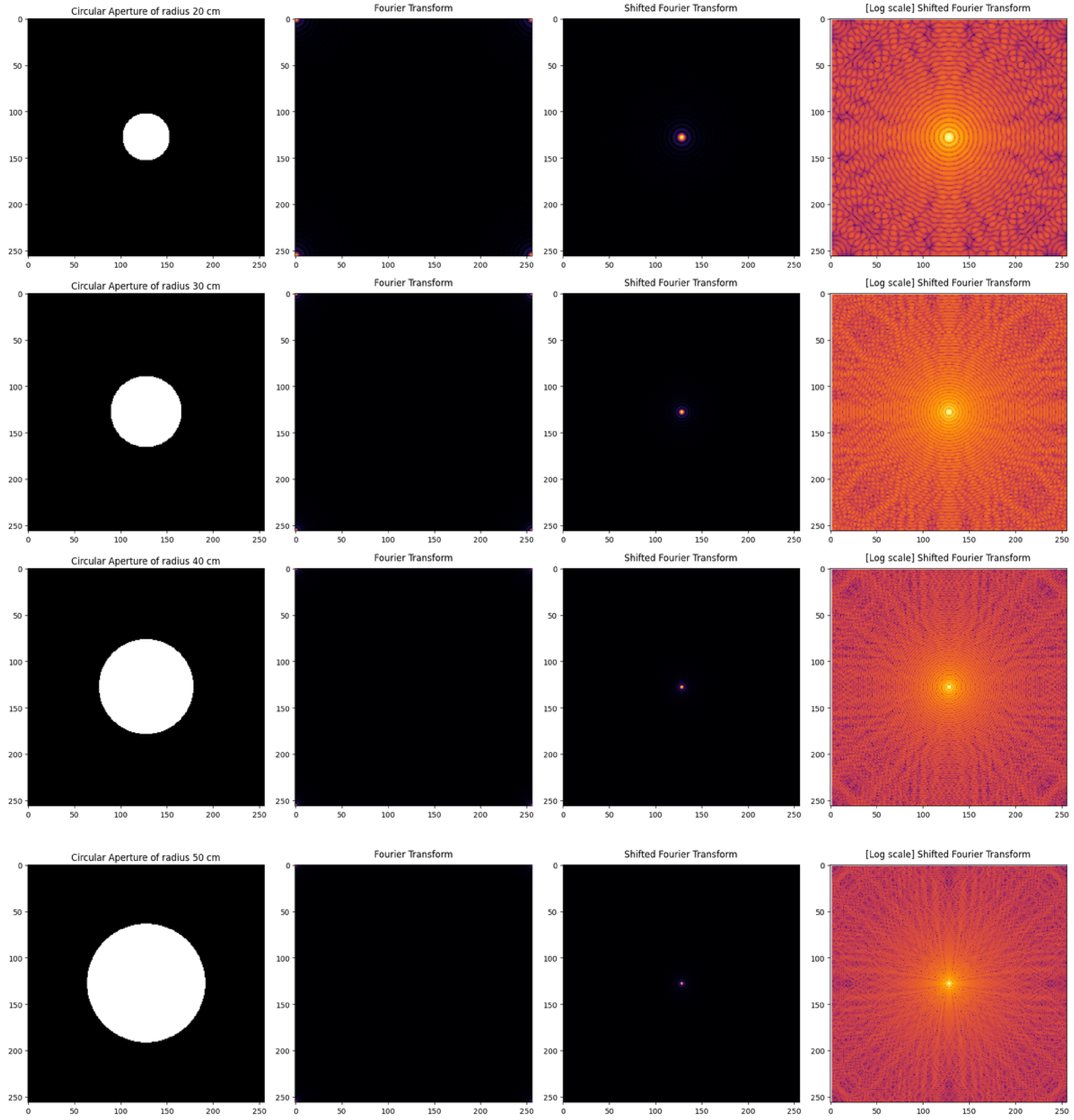
For this part of the activity, I applied numpy's FFT function on a circular aperture. The circular aperture is centered in a square matrix with a side length of 100 cm. The following are the results:



In this part of the activity, I made binary images containing a circle with varying radii and are centered at the center of the matrix. The radii that I used were 10 cm, 20 cm, 30 cm, 40 cm, and 50 cm. Looking at their respective shifted Fourier transforms, it can be seen that their pattern represents a Bessel function. Anamorphic properties in the Fourier transform can also be observed. A larger radius in the circular aperture resulted in a smaller radius in the Bessel function and a smaller radius in the aperture resulted in a larger radius in the Bessel function.

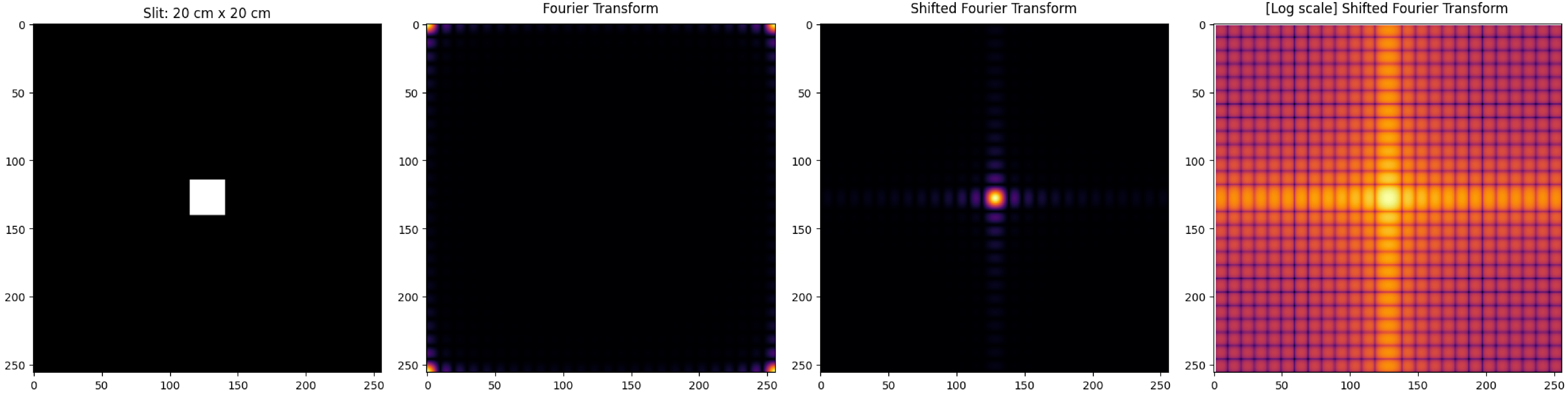
One physical interpretation for this part of the activity is that the circle in the image represents the aperture of a camera. By applying an FFT on the aperture, you get the point spread function (PSF) of the aperture, which is useful in Fourier optics, medical imaging, photography, and other fields that uses imaging. The PSF is useful because it contains information of how the points of an image are spread-out. By knowing how spread-out the points are, images can be enhanced through deblurring [1].

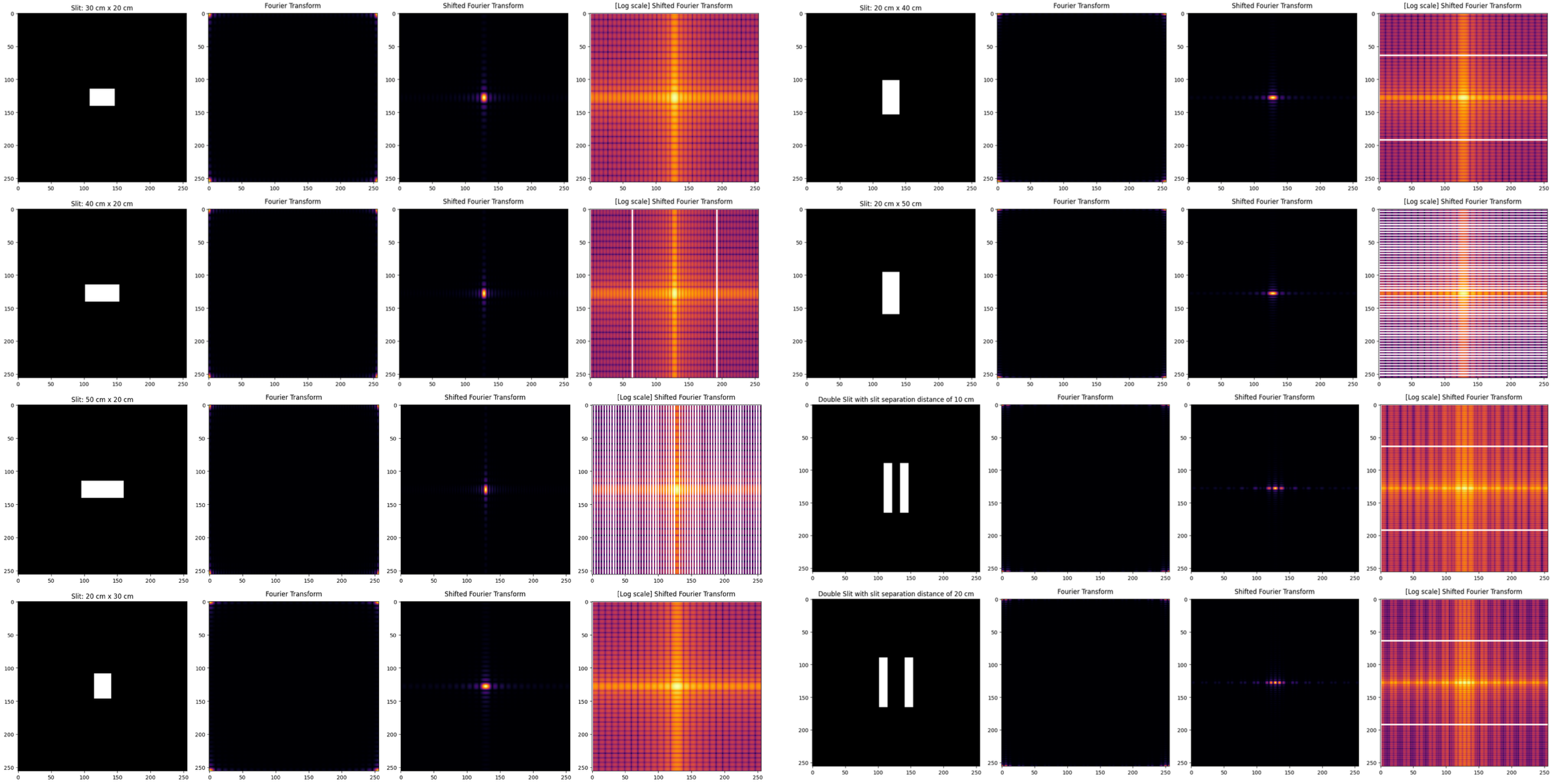
Also, I showed the log scale of the shifted Fourier transforms. These graphs highlight the difference betweeen the values, because the change in color in the color map would be in orders of 10.



