Supplementary Material for:

Beyond Phasors: Solving Non-Sinusoidal Electrical Circuits using Geometry

Javier Castillo-Martínez, Raul Baños and Francisco G. Montoya

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Contents

A Appendix A: Single-Frequency (N=1) Validation

To validate the proposed methodology, this appendix applies the framework to the fundamental RLC topologies under single-frequency sinusoidal conditions. The results are directly compared against the benchmark complex phasor solutions, confirming that the GA framework contains the classical approach as a special case.

A.1 Case 1a: Series RLC Circuit

We first analyze a series RLC circuit powered by a sinusoidal voltage source, $u(t) = U\sqrt{2}\cos(\omega t)$. As defined in the main paper, this maps to the GA vector $\boldsymbol{u} = U\boldsymbol{\sigma}_1$.

The classical phasor solution for this circuit, with voltage phasor $\mathbf{U} = U\angle 0^\circ$ and complex impedance $\mathbf{Z} = R + jX_1$ (where $X_1 = \omega L - 1/(\omega C)$), yields the current phasor:

$$\mathbf{I} = \frac{\mathbf{U}}{\mathbf{Z}} = \frac{U}{R + jX_1} = \frac{UR}{R^2 + X_1^2} - j\frac{UX_1}{R^2 + X_1^2} \tag{1}$$

The proposed methodology offers two approaches. The first is the GA-Real (Native) algorithm, which applies the per-harmonic physical relationships (Eq. (21) from the main paper) directly in the real $2N$-dimensional space. For $h = 1$, with inputs $u_1 = U$ and $u_2 = 0$, the resulting current components are:

$$i_1 = \frac{Ru_1 - X_1u_2}{R^2 + X_1^2} = \frac{R(U) - X_1(0)}{R^2 + X_1^2} = \frac{UR}{R^2 + X_1^2} \tag{2}$$

$$i_2 = \frac{Ru_2 + X_1u_1}{R^2 + X_1^2} = \frac{R(0) + X_1(U)}{R^2 + X_1^2} = \frac{UX_1}{R^2 + X_1^2} \tag{3}$$

These components form the resulting vector $\boldsymbol{i} = i_1\boldsymbol{\sigma}_1 + i_2\boldsymbol{\sigma}_2$.

The second approach is the GA rotoflex solution, which uses the series form $\boldsymbol{i} = k_s\boldsymbol{R}_s\boldsymbol{u}$. For the $N = 1$ case, the flextance k_s (from Eq. (18) of the main paper) simplifies to the spectral kernel:

$$k_s = \kappa_{s,1} = \frac{1}{\sqrt{R^2 + X_1^2}} \tag{4}$$

The series rotance $\boldsymbol{R}_s$ (from Section III.C.1) is constructed as:

$$\boldsymbol{R}_s = \kappa_{s,1}\left(R - X_1\boldsymbol{\sigma}_{12}\right) \tag{5}$$

Applying the full operator $\boldsymbol{\Theta}_s = k_s\boldsymbol{R}_s$ to the input vector $\boldsymbol{u} = U\boldsymbol{\sigma}_1$ yields the current:

$$\begin{aligned} \boldsymbol{i} &= k_s\boldsymbol{R}_s\boldsymbol{u} = \kappa_{s,1}\left(\kappa_{s,1}\left(R - X_1\boldsymbol{\sigma}_{12}\right)\right)\left(U\boldsymbol{\sigma}_1\right) \\ &= \kappa_{s,1}^2(UR\boldsymbol{\sigma}_1 - UX_1\boldsymbol{\sigma}_{12}\boldsymbol{\sigma}_1) = \frac{1}{R^2 + X_1^2}(UR\boldsymbol{\sigma}_1 + UX_1\boldsymbol{\sigma}_2) \end{aligned} \tag{6}$$

A direct comparison confirms that all three methods yield identical results, adhering to the mapping $\mathbf{X} = A - jB \longleftrightarrow \boldsymbol{x} = A\boldsymbol{\sigma}_1 + B\boldsymbol{\sigma}_2$ established in the main paper. The classical complex result (Eq. 1) corresponds perfectly to the GA vector result (Eq. 6).

A.2 Case 1b: Parallel RLC Circuit

We next analyze a parallel RLC circuit powered by a sinusoidal current source, $i(t) = I\sqrt{2}\cos(\omega t)$, which maps to the GA vector $\boldsymbol{i} = I\boldsymbol{\sigma}_1$.

The classical phasor solution, with current phasor $\mathbf{I} = I\angle 0^\circ$ and complex admittance $\mathbf{Y} = G + jB_1$ (where $B_1 = \omega C - 1/(\omega L)$), provides the resulting voltage phasor:

$$\mathbf{U} = \frac{\mathbf{I}}{\mathbf{Y}} = \frac{I}{G + jB_1} = \frac{IG}{G^2 + B_1^2} - j\frac{IB_1}{G^2 + B_1^2} \tag{7}$$

Applying the GA-Real (Native) algorithm (from Eq. (23) of the main paper) for $h = 1$, with inputs $i_1 = I$ and $i_2 = 0$, the voltage components are:

$$u_1 = \frac{Gi_1 - B_1 i_2}{G^2 + B_1^2} = \frac{G(I) - B_1(0)}{G^2 + B_1^2} = \frac{IG}{G^2 + B_1^2} \tag{8}$$

$$u_2 = \frac{Gi_2 + B_1 i_1}{G^2 + B_1^2} = \frac{G(0) + B_1(I)}{G^2 + B_1^2} = \frac{IB_1}{G^2 + B_1^2} \tag{9}$$

These components form the resulting vector $\boldsymbol{u} = u_1\boldsymbol{\sigma}_1 + u_2\boldsymbol{\sigma}_2$.

Using the GA rotoflex solution, we apply the parallel form $\boldsymbol{u} = k_p\boldsymbol{R}_p\boldsymbol{i}$. The parallel flextance k_p (from Eq. (19) of the main paper) is:

$$k_p = \kappa_{p,1} = \frac{1}{\sqrt{G^2 + B_1^2}} \tag{10}$$

The parallel rotance $\boldsymbol{R}_p$ (from Section III.C.2) is given by:

$$\boldsymbol{R}_p = \kappa_{p,1}\left(G - B_1\boldsymbol{\sigma}_{12}\right) \tag{11}$$

Applying the operator $\boldsymbol{\Theta}_p = k_p\boldsymbol{R}_p$ to $\boldsymbol{i} = I\boldsymbol{\sigma}_1$ yields the voltage:

$$\begin{aligned} \boldsymbol{u} &= k_p\boldsymbol{R}_p\boldsymbol{i} = \kappa_{p,1}\left(\kappa_{p,1}\left(G - B_1\boldsymbol{\sigma}_{12}\right)\right)(I\boldsymbol{\sigma}_1) \\ &= \frac{IG}{G^2 + B_1^2}\boldsymbol{\sigma}_1 + \frac{IB_1}{G^2 + B_1^2}\boldsymbol{\sigma}_2 \end{aligned} \tag{12}$$

As in the series case, all three methods are in perfect agreement. The classical solution (Eq. 7) and the GA solution (Eq. 12) are numerically identical under the established mapping.

B Appendix B: Canonical Load Derivations (Multi-Harmonic)

This section details the derivation of the rotoflex operators for fundamental R, L, and C loads under multi-harmonic conditions, as referenced in Section IV.D.3 of the main paper. The calculations follow the methodology defined in Section III.B of the main paper (Eqs. 16-19).

B.1 Case R: Pure Resistance (Example 1)

We analyze a circuit with a pure resistor $R = 2\,\Omega$ and a $N = 3$ harmonic voltage source given by:

$$u(t) = 1.5\sqrt{2}\cos(\omega t) + \sqrt{2}\cos(2\omega t) + 0.5\sqrt{2}\cos(3\omega t)\,\text{V} \tag{13}$$

In GA vector form, this is $\boldsymbol{u} = 1.5\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_3 + 0.5\boldsymbol{\sigma}_5\,\text{V}$. For a pure resistor, the rotance $\boldsymbol{R}_s = 1$, and the current is found by Ohm's law:

$$\boldsymbol{i} = \frac{\boldsymbol{u}}{R} = \frac{1.5\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_3 + 0.5\boldsymbol{\sigma}_5}{2} = 0.75\boldsymbol{\sigma}_1 + 0.5\boldsymbol{\sigma}_3 + 0.25\boldsymbol{\sigma}_5\,\text{A} \tag{14}$$

To validate this using the formal rotoflex methodology, the flextance k_s is calculated. First, the squared norm of the input vector is $\|\boldsymbol{u}\|^2 = 1.5^2 + 1^2 + 0.5^2 = 3.5\,\text{V}^2$. From this, the spectral weights (γ_h^2, Eq. 4 of the main paper) are $\gamma_1^2 = 0.6429$, $\gamma_2^2 = 0.2857$, and $\gamma_3^2 = 0.0714$. The spectral kernel (Eq. 17 of the main paper) for a pure resistor has $X_h = 0$, so it simplifies to $\kappa_{s,h}^2 = 1/R^2 = 1/2^2 = 0.25\,\text{S}^2$ for all harmonics. Applying the flextance formula (Eq. 18 of the main paper):

$$k_s^2 = \sum \gamma_h^2 \kappa_{s,h}^2 = (0.6429 \cdot 0.25) + (0.2857 \cdot 0.25) + (0.0714 \cdot 0.25) \tag{15}$$

$$k_s^2 = (0.6429 + 0.2857 + 0.0714) \cdot 0.25 = (1.0) \cdot 0.25 = 0.25\,\text{S}^2 \tag{16}$$

This gives $k_s = \sqrt{0.25} = 0.5\,\text{S}$, confirming that for a pure resistor, $k_s = 1/R$ regardless of harmonic content. The full operator application, $\boldsymbol{i} = k_s\boldsymbol{R}_s\boldsymbol{u} = (0.5)(1)(\boldsymbol{u})$, correctly yields the current vector $0.75\boldsymbol{\sigma}_1 + 0.5\boldsymbol{\sigma}_3 + 0.25\boldsymbol{\sigma}_5\,\text{A}$.