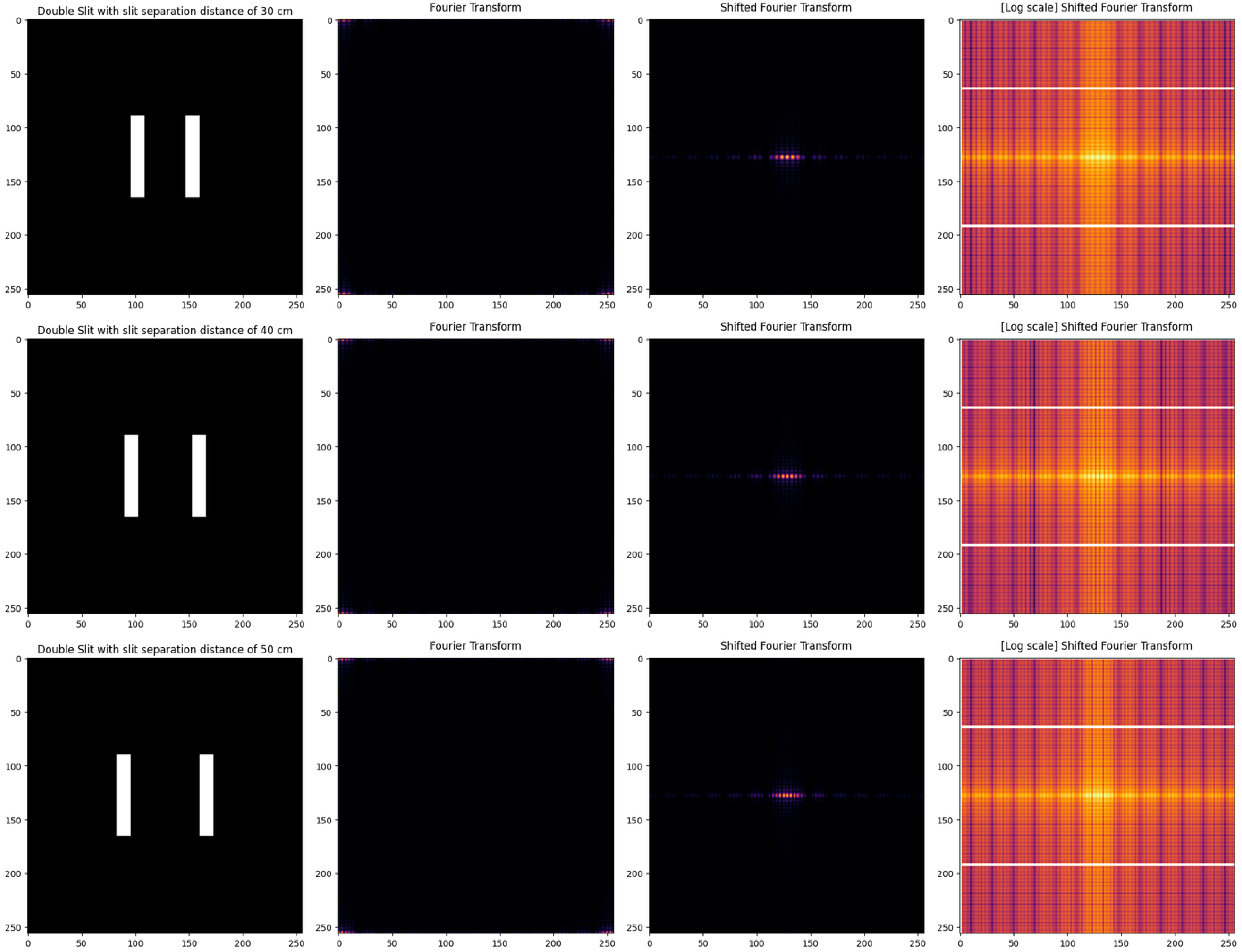
Single Slit and Double Slit

Additionally, I tried getting the Fourier Transform of a single slit and double slit. For the single slit, I varied its length while keeping its width constant at 20 cm. I also varied its width by keeping its length constant at 20 cm. For the double slit, I used two 10 cm by 60 cm slits that are symmetrical around the vertical axis of the matrix. I then varied the slit separation distance of the double slit.




Slit: 30 cm x 20 cm
Fourier Transform
Shifted Fourier Transform
[Log scale] Shifted Fourier Transform
Slit: 20 cm x 40 cm
Fourier Transform
Shifted Fourier Transform
[Log scale] Shifted Fourier Transform
Slit: 40 cm x 20 cm
Fourier Transform
Shifted Fourier Transform
[Log scale] Shifted Fourier Transform
Slit: 20 cm x 50 cm
Fourier Transform
Shifted Fourier Transform
[Log scale] Shifted Fourier Transform
Slit: 50 cm x 20 cm
Fourier Transform
Shifted Fourier Transform
[Log scale] Shifted Fourier Transform
Double Slit with slit separation distance of 10 cm
Fourier Transform
Shifted Fourier Transform
[Log scale] Shifted Fourier Transform
Slit: 20 cm x 30 cm
Fourier Transform
Shifted Fourier Transform
[Log scale] Shifted Fourier Transform
Double Slit with slit separation distance of 20 cm
Fourier Transform
Shifted Fourier Transform
[Log scale] Shifted Fourier Transform

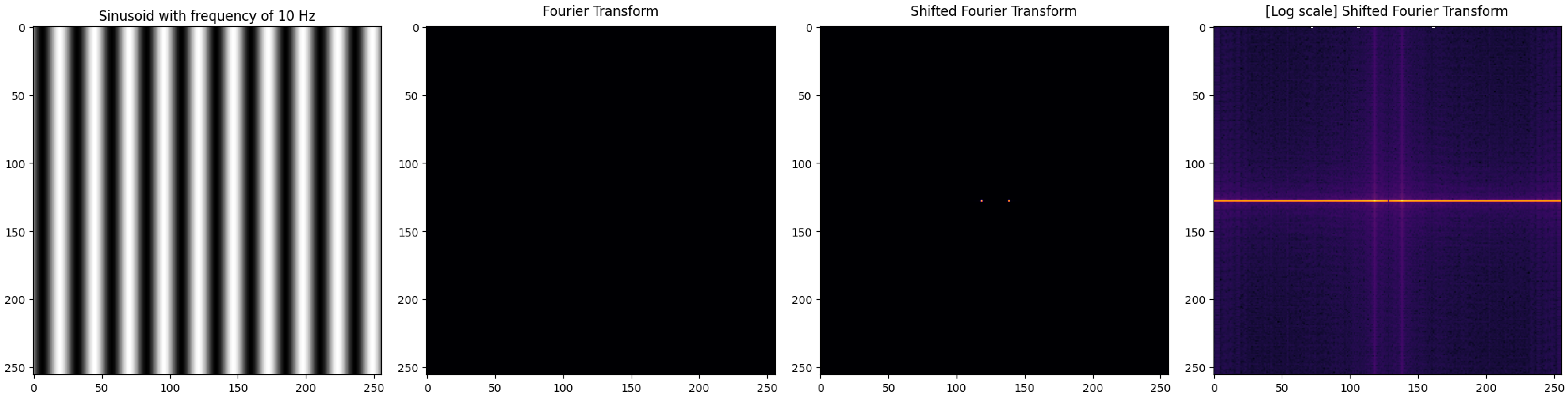
For the single slit, it can be seen that its Fourier transform shows the pattern of the bessel function. Except, the central maximum has a rectangular shape. The Fourier transform also shows anamorphism. A wider slit resulted in a thinner FFT and a taller slit resulted in a shorter FFT.

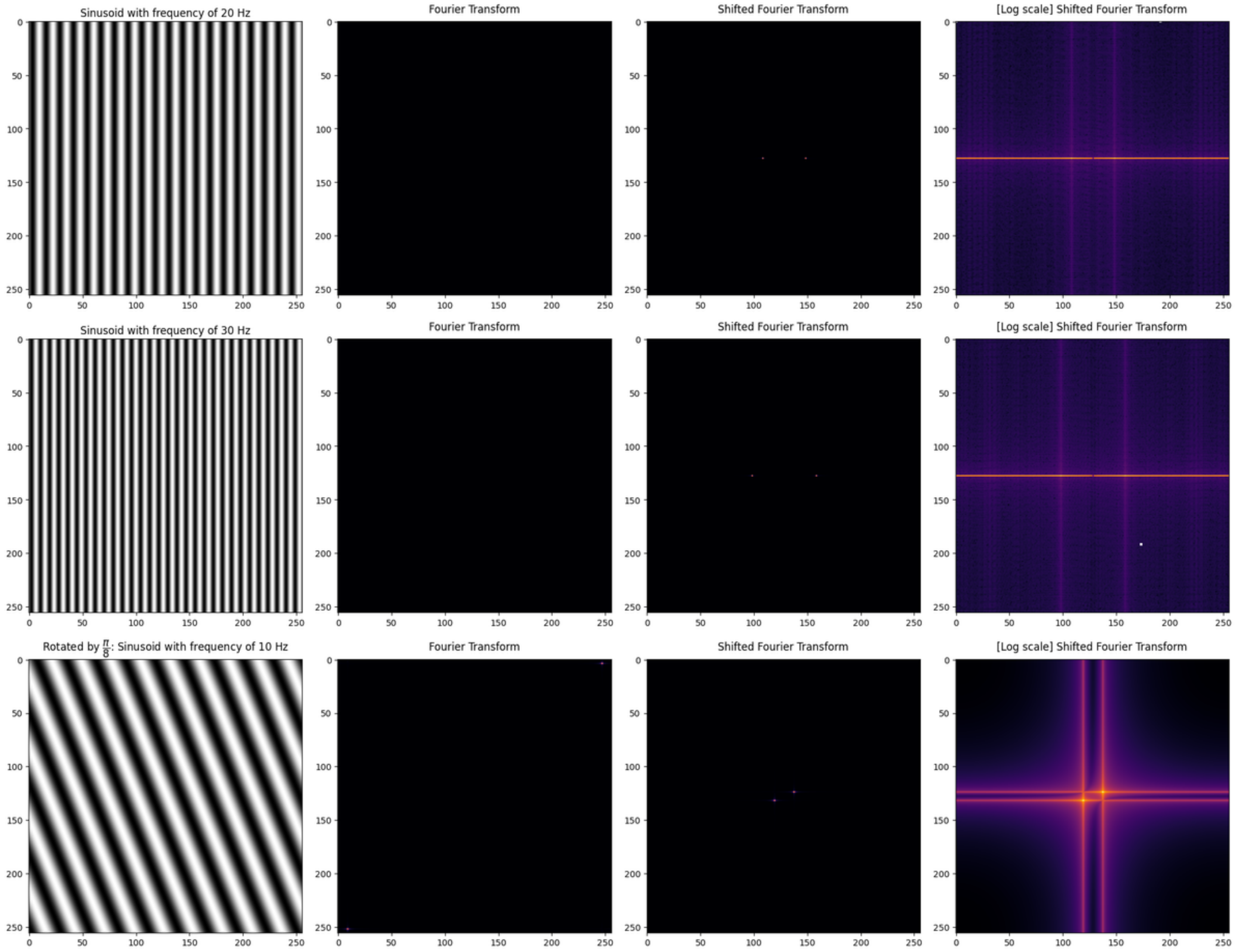


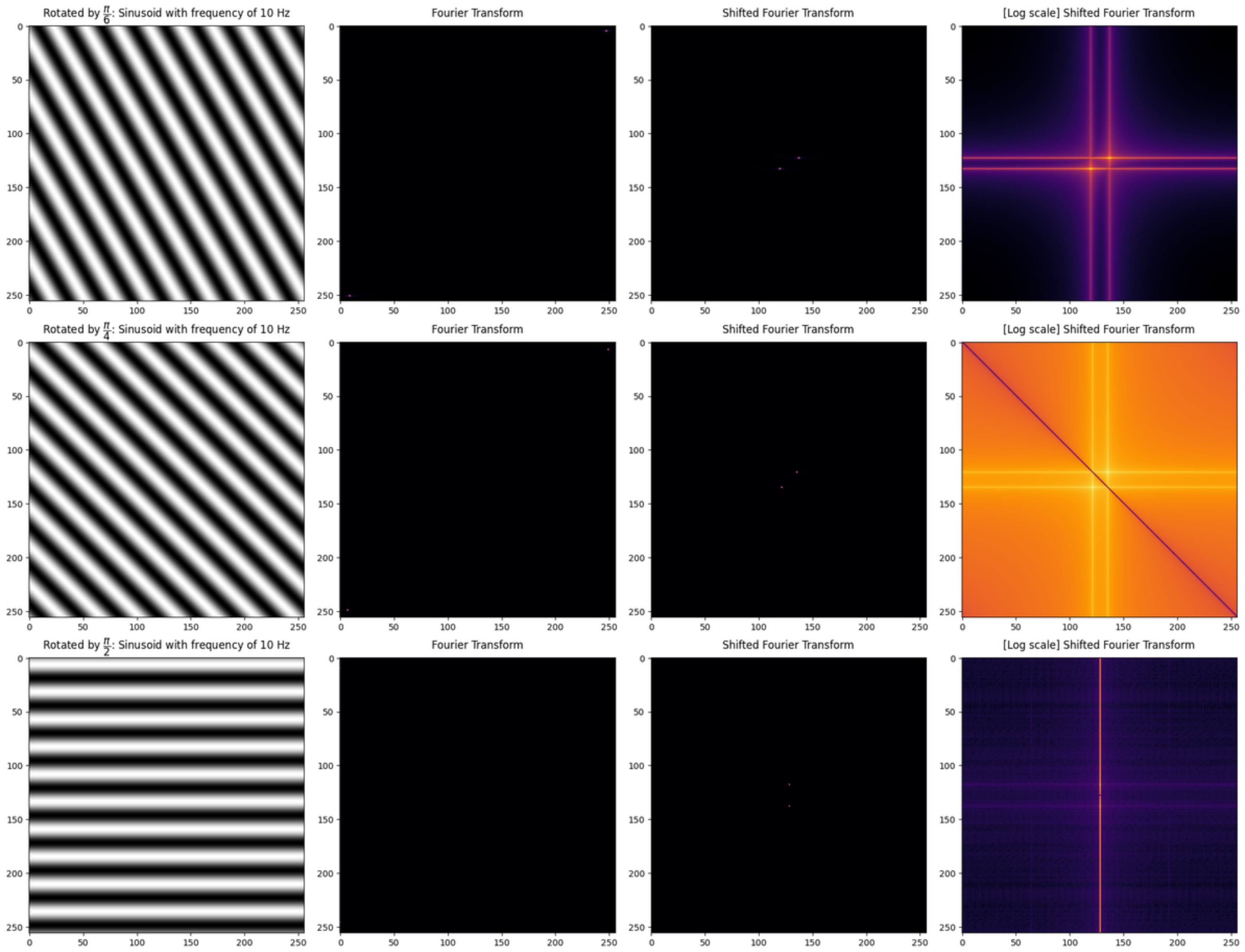
For the double slit, as the distance between the two slits increased, the number of partitions in the maxima also increased. One physical interpretation for this is that the FFT of the single slit represents the diffraction pattern and the double slit represents the interference pattern if an incident wave were to pass through the slit/s. This is because the Fourier transform of the two slits converts it from the coordinate-space to a momentum density and a momentum distribution function [2]. Additionally, the white lines in the log scale shows that the program tried to divide by 0 in those matrix elements.

Sinusoid

I got the Fourier transform of a sinusoid and tried to vary its frequency. I also rotated the sinusoid clock-wise and got its Fourier transform. The following are my results:





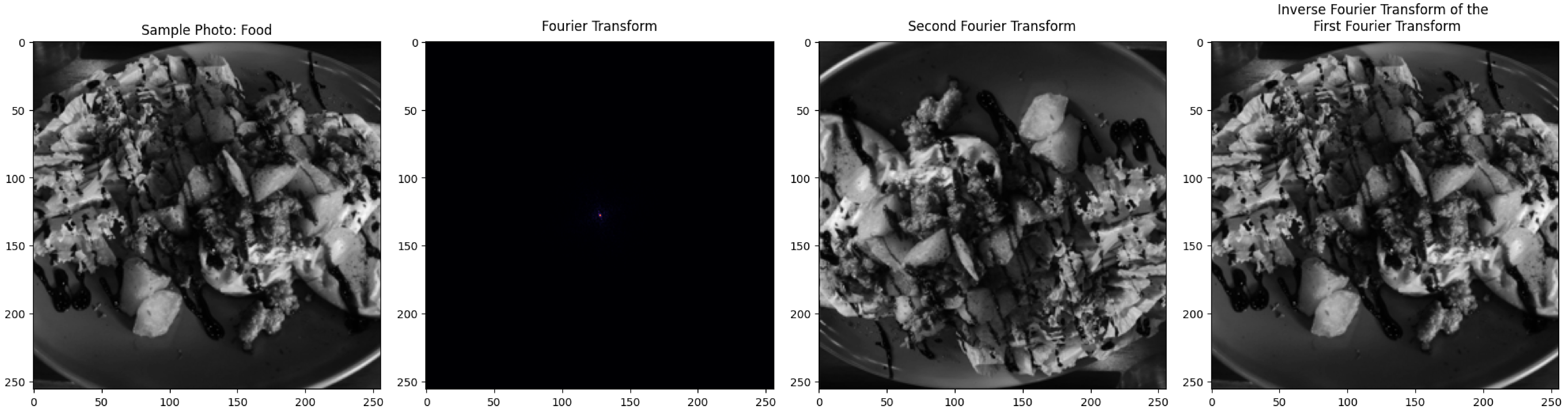


Looking at the Fourier transforms of the sinusoids, it can be seen that there are two peaks near the center of the image. And as the frequency of the sinusoid increases, the distance between the peaks increases. This is to be expected because the sinusoid has only one frequency, so its Fourier transform should only have one peak. But since the Fourier transform is symmetrical about the origin (which is at the center of the matrix), there is a mirror reflection of the peak across the y-axis.