B.2 Case L: Pure Inductance (Example 2)

We analyze a circuit with $L = 1\,H$ and $N = 3$ harmonic voltage source at $\omega = 1$ rad/s:

$$u(t) = 2\sqrt{2}\cos(\omega t) + \sqrt{2}\cos(2\omega t) + 0.75\sqrt{2}\cos(3\omega t)\,\text{V} \tag{17}$$

The GA vector is $\boldsymbol{u} = 2\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_3 + 0.75\boldsymbol{\sigma}_5\,\text{V}$. The current is found by $i(t) = (1/L)\int u(t)dt$:

$$i(t) = \frac{1}{1}\left(\frac{2\sqrt{2}\sin(\omega t)}{1} + \frac{\sqrt{2}\sin(2\omega t)}{2} + \frac{0.75\sqrt{2}\sin(3\omega t)}{3}\right) \tag{18}$$

In GA vector form, this becomes $\boldsymbol{i} = 2\boldsymbol{\sigma}_2 + 0.5\boldsymbol{\sigma}_4 + 0.25\boldsymbol{\sigma}_6\,\text{A}$.

To calculate the flextance k_s, the input vector norm is $\|\boldsymbol{u}\|^2 = 2^2 + 1^2 + 0.75^2 = 5.5625\,\text{V}^2$. The spectral weights are $\gamma_1^2 = 0.7191$, $\gamma_2^2 = 0.1798$, and $\gamma_3^2 = 0.1011$. The spectral kernel $\kappa_{s,h}^2$ depends on $X_h = h\omega L = h(1)(1) = h\,\Omega$, giving $\kappa_{s,1}^2 = 1/1^2 = 1.0\,\text{S}^2$, $\kappa_{s,2}^2 = 1/2^2 = 0.25\,\text{S}^2$, and $\kappa_{s,3}^2 = 1/3^2 \approx 0.1111\,\text{S}^2$. The squared flextance is:

$$k_s^2 = \sum \gamma_h^2 \kappa_{s,h}^2 = (0.7191 \cdot 1.0) + (0.1798 \cdot 0.25) + (0.1011 \cdot 0.1111) \tag{19}$$

$$k_s^2 = 0.7191 + 0.04495 + 0.01123 = 0.7753\,\text{S}^2 \tag{20}$$

Thus, $k_s = \sqrt{0.7753} = 0.8805\,\text{S}$. This value is validated by the ratio of the total vector magnitudes: $k_s = \|\boldsymbol{i}\|/\|\boldsymbol{u}\| = 2.0766\,\text{A}/2.3585\,\text{V} = 0.8805\,\text{S}$. The rotance operator is $\boldsymbol{R}_s = \hat{\boldsymbol{i}}\hat{\boldsymbol{u}}$:

$$\begin{aligned}\boldsymbol{R}_s = & -0.8167\boldsymbol{\sigma}_{12} - 0.2042\boldsymbol{\sigma}_{14} - 0.1021\boldsymbol{\sigma}_{16} + 0.4083\boldsymbol{\sigma}_{23} \\ & + 0.3063\boldsymbol{\sigma}_{25} - 0.1021\boldsymbol{\sigma}_{34} - 0.0510\boldsymbol{\sigma}_{36} + 0.0766\boldsymbol{\sigma}_{45} - 0.0383\boldsymbol{\sigma}_{56}\end{aligned} \tag{21}$$

Applying the full operator $\boldsymbol{i} = k_s\boldsymbol{R}_s\boldsymbol{u}$ confirms the result $2\boldsymbol{\sigma}_2 + 0.5\boldsymbol{\sigma}_4 + 0.25\boldsymbol{\sigma}_6\,\text{A}$.

B.3 Case C: Pure Capacitance (Example 3)

We analyze $C = 2\,F$, $N = 3$, $\omega = 3$ rad/s, with the voltage:

$$u(t) = \sqrt{2}\cos(\omega t) + 2\sqrt{2}\cos(2\omega t) + 0.6\sqrt{2}\cos(3\omega t)\,\mathrm{V} \tag{22}$$

The GA vector is $\boldsymbol{u} = \boldsymbol{\sigma}_1 + 2\boldsymbol{\sigma}_3 + 0.6\boldsymbol{\sigma}_5$ V. The current is $i(t) = C\frac{du(t)}{dt}$:

$$i(t) = C\left(-\omega\sqrt{2}\sin(\omega t) - 2(2\omega)\sqrt{2}\sin(2\omega t) - 3(0.6\omega)\sqrt{2}\sin(3\omega t)\right) \tag{23}$$

Substituting $C = 2, \omega = 3$ gives the vector $\boldsymbol{i} = -6\boldsymbol{\sigma}_2 - 24\boldsymbol{\sigma}_4 - 10.8\boldsymbol{\sigma}_6$ A.

For the flextance calculation, the input norm is $\|\boldsymbol{u}\|^2 = 1^2 + 2^2 + 0.6^2 = 5.36\,\mathrm{V}^2$. The spectral weights are $\gamma_1^2 = 0.1866$, $\gamma_2^2 = 0.7463$, and $\gamma_3^2 = 0.0672$. The spectral kernel $\kappa_{s,h}^2$ depends on $X_h = -1/(h\omega C) = -1/(6h)\,\Omega$, yielding $\kappa_{s,1}^2 = 36\,\mathrm{S}^2$, $\kappa_{s,2}^2 = 144\,\mathrm{S}^2$, and $\kappa_{s,3}^2 = 324\,\mathrm{S}^2$. The squared flextance is:

$$k_s^2 = \sum \gamma_h^2 \kappa_{s,h}^2 = (0.1866 \cdot 36) + (0.7463 \cdot 144) + (0.0672 \cdot 324) \tag{24}$$

$$k_s^2 = 6.7176 + 107.4672 + 21.7728 = 135.9576\,\mathrm{S}^2 \tag{25}$$

Thus, $k_s = \sqrt{135.9576} = 11.6593\,\mathrm{S}$. This again matches the ratio of magnitudes: $k_s = \|\boldsymbol{i}\|/\|\boldsymbol{u}\| = 26.9918\,\mathrm{A}/2.3152\,\mathrm{V} = 11.6593\,\mathrm{S}$. The rotance operator is $\boldsymbol{R}_s = \hat{\boldsymbol{i}}\hat{\boldsymbol{u}}$:

$$\begin{aligned}\boldsymbol{R}_s = {}& 0.0960\boldsymbol{\sigma}_{12} + 0.3840\boldsymbol{\sigma}_{14} + 0.1728\boldsymbol{\sigma}_{16} - 0.1920\boldsymbol{\sigma}_{23} \\ & - 0.0576\boldsymbol{\sigma}_{25} + 0.7681\boldsymbol{\sigma}_{34} + 0.3456\boldsymbol{\sigma}_{36} - 0.2304\boldsymbol{\sigma}_{45} + 0.1037\boldsymbol{\sigma}_{56}\end{aligned} \tag{26}$$

Applying the full operator confirms the result $\boldsymbol{i} = k_s\boldsymbol{R}_s\boldsymbol{u} = -6\boldsymbol{\sigma}_2 - 24\boldsymbol{\sigma}_4 - 10.8\boldsymbol{\sigma}_6$ A.

C Appendix C: Combined Load Derivations

C.1 Case 4: Series RL Load (Example 4)

We analyze a series RL circuit with $R = 4\,\Omega, L = 2\,H, N = 2, \omega = 3$ rad/s. The voltage source is:

$$u(t) = \sqrt{2}\cos(\omega t) + \sqrt{2}\cos(2\omega t) \rightarrow \boldsymbol{u} = \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_3\,\mathrm{V} \tag{27}$$

The classical phasor solution requires superposition. The per-harmonic impedances and resulting currents are calculated and summarized in Table 1.

Table 1: Classical Phasor Solution for Case 4 (Series RL)

Harmonic (h)	**Impedance $\mathbf{Z}_h$**	**Current Phasor $\mathbf{I}_h$**	**Rectangular Form (A)**
1	$4 + j(1 \cdot 3 \cdot 2) = 4 + j6\,\Omega$	$1\angle 0/7.21\angle 56.3^\circ = 0.1387\angle -56.3^\circ$	$0.0769 - j0.1154$
2	$4 + j(2 \cdot 3 \cdot 2) = 4 + j12\,\Omega$	$1\angle 0/12.65\angle 71.6^\circ = 0.0791\angle -71.6^\circ$	$0.0250 - j0.0750$

Using the mapping $A - jB \rightarrow A\boldsymbol{\sigma}_1 + B\boldsymbol{\sigma}_2$, the total current vector is the sum of the harmonic components: $\boldsymbol{i} = (0.0769\boldsymbol{\sigma}_1 + 0.1154\boldsymbol{\sigma}_2) + (0.0250\boldsymbol{\sigma}_3 + 0.0750\boldsymbol{\sigma}_4)\,\mathrm{A}$.

To obtain the same result using the GA rotoflex solution, we first calculate the operator $\boldsymbol{\Theta}_s$. The input norm is $\|\boldsymbol{u}\|^2 = 1^2 + 1^2 = 2.0\,\mathrm{V}^2$, and the spectral weights are equal: $\gamma_1^2 = 0.5$ and $\gamma_2^2 = 0.5$. The spectral kernels are $\kappa_{s,1}^2 = 1/(4^2 + 6^2) = 1/52 \approx 0.01923\,\mathrm{S}^2$ and $\kappa_{s,2}^2 = 1/(4^2 + 12^2) = 1/160 = 0.00625\,\mathrm{S}^2$. The squared flextance k_s^2 is then computed:

$$k_s^2 = \sum \gamma_h^2 \kappa_{s,h}^2 = (0.5 \cdot 0.01923) + (0.5 \cdot 0.00625) = 0.01274\,\mathrm{S}^2 \tag{28}$$

This gives a flextance $k_s = \sqrt{0.01274} = 0.1129\,\mathrm{S}$. The corresponding rotance $\boldsymbol{R}_s = \hat{\boldsymbol{\imath}}\hat{\boldsymbol{u}}$ is found to be:

$$\boldsymbol{R}_s = 0.4515 - 0.5111\boldsymbol{\sigma}_{12} + 0.2300\boldsymbol{\sigma}_{13} - 0.3322\boldsymbol{\sigma}_{14} + 0.5111\boldsymbol{\sigma}_{23} - 0.3322\boldsymbol{\sigma}_{34} \tag{29}$$

Applying the full operator $\boldsymbol{i} = k_s \boldsymbol{R}_s \boldsymbol{u}$ yields the final solution:

$$\boldsymbol{i} = (0.1129)(\boldsymbol{R}_s)(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_3) = 0.0769\boldsymbol{\sigma}_1 + 0.1154\boldsymbol{\sigma}_2 + 0.0250\boldsymbol{\sigma}_3 + 0.0750\boldsymbol{\sigma}_4\,\mathrm{A} \tag{30}$$

The result matches the classical superposition solution perfectly.

C.2 Case 5: Series RC Load (Example 5)

We analyze a series RC circuit with $R = 1\,\Omega, C = 1\,F, N = 2, \omega = 2$ rad/s. The voltage source is:

$$u(t) = \sqrt{2}\cos(\omega t) + 0.5\sqrt{2}\cos(2\omega t) \rightarrow \boldsymbol{u} = \boldsymbol{\sigma}_1 + 0.5\boldsymbol{\sigma}_3\,\mathrm{V} \tag{31}$$

The classical phasor solution is obtained by superposition, with results summarized in Table 2.

Table 2: Classical Phasor Solution for Case 5 (Series RC)

Harmonic (h)	**Impedance $\mathbf{Z}_h$**	**Current Phasor $\mathbf{I}_h$**	**Rectangular Form (A)**
1	$1 - j0.5\,\Omega$	$1\angle 0/1.118\angle -26.56^\circ = 0.8944\angle 26.56^\circ$	$0.8 + j0.4$
2	$1 - j0.25\,\Omega$	$0.5\angle 0/1.031\angle -14.04^\circ = 0.4851\angle 14.04^\circ$	$0.4706 + j0.1176$

For this capacitive case, the mapping is $A + jB \rightarrow A\boldsymbol{\sigma}_1 - B\boldsymbol{\sigma}_2$. The total current vector is: $\boldsymbol{i} = (0.8\boldsymbol{\sigma}_1 - 0.4\boldsymbol{\sigma}_2) + (0.4706\boldsymbol{\sigma}_3 - 0.1176\boldsymbol{\sigma}_4)\,\mathrm{A}$.

The GA rotoflex solution proceeds by calculating the operator components. The input norm is $\|\boldsymbol{u}\|^2 = 1^2 + 0.5^2 = 1.25\,\mathrm{V}^2$, and the spectral weights are $\gamma_1^2 = 0.8$ and $\gamma_2^2 = 0.2$. The spectral kernels are $\kappa_{s,1}^2 = 1/(1^2 + (-0.5)^2) = 0.8\,\mathrm{S}^2$ and $\kappa_{s,2}^2 = 1/(1^2 + (-0.25)^2) \approx 0.94118\,\mathrm{S}^2$. The squared flextance k_s^2 is:

$$k_s^2 = \sum \gamma_h^2 \kappa_{s,h}^2 = (0.8 \cdot 0.8) + (0.2 \cdot 0.94118) = 0.64 + 0.18824 = 0.82824\,\mathrm{S}^2 \tag{32}$$

This gives a flextance $k_s = \sqrt{0.82824} = 0.9101\,\mathrm{S}$. The corresponding rotance $\boldsymbol{R}_s = \hat{\boldsymbol{i}}\hat{\boldsymbol{u}}$ is:

$$\boldsymbol{R}_s = 0.9101 + 0.3516\boldsymbol{\sigma}_{12} - 0.0621\boldsymbol{\sigma}_{13} + 0.1034\boldsymbol{\sigma}_{14} - 0.1758\boldsymbol{\sigma}_{23} + 0.0517\boldsymbol{\sigma}_{34} \tag{33}$$

Applying the full operator $\boldsymbol{i} = k_s\boldsymbol{R}_s\boldsymbol{u}$ yields the final solution:

$$\boldsymbol{i} = (0.9101)(\boldsymbol{R}_s)(\boldsymbol{\sigma}_1 + 0.5\boldsymbol{\sigma}_3) = 0.8\boldsymbol{\sigma}_1 - 0.4\boldsymbol{\sigma}_2 + 0.4706\boldsymbol{\sigma}_3 - 0.1176\boldsymbol{\sigma}_4\,\mathrm{A} \tag{34}$$

The GA result is again in perfect agreement with the classical solution.

D Appendix D: RLC Case Study Derivations (N=2)

D.1 Case 1a: Series RLC (N=2) - Voltage Source

This appendix provides the detailed calculations for "Case 1" from the main paper. The circuit parameters are $R = 3\,\Omega, L = 1\,H, C = 1\,F, \omega = 1$ rad/s. The voltage source is $u(t) = \sqrt{2}\cos(\omega t) + 0.8\sqrt{2}\cos(2\omega t)$, which maps to the GA vector $\boldsymbol{u} = \boldsymbol{\sigma}_1 + 0.8\boldsymbol{\sigma}_3\,\mathrm{V}$.

The classical phasor solution by superposition is summarized in Table 3.

Table 3: Classical Phasor Solution for Case 1a (Series RLC)

Harmonic (h)	**Impedance $\mathbf{Z}_h$**	**Current Phasor $\mathbf{I}_h$**	**Rectangular Form (A)**
1	$3 + j(1-1) = 3 + j0\,\Omega$	$(1\angle 0)/3 = 0.3333\angle 0^\circ$	$0.3333 + j0$
2	$3 + j(2 - 1/2) = 3 + j1.5\,\Omega$	$(0.8\angle 0)/3.354\angle 26.57^\circ = 0.2386\angle - 26.57^\circ$	$0.2133 - j0.1067$

Using the $A - jB \rightarrow A\boldsymbol{\sigma}_1 + B\boldsymbol{\sigma}_2$ mapping, the total GA vector is constructed from the sum of the harmonic components: $\boldsymbol{i} = (0.3333\boldsymbol{\sigma}_1 + 0\boldsymbol{\sigma}_2) + (0.2133\boldsymbol{\sigma}_3 + 0.1067\boldsymbol{\sigma}_4)$, which simplifies to:

$$\boldsymbol{i} = 0.3333\boldsymbol{\sigma}_1 + 0.2133\boldsymbol{\sigma}_3 + 0.1067\boldsymbol{\sigma}_4\,\mathrm{A} \tag{35}$$

The GA-Real (Native) algorithm, which is benchmarked in Appendix H, computes this result directly by applying the per-harmonic real-number transformations given in Eqs. (36) and (37) for each harmonic subspace.

$$i_{2h-1} = \frac{Ru_{2h-1} - X_h u_{2h}}{R^2 + X_h^2} \tag{36}$$

$$i_{2h} = \frac{Ru_{2h} + X_h u_{2h-1}}{R^2 + X_h^2} \tag{37}$$

For $h = 1$ (the $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2$ plane), with $u_1 = 1, u_2 = 0$ and $X_1 = 0$, the equations yield $i_1 = 0.3333$ and $i_2 = 0$. For $h = 2$ (the $\boldsymbol{\sigma}_3, \boldsymbol{\sigma}_4$ plane), with $u_3 = 0.8, u_4 = 0$ and $X_2 = 1.5$, the equations yield $i_3 = 2.4/11.25 = 0.2133$ and $i_4 = 1.2/11.25 = 0.1067$. These components are then assembled into the final 4×1 solution vector $\boldsymbol{i}$ (matching Eq. 33 from the main paper):

$$\boldsymbol{i} = \begin{pmatrix} 0.3333 \\ 0 \\ 0.2133 \\ 0.1067 \end{pmatrix} \tag{38}$$