When I rotated the sinusoid counter-clockwise, the peaks in the Fourier transform were also rotated clock-wise with an axis of rotation at the origin. Thus, a rotation in the coordinate-space of the sinusoid is also seen in the Fourier space.

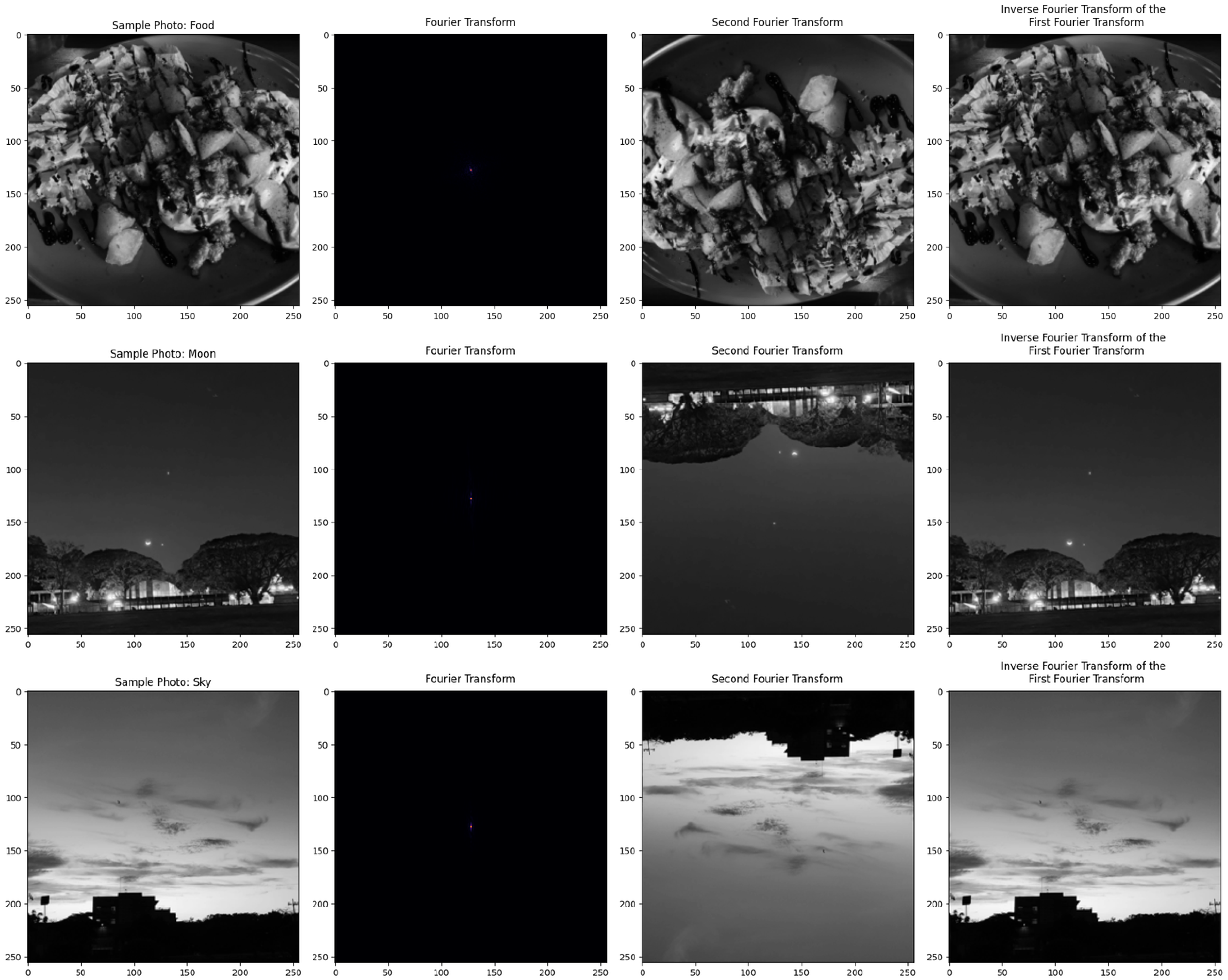
Applying FFT twice

I got the FFT of my sample images and I then applied FFT a seconds time. I also applied IFFT on the first FFT of the image and compared my results. The results are as follows.



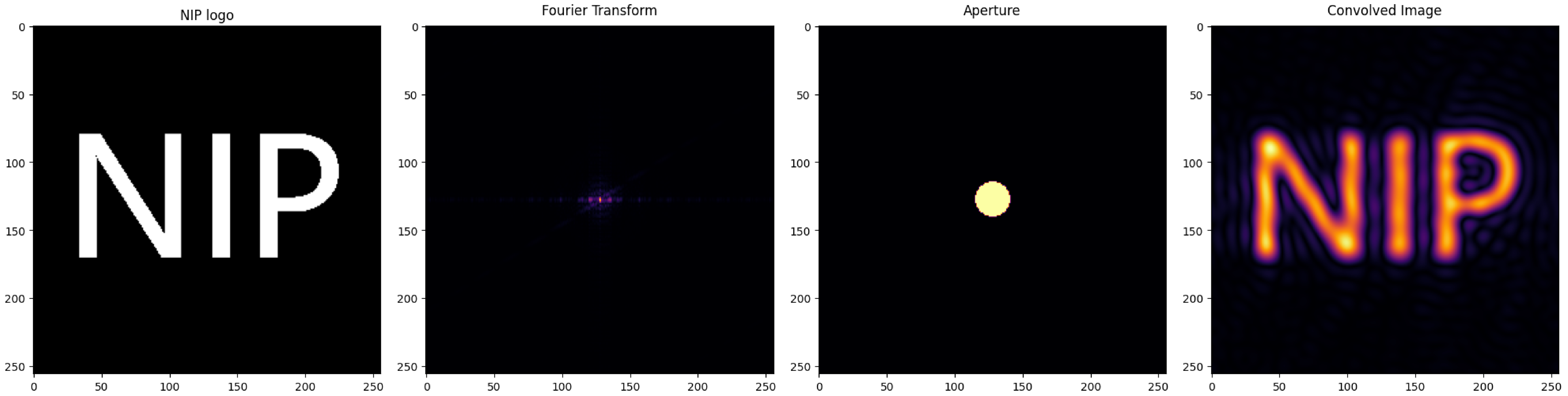
First looking at the first Fourier transform of the image, it can be seen that it is approximately a small dot at the center of the matrix. This means that most of the information of the image is stored in low frequencies. As for the second Fourier transform, it shows that the image is recovered but its orientation is flipped across its horizontal axis. However, looking at the IFFT, it can be seen that the image is perfectly recovered (or at least there are no anomalies that can be visualy observed).

Additionally, I was able to reconstruct the image by getting its second Fourier transform because of how the Fourier transforms is applied. The Fourier transform and the inverse Fourier transform basically has the same integral, but all instances of *xy* variables are swapped with frequency variables and all frequency variables are swapped with *xy* variables. Also, the image is inverted because of how the Fourier transform shifts the quadrants of the image. Basically, if FFT shifts the image's quadrants a certain way, then IFFT shifts them in the opposite way.



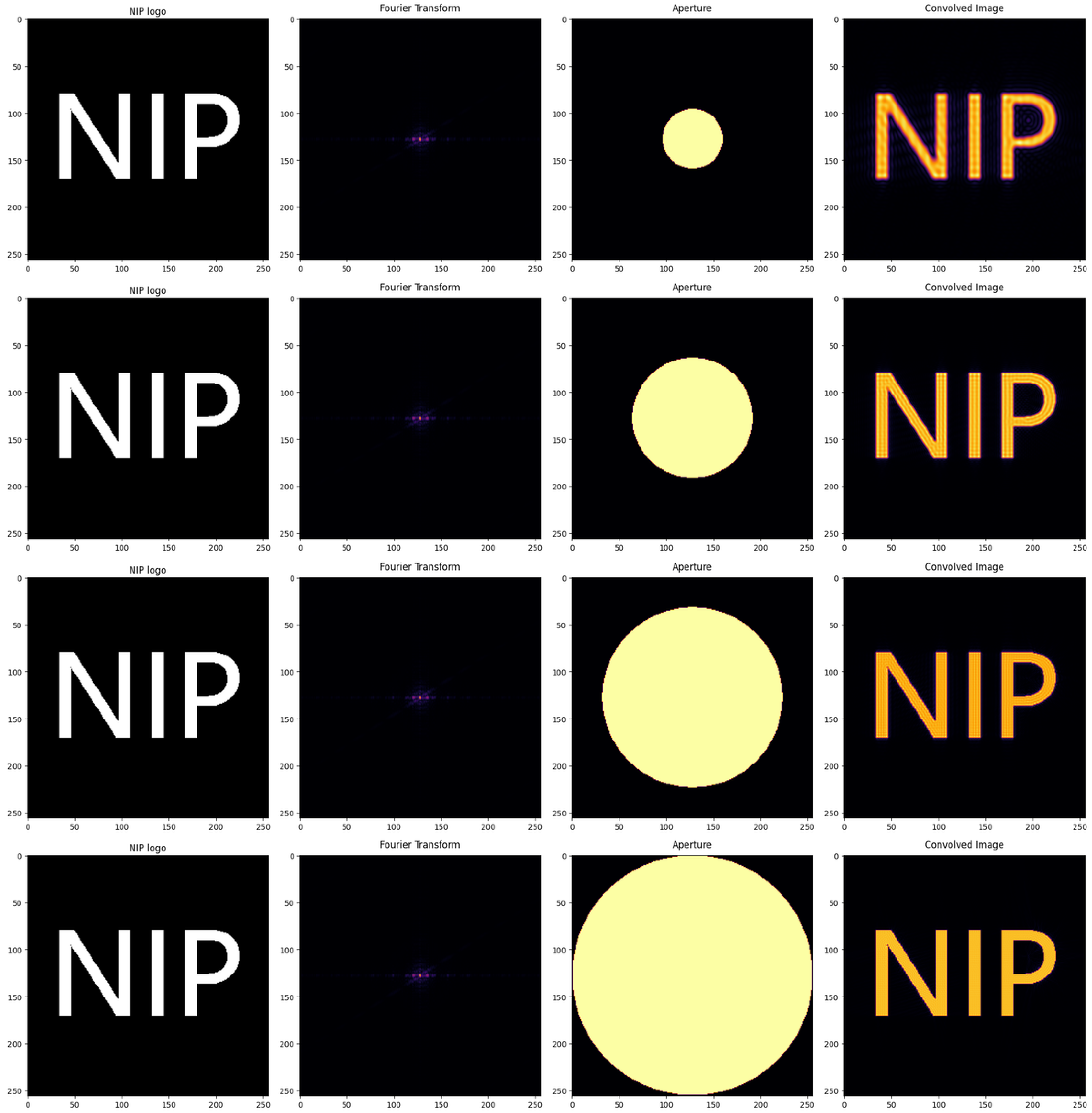
Activity 2.2 Simulation of an imaging system

In this part of the activity, I multiplied (element per element multiplication) the FFT of an image containing the word "NIP" and a circular aperture. I varied the size of my aperture and the following are my results:



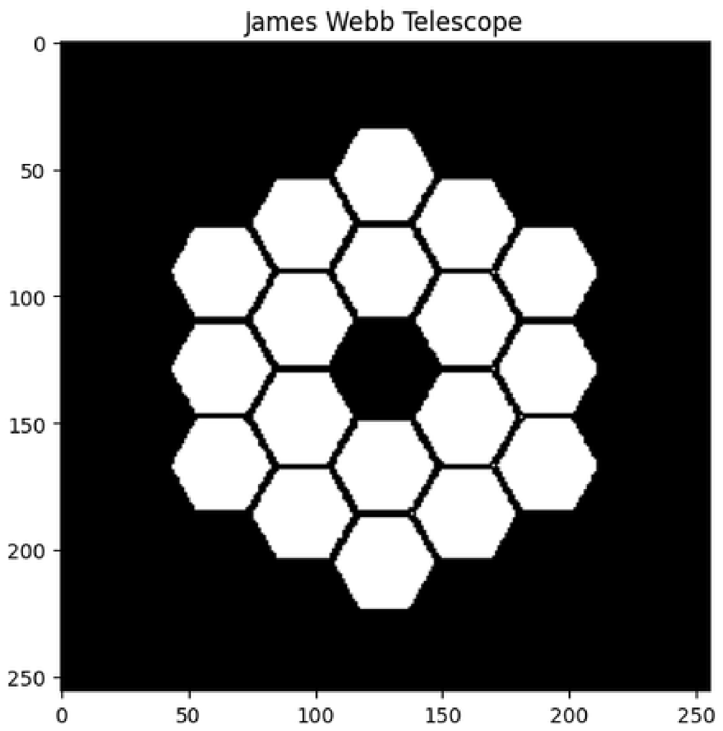
By convolving the FFT of the NIP image with the circular aperture and applying IFFT on the result, we can see that we got the reconstruction of the NIP image. But the reconstruction appears to have an Airy pattern around the letters. This is to be expected, because convolving two images in the Fourier-space is equal to the element per element multiplication of the aperture and the FFT of the NIP image [3]. This is basically a simulation of taking a picture by using a camera with a circular aperture. Because of the circular aperture's PSF (which I showed in Act. 1.1), the NIP image is blurred when its photo is taken. Applying a deconvolution on the reconstructed image should remove the Airy pattern [4].

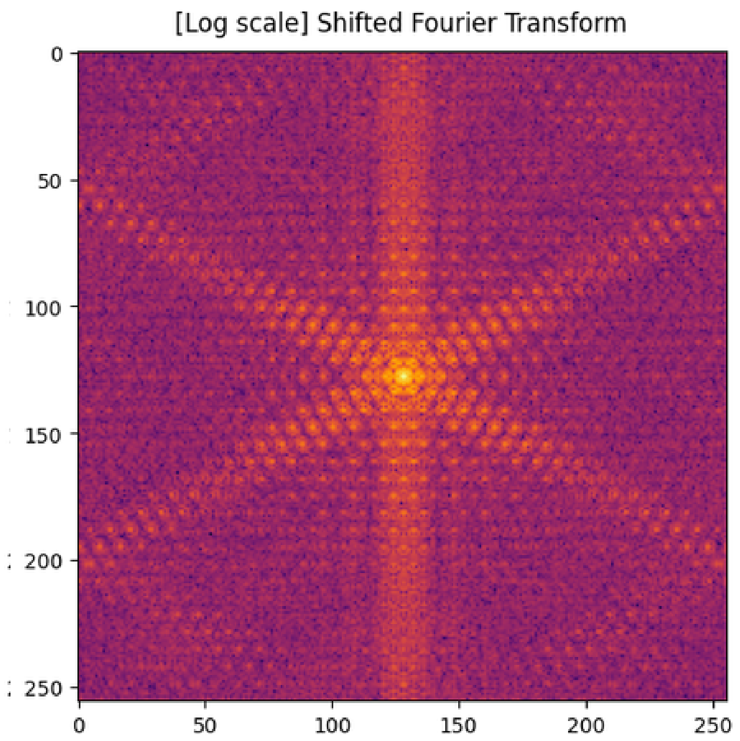
Additionally, it can be seen that as the aperture increases, the reconstructed image becomes sharper. This is because the PSF of the aperture becomes more condensed in the center, (this can be seen in Act. 1.1) which results in a sharper image.