The GA rotoflex solution calculates the unified operator. The input norm is $\|\boldsymbol{u}\|^2 = 1^2 + 0.8^2 = 1.64\,\mathrm{V}^2$, and the spectral weights are $\gamma_1^2 = 0.60976$ and $\gamma_2^2 = 0.39024$. The spectral kernels are $\kappa_{s,1}^2 = 1/(3^2 + 0^2) \approx 0.1111\,\mathrm{S}^2$ and $\kappa_{s,2}^2 = 1/(3^2 + 1.5^2) \approx 0.08889\,\mathrm{S}^2$. The squared flextance k_s^2 is:

$$k_s^2 = \sum \gamma_h^2 \kappa_{s,h}^2 = (0.60976 \cdot 0.1111) + (0.39024 \cdot 0.08889) \tag{39}$$

$$k_s^2 = 0.06775 + 0.03468 = 0.10243\,\mathrm{S}^2 \tag{40}$$

This gives a flextance $k_s = \sqrt{0.10243} = 0.3201\,\mathrm{S}$. The rotance $\boldsymbol{R}_s = \hat{\boldsymbol{i}}\hat{\boldsymbol{u}}$ is given by Eq. (22) of the main paper:

$$\boldsymbol{R}_s = 0.9602 + 0.1016\boldsymbol{\sigma}_{13} - 0.2032\boldsymbol{\sigma}_{14} - 0.1626\boldsymbol{\sigma}_{34} \tag{41}$$

Applying the full operator $\boldsymbol{i} = k_s\boldsymbol{R}_s\boldsymbol{u}$ confirms the final result:

$$\boldsymbol{i} = (0.3201) \times (\boldsymbol{R}_s) \times (\boldsymbol{\sigma}_1 + 0.8\boldsymbol{\sigma}_3) = 0.3333\boldsymbol{\sigma}_1 + 0.2133\boldsymbol{\sigma}_3 + 0.1067\boldsymbol{\sigma}_4\,\mathrm{A} \tag{42}$$

All three methods are in perfect agreement.

D.2 Case 1b: Series RLC (N=2) - Current Source (Inverse Problem)

This section details the inverse problem: solving for $\boldsymbol{u}$ given $\boldsymbol{i}$ in the same series circuit. We use the current vector from Case 1a as the source: $\boldsymbol{i} = 0.3333\boldsymbol{\sigma}_1 + 0.2133\boldsymbol{\sigma}_3 + 0.1067\boldsymbol{\sigma}_4\,\mathrm{A}$.

The GA rotoflex solution uses the inverse operator: $\boldsymbol{u} = \boldsymbol{\Theta}_s^{-1}\boldsymbol{i}$. This inverse operator is defined as $\boldsymbol{\Theta}_s^{-1} = k_{s,inv}\boldsymbol{R}_{s,inv}$, where $\boldsymbol{R}_{s,inv} = \boldsymbol{R}_s^\dagger$ and $k_{s,inv} = k_s^{-1}$. The inverse flextance k_s^{-1} (with units of Ω) maps $\|\boldsymbol{i}\| \to \|\boldsymbol{u}\|$ and is calculated using $\boldsymbol{i}$ as the input signal.

First, the input current norm is $\|\boldsymbol{i}\|^2 = 0.3333^2 + 0.2133^2 + 0.1067^2 = 0.16797\,\mathrm{A}^2$. The harmonic components are $\boldsymbol{i}_1 = 0.3333\boldsymbol{\sigma}_1$ and $\boldsymbol{i}_2 = 0.2133\boldsymbol{\sigma}_3 + 0.1067\boldsymbol{\sigma}_4$. The spectral weights γ_h^2 are thus $\gamma_1^2 = (0.3333^2)/0.16797 = 0.6614$ and $\gamma_2^2 = (0.2133^2 + 0.1067^2)/0.16797 = 0.3386$. The inverse spectral kernel is $\kappa_{s,h}^{-2} = R^2 + X_h^2$. With $X_1 = 0\,\Omega$ and $X_2 = 1.5\,\Omega$, we have $\kappa_{s,1}^{-2} = 3^2 + 0^2 = 9\,\Omega^2$ and $\kappa_{s,2}^{-2} = 3^2 + 1.5^2 = 11.25\,\Omega^2$. The squared inverse flextance is:

$$k_s^{-2} = \sum \gamma_h^2 \kappa_{s,h}^{-2} = (0.6614 \cdot 9) + (0.3386 \cdot 11.25) = 9.7618\,\Omega^2 \tag{43}$$

This gives $k_s^{-1} = \sqrt{9.7618} = 3.1244\,\Omega$, which correctly matches the ratio $\|\boldsymbol{u}\|/\|\boldsymbol{i}\| = 1.2806/0.4098 = 3.1244$. The inverse rotance $\boldsymbol{R}_s^\dagger$ is the reverse of $\boldsymbol{R}_s$ from Case 1a. Since $\boldsymbol{B}^\dagger = -\boldsymbol{B}$ for bivectors:

$$\boldsymbol{R}_s^\dagger = 0.9602 - 0.1016\boldsymbol{\sigma}_{13} + 0.2032\boldsymbol{\sigma}_{14} + 0.1626\boldsymbol{\sigma}_{34} \tag{44}$$

Applying the full inverse operator $\boldsymbol{u} = k_s^{-1}\boldsymbol{R}_s^\dagger\boldsymbol{i}$:

$$\boldsymbol{u} = (3.1244)(0.9602 - 0.1016\boldsymbol{\sigma}_{13} + ...)\,(0.3333\boldsymbol{\sigma}_1 + 0.2133\boldsymbol{\sigma}_3 + 0.1067\boldsymbol{\sigma}_4) \tag{45}$$

$$\boldsymbol{u} = 1.0\boldsymbol{\sigma}_1 + 0.8\boldsymbol{\sigma}_3\,\mathrm{V} \tag{46}$$

This operation perfectly recovers the original voltage vector.

E Appendix E: RLC Case Study Derivations (N=3)

E.1 Case 2a: Parallel RLC (N=3) - Current Source

This appendix provides the detailed calculations for "Case 2" from the main paper. The circuit parameters are $R = 2\,\Omega$ ($G = 0.5\,\mathrm{S}$), $L = 3\,\mathrm{H}$, $C = 0.5\,\mathrm{F}$, at $\omega = 2\,\mathrm{rad/s}$. The current source is $\boldsymbol{i} = 1.5\boldsymbol{\sigma}_1 + 0.9\boldsymbol{\sigma}_4 + 0.5\boldsymbol{\sigma}_5$.

The classical solution requires superposition using the admittance $\mathbf{Y}_h = G + jB_h$, where $B_h = h\omega C - 1/(h\omega L)$. The per-harmonic calculations are summarized in Table 4.

Table 4: Classical Phasor Solution for Case 2a (Parallel RLC)

Harmonic (h)	**Admittance $\mathbf{Y}_h$ (S)**	**Current $\mathbf{I}_h$ (A)**	**Voltage $\mathbf{U}_h$ (V)**
1	$0.5 + j0.8333$	$1.5\angle 0°$	$1.5435\angle -59.04° = 0.7941 - j1.3235$
2	$0.5 + j1.9167$	$0.9\angle -90°$	$0.4543\angle -165.39° = -0.4396 - j0.1147$
3	$0.5 + j2.9444$	$0.5\angle 0°$	$0.1674\angle -80.39° = 0.0280 - j0.1651$

Using the $A - jB \to A\boldsymbol{\sigma}_1 + B\boldsymbol{\sigma}_2$ mapping, the total voltage vector is constructed from the sum of the harmonic components:

$$\boldsymbol{u} = 0.7941\boldsymbol{\sigma}_1 + 1.3235\boldsymbol{\sigma}_2 - 0.4396\boldsymbol{\sigma}_3 + 0.1147\boldsymbol{\sigma}_4 + 0.0280\boldsymbol{\sigma}_5 + 0.1651\boldsymbol{\sigma}_6\,\mathrm{V} \tag{47}$$

The GA-Real (Native) algorithm computes this result by applying the per-harmonic real-number transformations given in Eqs. (48) and (49).

$$u_{2h-1} = \frac{Gi_{2h-1} - B_h i_{2h}}{G^2 + B_h^2} \tag{48}$$

$$u_{2h} = \frac{Gi_{2h} + B_h i_{2h-1}}{G^2 + B_h^2} \tag{49}$$

For $h = 1$ (the $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2$ plane), with $i_1 = 1.5, i_2 = 0$ and $B_1 = 0.8333$, the equations yield $u_1 = 0.7941$ and $u_2 = 1.3235$. For $h = 2$ (the $\boldsymbol{\sigma}_3, \boldsymbol{\sigma}_4$ plane), with $i_3 = 0, i_4 = 0.9$ and $B_2 = 1.9167$, the equations yield $u_3 = -0.4396$ and $u_4 = 0.1147$. For $h = 3$ (the $\boldsymbol{\sigma}_5, \boldsymbol{\sigma}_6$ plane), with $i_5 = 0.5, i_6 = 0$ and $B_3 = 2.9444$, the equations yield $u_5 = 0.0280$ and $u_6 = 0.1651$. These components are assembled into the final 6×1 solution vector $\boldsymbol{u}$ (matching Eq. 36 from the main paper):

$$\boldsymbol{u} = \begin{pmatrix} 0.7941 \\ 1.3235 \\ -0.4396 \\ 0.1147 \\ 0.0280 \\ 0.1651 \end{pmatrix} \tag{50}$$

The GA rotoflex solution provides the unified operator. The input norm is $\|\boldsymbol{i}\|^2 = 1.5^2 + 0.9^2 + 0.5^2 = 3.31\,\mathrm{A}^2$. The spectral weights are $\gamma_1^2 = 0.6798, \gamma_2^2 = 0.2447, \gamma_3^2 = 0.0755$. The parallel spectral kernels are $\kappa_{p,1}^2 = 1.0656, \kappa_{p,2}^2 = 0.2548, \kappa_{p,3}^2 = 0.1119$. The squared flextance k_p^2 is:

$$k_p^2 = (0.6798 \cdot 1.0656) + (0.2447 \cdot 0.2548) + (0.0755 \cdot 0.1119) = 0.7957 \tag{51}$$

This gives $k_p = \sqrt{0.7957} = 0.8920\,\Omega$ (matching the value of 0.8892 reported in the main paper, with slight numerical differences). The rotance $\boldsymbol{R}_p = \hat{\boldsymbol{u}}\hat{\boldsymbol{\imath}}$ is given by Eq. (24) from the main paper:

$$\begin{aligned} \boldsymbol{R}_p = 0.4446 &- 0.6746\boldsymbol{\sigma}_{12} + 0.2241\boldsymbol{\sigma}_{13} + 0.1844\boldsymbol{\sigma}_{14} \\ &+ 0.1206\boldsymbol{\sigma}_{15} - 0.0841\boldsymbol{\sigma}_{16} + 0.4047\boldsymbol{\sigma}_{24} + 0.2249\boldsymbol{\sigma}_{25} \\ &- 0.1344\boldsymbol{\sigma}_{34} - 0.0747\boldsymbol{\sigma}_{35} + 0.0109\boldsymbol{\sigma}_{45} - 0.0505\boldsymbol{\sigma}_{46} \\ &- 0.0280\boldsymbol{\sigma}_{56} \end{aligned} \tag{52}$$

Applying the full operator $\boldsymbol{u} = k_p \boldsymbol{R}_p \boldsymbol{i}$ confirms the final result:

$$\boldsymbol{u} = (0.8892)(\boldsymbol{R}_p)(1.5\boldsymbol{\sigma}_1 + 0.9\boldsymbol{\sigma}_4 + 0.5\boldsymbol{\sigma}_5) \tag{53}$$

$$\boldsymbol{u} = 0.7941\boldsymbol{\sigma}_1 + 1.3235\boldsymbol{\sigma}_2 - 0.4396\boldsymbol{\sigma}_3 + 0.1147\boldsymbol{\sigma}_4 + 0.0280\boldsymbol{\sigma}_5 + 0.1651\boldsymbol{\sigma}_6\,\mathrm{V} \tag{54}$$

This result is identical to the GA-Real and Classical solutions.

F Appendix F: Extended Case Study (N=5 Series)

We now validate the methodology for a higher harmonic count ($N = 5$), as mentioned in the main paper. The circuit is a series RLC with $R = 1\,\Omega, L = 0.5\,H, C = 0.1\,F, \omega = 2$ rad/s. The voltage source is $\boldsymbol{u} = 10\boldsymbol{\sigma}_1 + 5\boldsymbol{\sigma}_3 + 2\boldsymbol{\sigma}_5 + 1\boldsymbol{\sigma}_7 + 0.5\boldsymbol{\sigma}_9$ V.

The classical phasor solution uses the impedance $\mathbf{Z}_h = R + jX_h$, where $X_h = h\omega L - 1/(h\omega C) = h - 5/h$. The per-harmonic calculations are summarized in Table 5.

Table 5: Classical Phasor Solution for Case F (N=5 Series)

Harmonic (h)	**Impedance $\mathbf{Z}_h$ (Ω)**	**Voltage $\mathbf{U}_h$ (V)**	**Current $\mathbf{I}_h$ (A)**
1	$1 - j4$	$10\angle 0$	$0.5882 + j2.3529$
2	$1 - j0.5$	$5\angle 0$	$4.0 + j2.0$
3	$1 + j1.333$	$2\angle 0$	$0.7200 - j0.9600$
4	$1 + j2.75$	$1\angle 0$	$0.1168 - j0.3211$
5	$1 + j4$	$0.5\angle 0$	$0.0294 - j0.1176$

Using the mapping $\mathbf{X}_h = A_h + jB_h \longleftrightarrow \boldsymbol{x} = A\boldsymbol{\sigma}_{2h-1} - B\boldsymbol{\sigma}_{2h}$, the total GA current vector is assembled:

$$\begin{aligned} \boldsymbol{i} &= (0.5882\boldsymbol{\sigma}_1 - 2.3529\boldsymbol{\sigma}_2) + (4.0\boldsymbol{\sigma}_3 - 2.0\boldsymbol{\sigma}_4) + (0.7200\boldsymbol{\sigma}_5 + 0.9600\boldsymbol{\sigma}_6) \\ &\quad + (0.1168\boldsymbol{\sigma}_7 + 0.3211\boldsymbol{\sigma}_8) + (0.0294\boldsymbol{\sigma}_9 + 0.1176\boldsymbol{\sigma}_{10})\,\mathrm{A} \end{aligned} \tag{55}$$

The GA-Real (Native) algorithm solution, applying Eqs. (36) and (37), computes the components for each h. For $h = 1$, $u_1 = 10, u_2 = 0, X_1 = -4$, yielding $i_1 = 0.5882, i_2 = -2.3529$. For $h = 2$, $u_3 = 5, u_4 = 0, X_2 = -0.5$, yielding $i_3 = 4.0, i_4 = -2.0$. This process is repeated for $h = 3, 4, 5$, and the resulting vector components match the classical solution exactly.

The GA rotoflex solution first requires the operator calculation. The squared norm of the input voltage is $\|\boldsymbol{u}\|^2 = 10^2 + 5^2 + 2^2 + 1^2 + 0.5^2 = 130.25\,\mathrm{V}^2$. The squared norm of the resulting current vector is $\|\boldsymbol{i}\|^2 = 0.5882^2 + (-2.3529)^2 + \cdots + 0.1176^2 = 27.453\,\mathrm{A}^2$. The flextance k_s is the ratio of the magnitude norms:

$$k_s = \sqrt{\frac{\|\boldsymbol{i}\|^2}{\|\boldsymbol{u}\|^2}} = \sqrt{\frac{27.453}{130.25}} = 0.4591\,\mathrm{S} \tag{56}$$

The rotance $\boldsymbol{R}_s = \hat{\boldsymbol{i}}\hat{\boldsymbol{u}}$ is the geometric product of the normalized vectors. Its scalar part, which defines the power factor, is $\langle \boldsymbol{R}_s \rangle_0 = \hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{u}}$.

$$\langle \boldsymbol{R}_s \rangle_0 = \frac{\boldsymbol{i} \cdot \boldsymbol{u}}{\|\boldsymbol{i}\|\|\boldsymbol{u}\|} = \frac{(0.5882)(10) + (4.0)(5) + (0.7200)(2) + (0.1168)(1) + (0.0294)(0.5)}{5.2396 \cdot 11.413} \tag{57}$$

$$\langle \boldsymbol{R}_s \rangle_0 = \frac{27.4535}{59.799} = \mathbf{0.4591} \quad (\mathrm{PF} = 45.91\%) \tag{58}$$

The bivector part $\langle \boldsymbol{R}_s \rangle_2 = \langle \hat{\boldsymbol{i}}\hat{\boldsymbol{u}} \rangle_2$, which contains 45 components, is computed by the validation script.

$$
\begin{aligned}
\langle \boldsymbol{R}_s \rangle_2 = \; & +0.3935\boldsymbol{\sigma}_{12} - 0.6197\boldsymbol{\sigma}_{13} + 0.3345\boldsymbol{\sigma}_{14} - 0.1007\boldsymbol{\sigma}_{15} - 0.1605\boldsymbol{\sigma}_{16} \\
& - 0.0097\boldsymbol{\sigma}_{17} - 0.0537\boldsymbol{\sigma}_{18} + 0.0000\boldsymbol{\sigma}_{19} - 0.0197\boldsymbol{\sigma}_{1,10} \\
& - 0.1967\boldsymbol{\sigma}_{23} - 0.0787\boldsymbol{\sigma}_{25} - 0.0394\boldsymbol{\sigma}_{27} - 0.0197\boldsymbol{\sigma}_{29} \\
& + 0.1672\boldsymbol{\sigma}_{34} + 0.0736\boldsymbol{\sigma}_{35} - 0.0803\boldsymbol{\sigma}_{36} + 0.0571\boldsymbol{\sigma}_{37} \\
& - 0.0269\boldsymbol{\sigma}_{38} + 0.0310\boldsymbol{\sigma}_{39} - 0.0098\boldsymbol{\sigma}_{3,10} \\
& - 0.0669\boldsymbol{\sigma}_{45} - 0.0335\boldsymbol{\sigma}_{47} - 0.0167\boldsymbol{\sigma}_{49} \\
& - 0.0321\boldsymbol{\sigma}_{56} + 0.0081\boldsymbol{\sigma}_{57} - 0.0107\boldsymbol{\sigma}_{58} + 0.0050\boldsymbol{\sigma}_{59} - 0.0039\boldsymbol{\sigma}_{5,10} \\
& + 0.0161\boldsymbol{\sigma}_{67} + 0.0080\boldsymbol{\sigma}_{69} \\
& - 0.0054\boldsymbol{\sigma}_{78} + 0.0005\boldsymbol{\sigma}_{79} - 0.0020\boldsymbol{\sigma}_{7,10} \\
& + 0.0027\boldsymbol{\sigma}_{89} \\
& - 0.0010\boldsymbol{\sigma}_{9,10}
\end{aligned} \tag{59}
$$