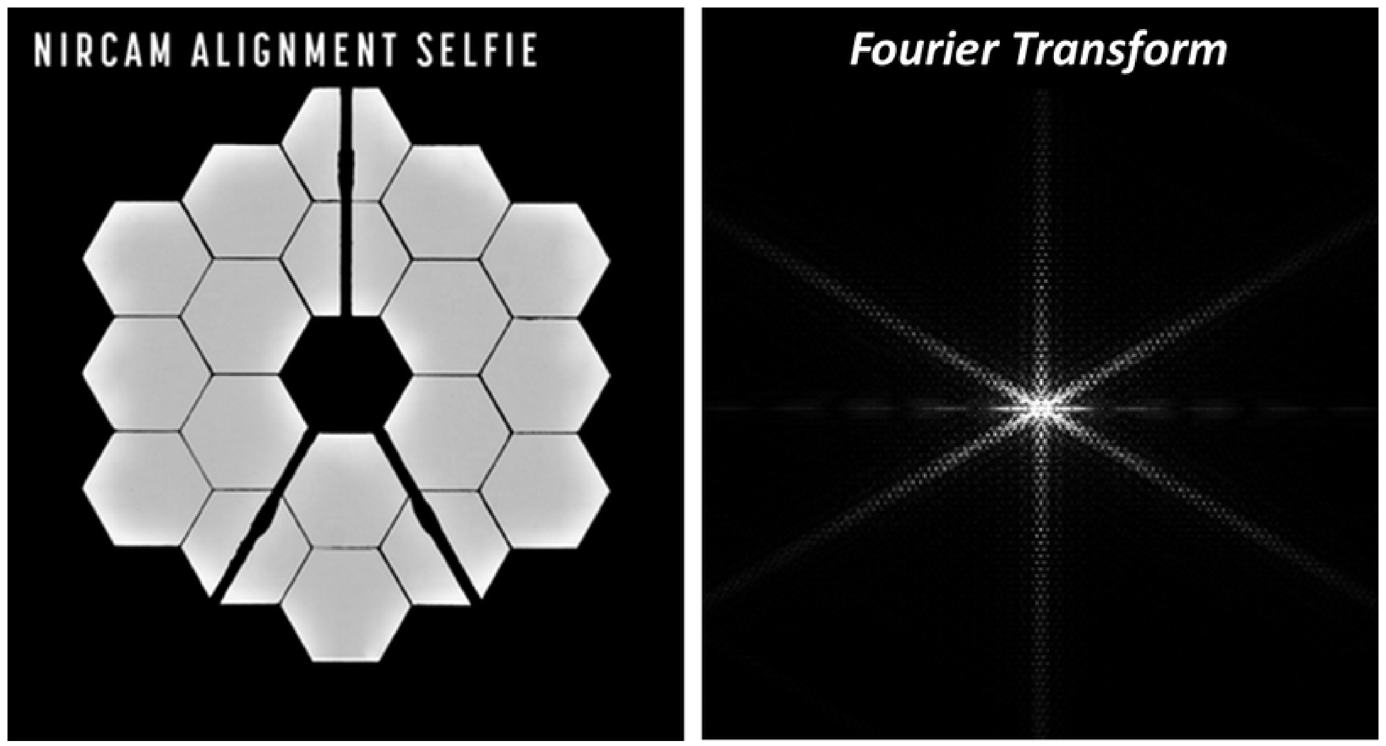


How the James Webb Space Telescope Sees a Star

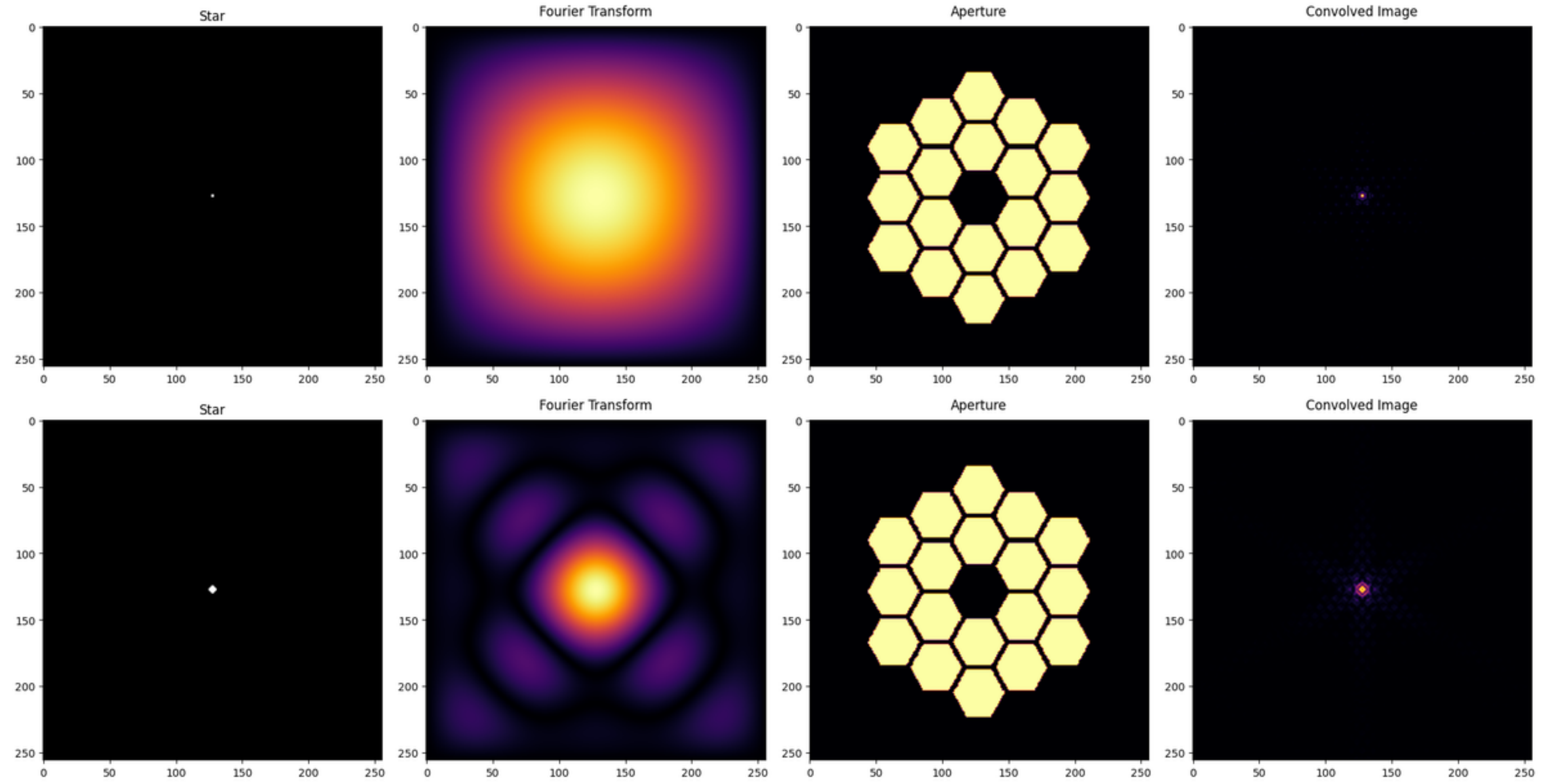
From the laboratory manual, the FFT of the James Webb telescope mirror is how the telescope sees a distant star. Below is the log scale of the FFT of the James Webb Space Telescope and the Nicram Alignment Selfie which was obtained from NASA's website. Comparing the two images, it can be seen that I got the same output, even though my image of the telescope's mirror does not have the beams holding up the secondary mirror.

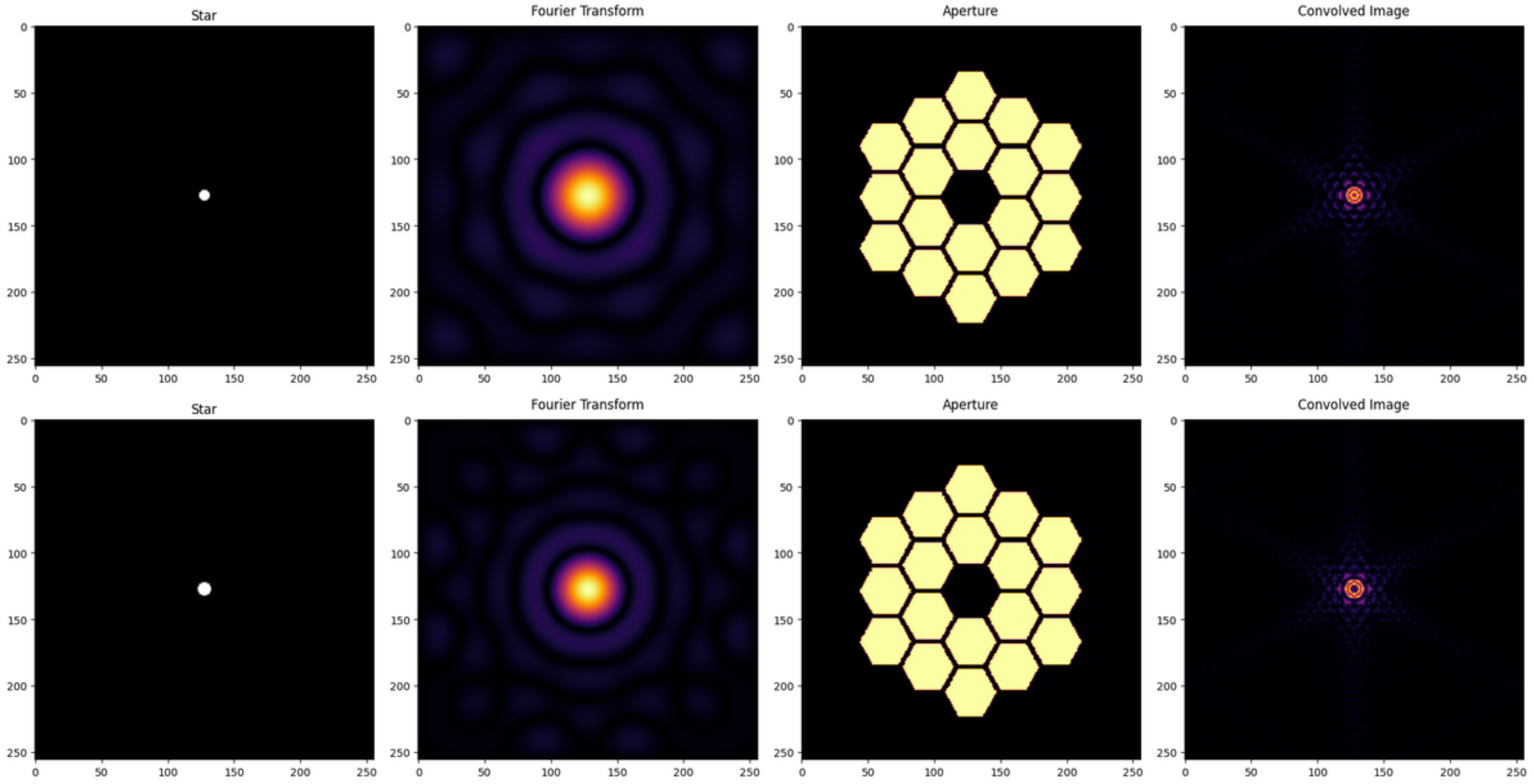






The image was taken from *https://www.nasa.gov/press-release/nasa-s-webb-reaches-alignment-milestone-optics-working-successfully*