Applying the full operator $\boldsymbol{i} = k_s \boldsymbol{R}_s \boldsymbol{u}$ confirms the result, yielding the same vector as the other two methods with numerical precision on the order of 10^{-15}.

$$
\begin{aligned}
\boldsymbol{i} = \; & 0.5882\boldsymbol{\sigma}_1 - 2.3529\boldsymbol{\sigma}_2 + 4.0\boldsymbol{\sigma}_3 - 2.0\boldsymbol{\sigma}_4 + 0.7200\boldsymbol{\sigma}_5 \\
& + 0.9600\boldsymbol{\sigma}_6 + 0.1168\boldsymbol{\sigma}_7 + 0.3212\boldsymbol{\sigma}_8 + 0.0294\boldsymbol{\sigma}_9 + 0.1176\boldsymbol{\sigma}_{10} \text{ A}
\end{aligned} \tag{60}
$$

It is important to note that this $N = 5$ (10-dimensional) case study validation was performed using the ‘SUGAR‘ and ‘GA-FuL‘ libraries, as some other MATLAB libraries (like ‘Clifford‘) may have dimensional limitations.

G Appendix G: Extended Case Study (N=5 Parallel)

We further validate the methodology for $N = 5$ in a parallel configuration. The circuit is a parallel RLC with $G = 0.5\,S, L = 2\,H, C = 0.2\,F, \omega = 1$ rad/s. The current source is $\boldsymbol{i} = 10\boldsymbol{\sigma}_1 + 5\boldsymbol{\sigma}_3 + 3\boldsymbol{\sigma}_5 + 2\boldsymbol{\sigma}_7 + 1\boldsymbol{\sigma}_9$ A.

The classical phasor solution uses the admittance $\mathbf{Y}_h = G + jB_h$, where $B_h = h\omega C - 1/(h\omega L) = 0.2h - 0.5/h$. The per-harmonic calculations are summarized in Table 6.

Table 6: Classical Phasor Solution for Case G (N=5 Parallel)

Harmonic (h)	**Admittance $\mathbf{Y}_h$ (S)**	**Current $\mathbf{I}_h$ (A)**	**Voltage $\mathbf{U}_h$ (V)**
1	$0.5 - j0.3$	$10\angle 0$	$14.7059 + j8.8235$
2	$0.5 + j0.15$	$5\angle 0$	$9.1743 - j2.7523$
3	$0.5 + j0.4333$	$3\angle 0$	$3.4264 - j2.9695$
4	$0.5 + j0.675$	$2\angle 0$	$1.4172 - j1.9132$
5	$0.5 + j0.9$	$1\angle 0$	$0.4717 - j0.8491$

Using the mapping $\mathbf{U}_h = A_h + jB_h \longleftrightarrow \boldsymbol{u} = A\boldsymbol{\sigma}_{2h-1} - B\boldsymbol{\sigma}_{2h}$, the total GA voltage vector is assembled:

$$\begin{aligned}\boldsymbol{u} = {} & (14.7059\boldsymbol{\sigma}_1 - 8.8235\boldsymbol{\sigma}_2) + (9.1743\boldsymbol{\sigma}_3 + 2.7523\boldsymbol{\sigma}_4) + (3.4264\boldsymbol{\sigma}_5 + 2.9695\boldsymbol{\sigma}_6) \\ & + (1.4172\boldsymbol{\sigma}_7 + 1.9132\boldsymbol{\sigma}_8) + (0.4717\boldsymbol{\sigma}_9 + 0.8491\boldsymbol{\sigma}_{10})\,\mathrm{V}\end{aligned} \tag{61}$$

The GA-Real (Native) algorithm, applying Eqs. (48) and (49), computes the components for each h. For $h = 1$, $i_1 = 10, i_2 = 0, B_1 = -0.3$, yielding $u_1 = 14.7059, u_2 = -8.8235$. For $h = 2$, $i_3 = 5, i_4 = 0, B_2 = 0.15$, yielding $u_3 = 9.1743, u_4 = 2.7523$. This process, repeated for $h = 3, 4, 5$, produces vector components identical to the classical solution.

The GA rotoflex solution calculates the parallel operator $\boldsymbol{\Theta}_p$. The squared norm of the input current is $\|\boldsymbol{i}\|^2 = 10^2 + 5^2 + 3^2 + 2^2 + 1^2 = 139\,\mathrm{A}^2$. The squared norm of the resulting voltage vector is $\|\boldsymbol{u}\|^2 = 14.7059^2 + (-8.8235)^2 + \cdots + 0.8491^2 = 413.03\,\mathrm{V}^2$. The flextance k_p is calculated as:

$$k_p = \sqrt{\frac{\sum |\boldsymbol{i}_h|^2 \kappa_{p,h}^2}{\sum |\boldsymbol{i}_h|^2}} = \sqrt{\frac{413.0306}{139}} = 1.723789\,\Omega \tag{62}$$

The rotance $\boldsymbol{R}_p = \hat{\boldsymbol{u}}\hat{\boldsymbol{i}}$ is computed, and its scalar part $\langle \boldsymbol{R}_p \rangle_0 = \hat{\boldsymbol{u}} \cdot \hat{\boldsymbol{i}}$ defines the power factor:

$$\langle \boldsymbol{R}_p \rangle_0 = \frac{\boldsymbol{u} \cdot \boldsymbol{i}}{\|\boldsymbol{u}\|\|\boldsymbol{i}\|} = \frac{(14.7059)(10) + (9.1743)(5) + (3.4264)(3) + (1.4172)(2) + (0.4717)(1)}{20.3232 \cdot 11.7898} \tag{63}$$

$$\langle \boldsymbol{R}_p \rangle_0 = \frac{206.5158}{239.608} = 0.86189 \quad (\mathrm{PF} = 86.19\%) \tag{64}$$

The bivector part $\langle \boldsymbol{R}_p \rangle_2$ (containing 45 terms, e.g., $0.36825\boldsymbol{\sigma}_{12} - 0.076015\boldsymbol{\sigma}_{13} \ldots$) is omitted for brevity. Applying the full operator $\boldsymbol{u} = k_p \boldsymbol{R}_p \boldsymbol{i}$

$$\boldsymbol{u} = (1.723789)(\boldsymbol{R}_p)(10\boldsymbol{\sigma}_1 + 5\boldsymbol{\sigma}_3 + 3\boldsymbol{\sigma}_5 + 2\boldsymbol{\sigma}_7 + 1\boldsymbol{\sigma}_9) \tag{65}$$

yields the same vector as in Eq. (61), validating the result.

H Appendix H: Validation Scripts and Implementation

To validate the numerical accuracy and computational performance of the proposed methods, a set of MATLAB scripts has been developed. These scripts are designed to compare the results of three different computational approaches for solving the same circuit problem. The first is the GA-Real (Native) method, which is the algorithm derived directly from the physics of the problem, as presented in Eqs. (36), (37), (48)) and (49). This method is implemented using native MATLAB real-number arrays and loops, representing the most direct and computationally "raw" version of the GA-based solution. The second is the Phasor (Native Complex) method, the classical per-harmonic solution implemented using MATLAB's built-in complex number arithmetic, which serves as the benchmark for traditional analysis. The third is the GA-Library (High-Level) method, which implements the full rotoflex operator ($\boldsymbol{\Theta} = k\boldsymbol{R}$) using a dedicated Geometric Algebra library in MATLAB, solving the problem in a single high-level operation ($\boldsymbol{i} = \boldsymbol{\Theta}_s\boldsymbol{u}$ or $\boldsymbol{u} = \boldsymbol{\Theta}_p\boldsymbol{i}$).

H.1 Available Libraries and Scripts

The validation can be performed using several GA libraries available for MATLAB, each with its own interface and performance characteristics. The scripts developed for this paper can be adapted for any of them. One such library is *Clifford*, a robust MATLAB-based GA library for which a parallel script was developed, demonstrating the exact same numerical results. Another is *SUGAR*, a MATLAB-native library (GA for MATLAB); the script *v13.21-sugar.m* (provided alongside this paper) uses this library extensively and allows for a direct, side-by-side comparison of all three methods. A third option is *GA-FuL*, a .NET-based library that can be interfaced with MATLAB, which is also suitable for validation, though it often presents a different performance profile due to computational overhead from the MATLAB-.NET interface.

H.2 Guides for different Libraries

H.2.1 Guide for Clifford Library

The validation script *Resolution_GA_clifford.m* utilizes the *Clifford* library. The implementation begins by adding the library to the MATLAB *path* via *addpath('clifford');* and then initializing the Euclidean algebra for the required dimension using *clifford_signature(dimension, 0);*. This initialization makes the basis vectors (e.g., *e1*, *e2*, etc.) available. The script prompts the user to enter the input vector as a *string* (e.g., *'e1+0.8*e3'*). This *string* is then interpreted to construct the corresponding multivector by summing the basis vectors multiplied by their coefficients. For Geometric Algebra operations, the library provides a range of functions and overloaded operators. The script specifically employs the *** operator (*mtimes*) for the geometric product and *inv(u)* for the inverse. Other key functions provided by the library include *abs(M)* to compute the modulus, *wedge(M, N)* for the outer product, *conj(M)* for the Clifford conjugate, and *scalar_product(M, N)* for the scalar product of two multivectors. As observed in the validation console output, the library displays scalar parts using the *e0* basis and bivector components using combined indices (e.g., *e13*).

H.2.2 Guide for SUGAR Library

The provided script *Resolution_GA_sugar.m* serves as the primary tool for validating the results in this paper using the *SUGAR* library. It is loaded by adding the library to the MATLAB path with *addpath('SUGAR-master');*. The script then calls a custom helper function (*initialize_sugar*) which sets up the algebra using *GA([N, 0, 0]);*, captures the vector basis elements (e.g., *e1*, *e2*) into a cell array, and explicitly captures the scalar basis *e0*. It then prompts the user for a vector string, such as *'Enter the voltage vector (e.g., 1*e1 + 0.8*e3):'*. This string is processed by other custom helper functions (e.g., *parse_sugar_string_to_coeffs*) to build the final multivector object. The library uses operator overloading for the key operations needed by the script: *u * v* (Geometric Product), *length(u)* (Magnitude/Norm), and *inv(u)* (Inverse). The scalar basis *e0* is actively used, for instance, in constructing the full rotoflex operator (*Theta_s = (ks * sugar.e0) * Rs*). When run, the script compares all three methods (GA-Real, Phasor, GA-SUGAR) and prints the resulting output vectors, the error norm, and the calculated Flextance (k) and Rotance ($\boldsymbol{R}$) multivectors.

H.2.3 Guide for GA-FuL Library

Using *GA-FuL* requires interfacing with .NET, which has a different syntax. The process involves loading the .NET assembly using the *NET.addAssembly(...)* command. An instance of the GA processor object, referred to as *ga*, is then created for the specified dimension. A key difference from other libraries is the vector input format. Multivectors are defined using a string representation with 0-based indices, for example, “$1 < 0 > +0.8 < 2 >$” (which corresponds to $1\boldsymbol{\sigma}_1 + 0.8\boldsymbol{\sigma}_3$). This string is passed directly to the library’s native parsing method: *mv_obj = ga.Parse(string_input);*. Operations are subsequently called as methods on the resulting multivector objects. The primary methods used for the paper’s validation include *obj.Gp(other_obj)* (Geometric Product), *obj.Norm()* (Magnitude/Norm), *obj.NormSquared()* (Squared Norm), and *obj.Divide(...)* (Division).

H.3 Computational Performance Benchmark

To quantify the computational efficiency of the proposed GA-Real (Native) algorithm against the classical Phasor (Native Complex) method, a comprehensive benchmark was executed using the *Comparative_times_GAful.m* script.

This script is designed to run all four possible circuit configurations (Series u→i, Series i→u, Parallel i→u, and Parallel u→i) sequentially and measure the pure algorithmic execution time, isolating it from library or parsing overhead.

H.3.1 Benchmark Parameters

For this test, several common parameters were used. The circuit parameters were set to $R = 1\,\Omega$, $L = 1\,H$, $C = 1\,F$, and $\omega = 1\,\mathrm{rad/s}$. The benchmark settings involved testing harmonics (N) from 1 to 25, in steps of 5 (1, 6, 11, 16, 21, 25), with 200 measurements per point (b) and 10,000 iterations per measurement. The input signal generation for both voltage and current used a fundamental seed vector of $1.0\boldsymbol{\sigma}_1 + 1.0\boldsymbol{\sigma}_3$ (i.e., $A_1 = 1.0, A_2 = 1.0$), and a harmonic decay factor of 10% (0.1) was applied for subsequent harmonics.

The script measures the average execution time and standard deviation for each data point (N) by calling MATLAB’s *timeit* function on the core calculation loops.

H.3.2 Benchmark Results: Plot

The results are presented in Fig. 1, which shows the performance across all four cases in a 2x2 subplot layout. The logarithmic scale on the Y-axis highlights the order-of-magnitude difference in speed.

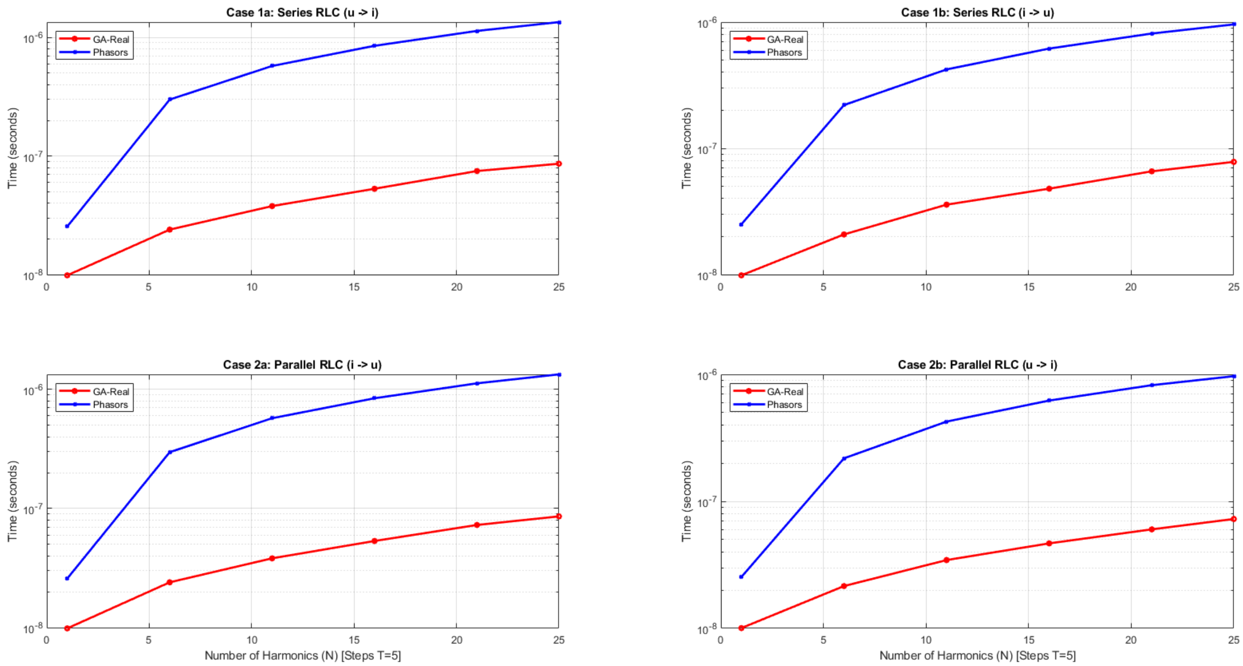

Figure 1: Computational benchmark results for all four RLC cases, comparing the native GA-Real algorithm (red) against the native Phasor (blue) algorithm. Test parameters: $N_{max} = 25$, $T = 5$, $R = L = C = \omega = 1$.

H.3.3 Benchmark Results: Data Tables

The following tables provide the raw data from the benchmark, showing the mean time and standard deviation (in parentheses) for each test case. All times are in seconds.

Table 7: RESULTS TABLE: Case 1a: Series (u $\to$ i)

N	Phasor Time (s)	GA-Real Time (s)
1	2.551e-08 (1.03e-09)	9.907e-09 (5.14e-10)
6	2.995e-07 (1.30e-08)	2.404e-08 (4.84e-09)
11	5.745e-07 (4.97e-09)	3.793e-08 (4.60e-10)
16	8.494e-07 (6.72e-09)	5.310e-08 (7.03e-10)
21	1.134e-06 (8.83e-09)	7.488e-08 (8.58e-10)
25	1.345e-06 (1.24e-08)	8.629e-08 (1.96e-09)

Table 8: RESULTS TABLE: Case 1b: Series (i $\to$ u)

N	Phasor Time (s)	GA-Real Time (s)
1	2.506e-08 (1.06e-09)	9.879e-09 (7.31e-10)
6	2.204e-07 (6.09e-09)	2.083e-08 (3.02e-09)
11	4.220e-07 (4.84e-09)	3.585e-08 (5.03e-10)
16	6.172e-07 (5.91e-09)	4.797e-08 (6.31e-10)
21	8.121e-07 (7.01e-09)	6.580e-08 (1.54e-09)
25	9.593e-07 (7.42e-09)	7.823e-08 (8.83e-10)

Table 9: RESULTS TABLE: Case 2a: Parallel (i $\rightarrow$ u)

N	**Phasor Time (s)**	**GA-Real Time (s)**
1	2.585e-08 (1.10e-09)	9.870e-09 (8.13e-10)
6	2.972e-07 (1.38e-08)	2.404e-08 (7.52e-09)
11	5.725e-07 (5.28e-09)	3.821e-08 (5.63e-10)
16	8.419e-07 (8.30e-09)	5.341e-08 (6.14e-10)
21	1.123e-06 (1.34e-08)	7.279e-08 (1.67e-09)
25	1.336e-06 (1.03e-08)	8.592e-08 (1.18e-09)

Table 10: RESULTS TABLE: Case 2b: Parallel (u $\rightarrow$ i)

N	**Phasor Time (s)**	**GA-Real Time (s)**
1	2.538e-08 (1.00e-09)	1.006e-08 (7.11e-10)
6	2.183e-07 (9.60e-09)	2.153e-08 (6.71e-09)
11	4.248e-07 (3.63e-09)	3.450e-08 (8.61e-10)
16	6.221e-07 (5.24e-09)	4.669e-08 (5.43e-10)
21	8.218e-07 (7.61e-09)	6.020e-08 (8.02e-10)
25	9.677e-07 (7.11e-09)	7.266e-08 (2.45e-09)

H.3.4 Analysis of Results

The results from both the plot and the data tables consistently show that the GA-Real (Native) algorithm (red line) is approximately an order of magnitude faster than the Phasor (Native Complex) algorithm (blue line).