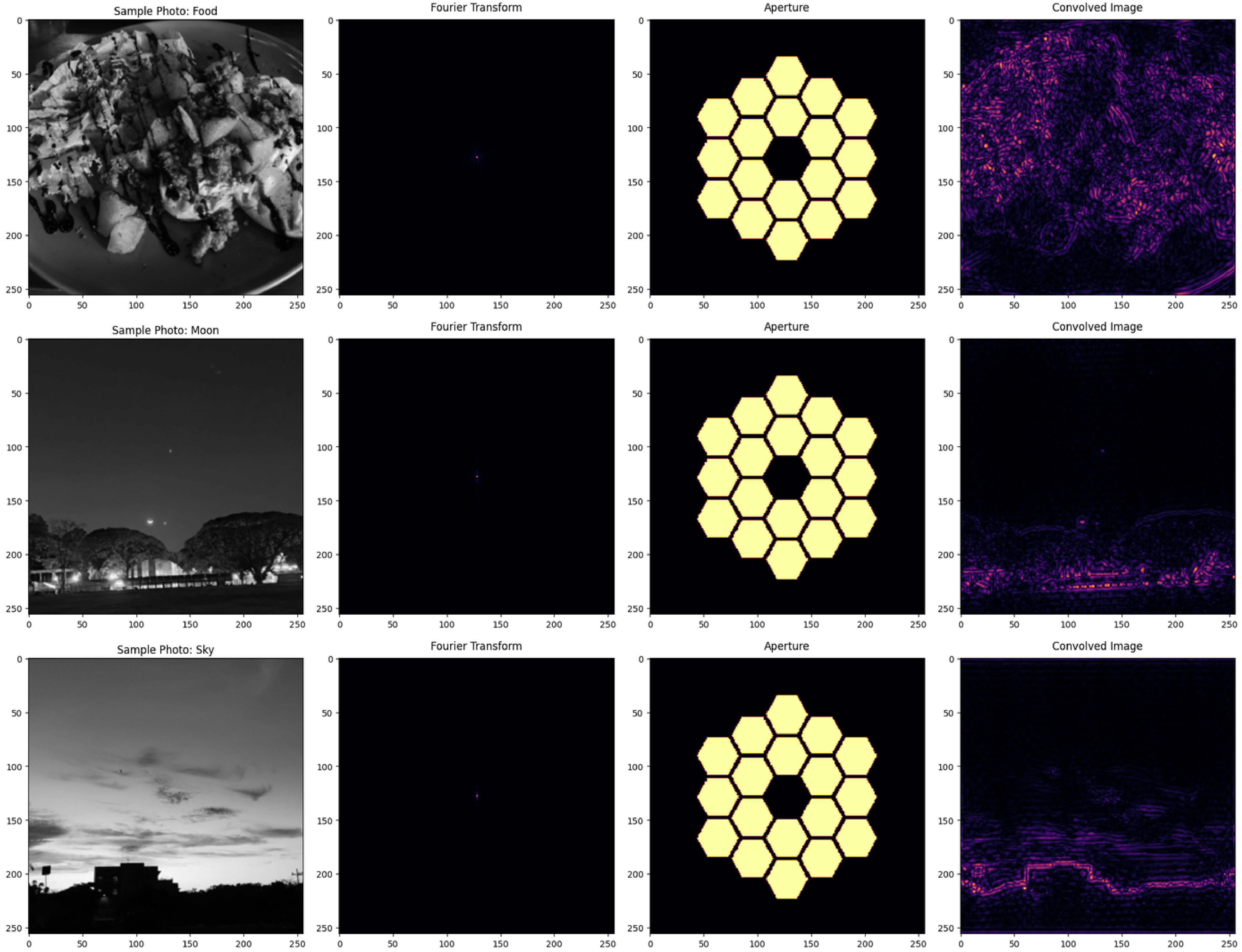


I then convolved the telescope's mirror with a circular aperture to simulate a star that is close to the telescope. I then varied the radius of the circle to simulate different distances.

From the results, it can be seen that as the radius of the circluar aperture increases, the reconstructed image's radius also increases. This means that an increase in radius of the circle (or the simulated star) can also be observed in the reconstructed image.

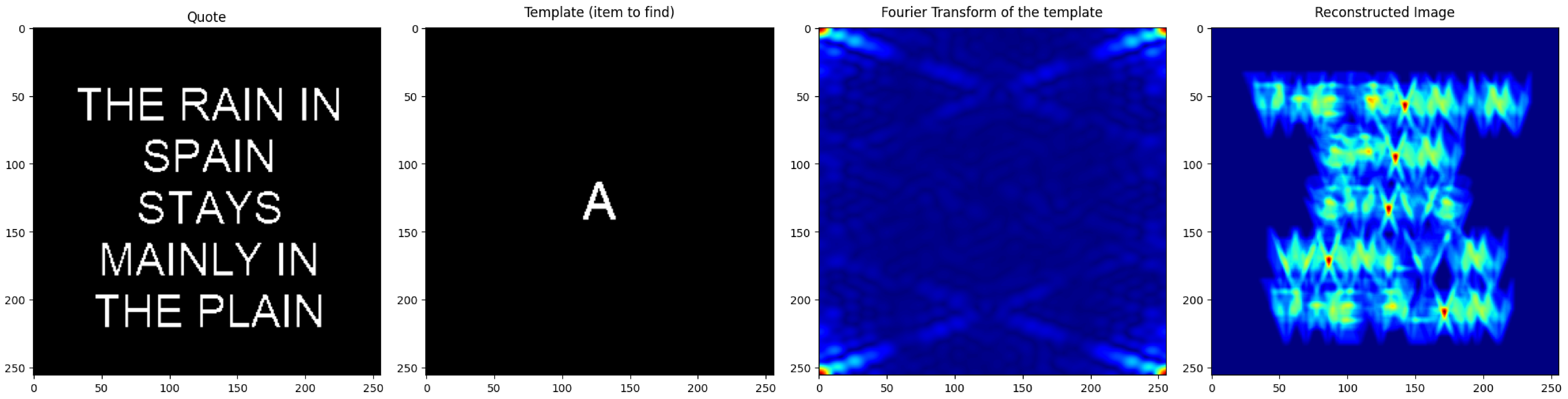
I was also curious on how the James Webb Space Telescope sees everyday images. So I convolved my sample images with the telescope's primary mirror.

Looking at the results, it looks like the telescope detects the edges of gray scale images. The edges themselves appear as black lines, but the parts around them are bright.