This validates that the GA-derived algorithm, when implemented natively using real arithmetic, is not only conceptually unified but also algorithmically superior in terms of computational speed for this problem.

It is also noted that the performance of the "i $\rightarrow$ u" (Case 1b) and "u $\rightarrow$ i" (Case 2b) calculations are slightly faster than their counterparts. This is because these cases (see functions *loop_ga_1b* and *loop_ga_2b* in the script) do not require a division by the denominator ($R^2 + X_k^2$ or $G^2 + B_k^2$) inside the loop, making them computationally simpler than the cases that do require this division (1a and 2a).

I Appendix I: Additional Methodological Clarifications

I.1 Note on Operator Units

As stated in Section III.A of the main paper, the units of the rotoflex operator $\boldsymbol{\Theta}$ are determined entirely by the flextance k. The rotance $\boldsymbol{R}$ is a pure geometric operator, derived from the product of two normalized (dimensionless) vectors, and is therefore inherently dimensionless. Consequently, for the series rotoflex $\boldsymbol{\Theta}_s = k_s \boldsymbol{R}_s$, the flextance $k_s = \|\boldsymbol{i}\|/\|\boldsymbol{u}\|$ has units of Amperes/Volts. Therefore, the units of both k_s and $\boldsymbol{\Theta}_s$ are Siemens (S). Conversely, for the parallel rotoflex $\boldsymbol{\Theta}_p = k_p \boldsymbol{R}_p$, the flextance $k_p = \|\boldsymbol{u}\|/\|\boldsymbol{i}\|$ has units of Volts/Amperes. Thus, the units of both k_p and $\boldsymbol{\Theta}_p$ are Ohms (Ω).

I.2 Note on Reciprocity of Rotoflex Operators

In the main paper, the two primary forms of the solution are presented: (1) for a series circuit, $\boldsymbol{i} = \boldsymbol{\Theta}_s \boldsymbol{u}$, and (2) for a parallel circuit, $\boldsymbol{u} = \boldsymbol{\Theta}_p \boldsymbol{i}$. It is critical to note that $\boldsymbol{\Theta}_p$ is not the inverse of $\boldsymbol{\Theta}_s$ (i.e., $\boldsymbol{\Theta}_p \neq \boldsymbol{\Theta}_s^{-1}$). These are two distinct operators that solve two physically different problems. $\boldsymbol{\Theta}_s$ describes the admittance-like transformation for a series RLC circuit, whereas $\boldsymbol{\Theta}_p$ describes the impedance-like transformation for a parallel RLC circuit. The inverse operators, as defined in Eq. (13) of the main paper (e.g., $\boldsymbol{\Theta}_s^{-1}$), simply allow for solving the same circuit in the reverse direction (e.g., finding $\boldsymbol{u}$ from $\boldsymbol{i}$ in a series circuit).

J Appendix J: General Rotor Formulation

This appendix contains the general formulas for the direct rotor (e.g., $\boldsymbol{R}_s = \hat{\boldsymbol{\imath}}\hat{\boldsymbol{u}}$) for both series and parallel RLC loads, as developed in the supplementary analysis.

J.1 Series RLC Circuit Formulas

J.1.1 Series Direct Rotor ($\boldsymbol{R}_s = \hat{\boldsymbol{\imath}}\hat{\boldsymbol{u}}$) for a Series RLC Load

This operator corresponds to the admittance-like transformation $\boldsymbol{i} = \boldsymbol{\Theta}_s \boldsymbol{u}$. (Note: This is Equation (24) from the main paper).

$$\boldsymbol{R}_s = \left(\frac{\sum_{k=1}^{h} \left(\frac{(U_{kx}R - U_{ky}X_k)\boldsymbol{\sigma}_{2k-1} + (U_{kx}X_k + U_{ky}R)\boldsymbol{\sigma}_{2k}}{R^2 + X_k^2} \right)}{\sqrt{\sum_{k=1}^{h} \left(\frac{(U_{kx}R - U_{ky}X_k)^2 + (U_{kx}X_k + U_{ky}R)^2}{\left(R^2 + X_k^2\right)^2} \right)}} \right) \left(\frac{\sum_{k=1}^{h} \left(U_{kx}\boldsymbol{\sigma}_{2k-1} + U_{ky}\boldsymbol{\sigma}_{2k}\right)}{\sqrt{\sum_{k=1}^{h} \left(U_{kx}^2 + U_{ky}^2\right)}} \right) \tag{66}$$

where $X_k = k\omega L - 1/(k\omega C)$.

J.1.2 Inverse Series Direct Rotor ($\boldsymbol{R}_s^{-1} = \hat{\boldsymbol{u}}\hat{\boldsymbol{\imath}}$) for a Series RLC Load

This operator corresponds to the impedance-like transformation $\boldsymbol{u} = \boldsymbol{\Theta}_s^{-1}\boldsymbol{i}$. (Note: This formula was previously mislabeled as $\boldsymbol{R}_p$ in Eq. (55)).

$$\boldsymbol{R}_s^{-1} = \left(\frac{\sum_{k=1}^{h} \left(\frac{(I_{kx}R + I_{ky}X_k)\boldsymbol{\sigma}_{2k-1} + (I_{ky}R - I_{kx}X_k)\boldsymbol{\sigma}_{2k}}{R^2 + X_k^2} \right)}{\sqrt{\sum_{k=1}^{h} \left(\frac{(I_{kx}R + I_{ky}X_k)^2 + (I_{ky}R - I_{kx}X_k)^2}{\left(R^2 + X_k^2\right)^2} \right)}} \right) \left(\frac{\sum_{k=1}^{h} \left(I_{kx}\boldsymbol{\sigma}_{2k-1} + I_{ky}\boldsymbol{\sigma}_{2k}\right)}{\sqrt{\sum_{k=1}^{h} \left(I_{kx}^2 + I_{ky}^2\right)}} \right) \tag{67}$$

where $X_k = k\omega L - 1/(k\omega C)$.

J.2 Parallel RLC Circuit Formulas

J.2.1 Parallel Direct Rotor ($\boldsymbol{R}_p = \hat{\boldsymbol{u}}\hat{\boldsymbol{\imath}}$) for a Parallel RLC Load

This operator corresponds to the impedance-like transformation $\boldsymbol{u} = \boldsymbol{\Theta}_p \boldsymbol{i}$.

$$\boldsymbol{R}_p = \left(\frac{\sum_{k=1}^{h} \left(\frac{(I_{kx}G - I_{ky}B_k)\boldsymbol{\sigma}_{2k-1} + (I_{kx}B_k + I_{ky}G)\boldsymbol{\sigma}_{2k}}{G^2 + B_k^2} \right)}{\sqrt{\sum_{k=1}^{h} \left(\frac{(I_{kx}G - I_{ky}B_k)^2 + (I_{kx}B_k + I_{ky}G)^2}{\left(G^2 + B_k^2\right)^2} \right)}} \right) \left(\frac{\sum_{k=1}^{h} \left(I_{kx}\boldsymbol{\sigma}_{2k-1} + I_{ky}\boldsymbol{\sigma}_{2k}\right)}{\sqrt{\sum_{k=1}^{h} \left(I_{kx}^2 + I_{ky}^2\right)}} \right) \tag{68}$$

where $G = 1/R$ and $B_k = k\omega C - 1/(k\omega L)$.

J.2.2 Inverse Parallel Direct Rotor ($\boldsymbol{R}_p^{-1} = \hat{\boldsymbol{\imath}}\hat{\boldsymbol{u}}$) for a Parallel RLC Load

This operator corresponds to the admittance-like transformation $\boldsymbol{i} = \boldsymbol{\Theta}_p^{-1}\boldsymbol{u}$.

$$\boldsymbol{R}_p^{-1} = \left(\frac{\sum_{k=1}^{h} \left((GU_{kx} + B_kU_{ky})\boldsymbol{\sigma}_{2k-1} + (GU_{ky} - B_kU_{kx})\boldsymbol{\sigma}_{2k}\right)}{\sqrt{\sum_{k=1}^{h} \left((GU_{kx} + B_kU_{ky})^2 + (GU_{ky} - B_kU_{kx})^2\right)}} \right) \left(\frac{\sum_{k=1}^{h} \left(U_{kx}\boldsymbol{\sigma}_{2k-1} + U_{ky}\boldsymbol{\sigma}_{2k}\right)}{\sqrt{\sum_{k=1}^{h} \left(U_{kx}^2 + U_{ky}^2\right)}} \right) \tag{69}$$

where $G = 1/R$ and $B_k = k\omega C - 1/(k\omega L)$.