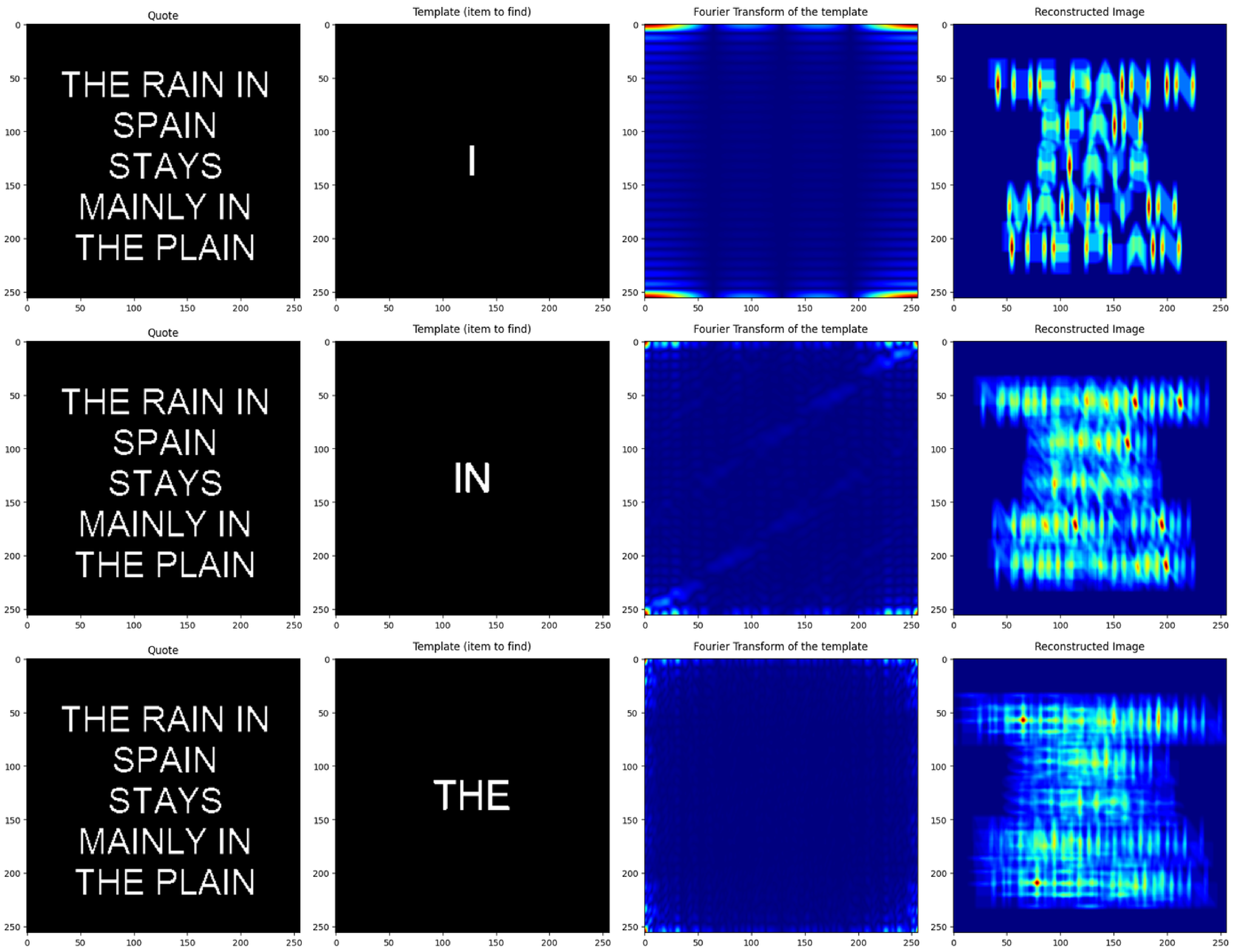


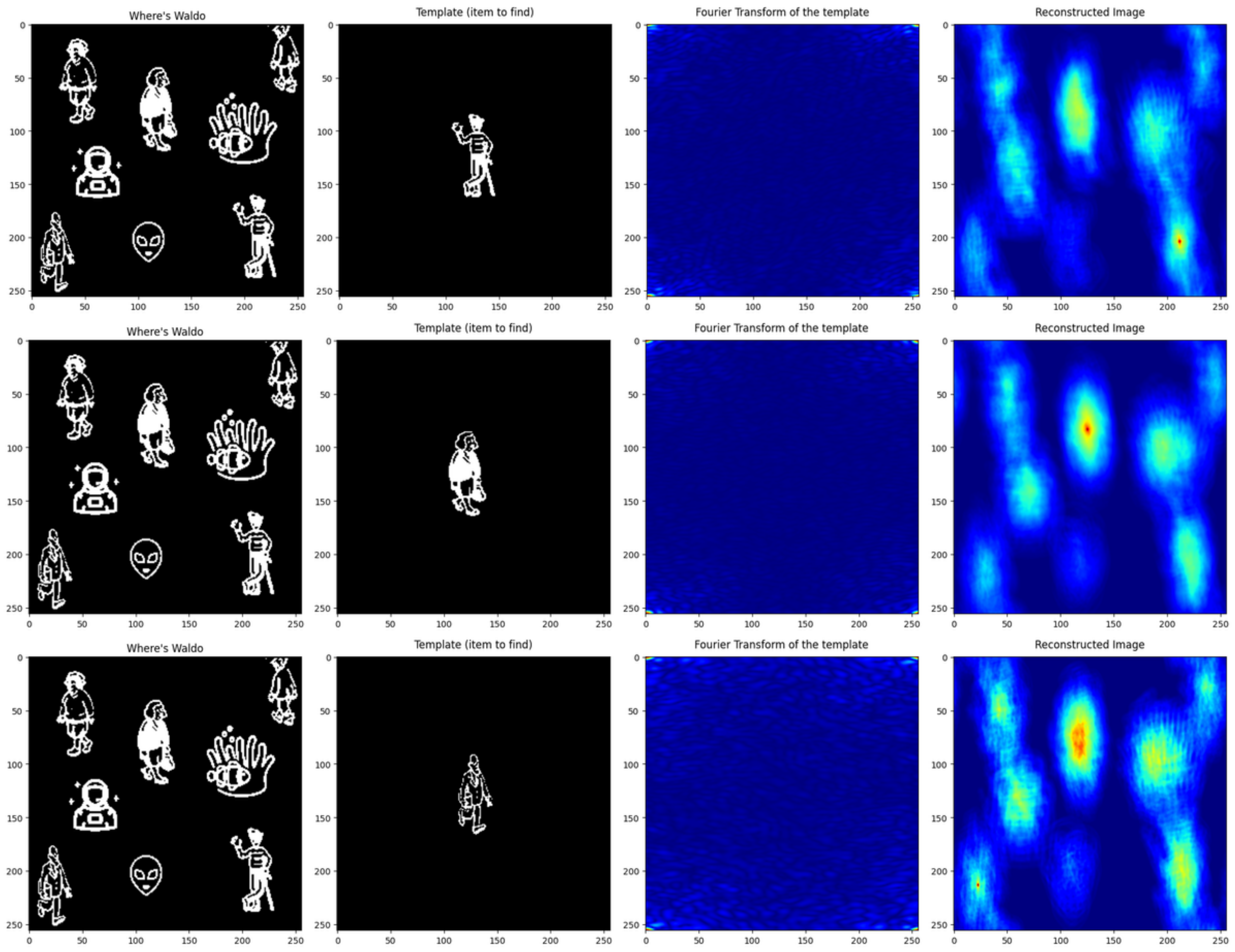
Activity 2.3. Template matching using correlation

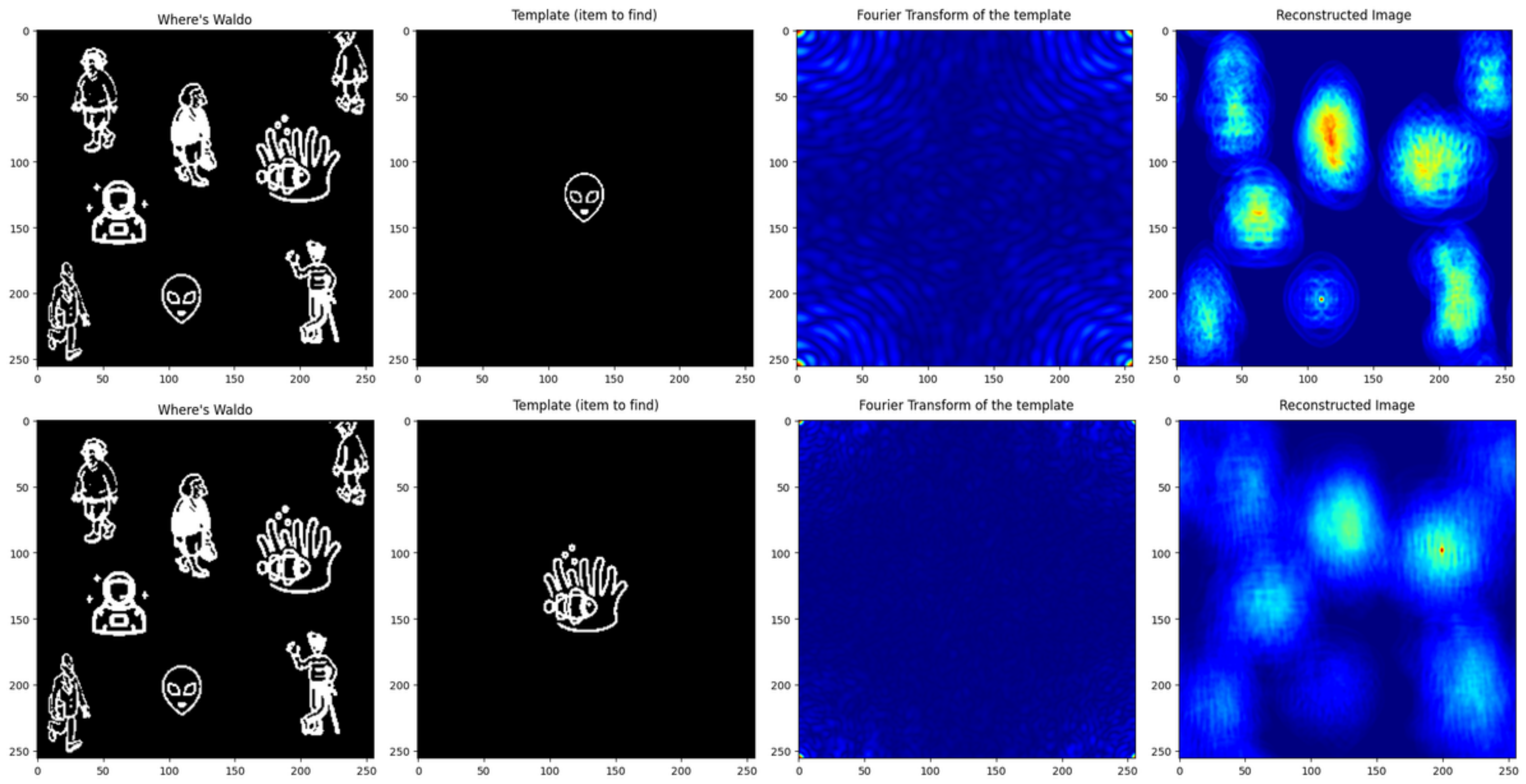
My image for the template matching is a quote that says "THE RAIN IN SPAIN STAYS MAINLY IN PLAIN" and I applied correlation by using the template "A", "I", "IN", and "THE". The following are my results.



From the results, it can be seen that there are peaks (red) in the reconstructed image where the template matches the image. For the template "A", there are peaks where all the letter "A" is in the image. For the template "I", there are peaks where there are vertical lines. This is because the template is simply a vertical line. Thus resulting in peaks in all vertical lines, which the letters: T, H, R, I, N, P, L, and M have. For the templates "IN", peaks show up in "RAIN", "IN", "SPIN", "MAINLY", and "PLAIN". This is because all of these words contain the pattern "IN". For the template "THE", the peaks are only on the word "THE", this is because no other words contain the template's pattern.







The figures of Waldo, the woman, and the man were taken from the Canadian Mental Health Association, and the other icons were taken from Microsoft Office Powerpoint.

I was interested with template matching, so I created an image which is a compilation of distinct figures. After applying correlation, it can be seen that there are peaks where the template matches the image. But for the man (third row, left set of images) and the alien, they appear to have a relatively high correlation to the woman. This is because the woman figure's design is mostly white, which results in almost any pattern having a relatively high correlation to it.

Reflection

I found this activity really fun and enjoyable. My first encounter with Fourier transforms is from Physics 117 and it was purely math, which I really did not enjoy. But in this activity, I can see the application and the results which gave me a deeper appreciation for Fourier transforms. For my results, I think they are correct. I compared them with my classmate's and asked for clariffications from my instructors and professor.

I'd like to thank my instructors, Sir Rene Principe Jr. and Sir Kenneth Leo, for guiding me throughout the activity. I would also like to thank my professor, Ma'am Jing, for guiding me in my coding while my classmates and I worked in R202. I would also like to acknowledge my classmates: Abdel, Johnenn, Jonabel, Richmond, Lovely, Hans, Genesis, Jeruine, Rusher, and Ron for helping me complete this activity.

Self Grade

Technical Correctness	I understood the lesson and met all the objectives.	35
Quality of Presentation	The images I added to this report are of good quality and all the graphs are properly labelled. My code is also properly organized and labelled.	35
Self Reflection	I got the expected results, and acknowledged the contributions of my peers while doing this activity. I also properly cited online references.	30
Initiative	Apart from doing the required tasks, I also applied what I learned to sample images that I took and compiled. I also helped my classmates with their code and helped them by cross-referencing my results with their's.	10
	Total	110

References

[1] Progress in Optics. Elsevier. 2008-01-25. p. 355. ISBN 978-0-08-055768-7.

[2] Rioux, F. (2023, January 11). 1.29: Single slit diffraction and the Fourier transform. Chemistry LibreTexts. Retrieved March 23, 2023, from https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Quantum_Tutorials_(Rioux)/01%3A_Quantum_Fundamentals/1.29%3A_Single_Slit_Diffraction_and_the_Fourier_Transform

[3] Applied Physics 157 Laboratory Manual. Activity 2. Fourier Transform Model of Image Formation (Part 1 of 2)

[4] Fortunato, Horacio & Oliveira, Manuel. (2011). A Gentle Introduction to Coded Computational Photography.. Proceedings - 24th SIBGRAPI Conference on Graphics, Patterns, and Images Tutorials, SIBGRAPI-T 2011. 39-55. 10.1109/SIBGRAPI-T.2011.13.