Final Project

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1 Project Overview

This is the project

2 Infrastructure Interdependence Analysis

Question 1

Given

$$x_i = o_i + f_i = \sum_j x_{ij} + f_i \tag{1}$$

$$x_{ij} = a_{ij}x_j \tag{2}$$

we obtain:

$$x_i = \sum_j a_{ij}x_j + f_i = a_i \boldsymbol{x} + f_i,$$

where a_i is a $1 \times i$ matrix and x is an $i \times 1$ vector. Similarly, for all cases of i, we obtain the matrix equation:

$$x = Ax + f, \tag{3}$$

where x is an $i \times 1$ vector, A is a $i \times j$ matrix and f is a $i \times 1$ vector. Note that A must be a square matrix, so $j = i$; its dimensions are $i \times i$.

Question 2

Table 2 in the given instructions sheet presents matrix A, which is the matrix of influence coefficients a_{ij}. These coefficients should be understood as the fraction of inoperability transmitted by the jth infrastructure to the ith infrastructure.

The last row of matrix A corresponds to the $i = 10$ infrastructure: satellite communication and navigation. Thereby, we must understand each coefficient a_{10j} for all j to be the fraction of inoperability transmitted by the jth infrastructure to the satellite communication and navigation infrastructure (10th).

We observe that the coefficients a_{10j} for all j are 0. This means that the failure of any j infrastructure does not transmit inoperability to the satellite communication and navigation infrastructure. On the other hand, all of the coefficients a_{i10} for all $i \neq 10$ are nonzero. In other words, the operability of the satellite communication and navigation infrastructure is independent of the operability of the other infrastructure, while the operability of the other infrastructure is dependent on the operability of the satellite and communication infrastructure.

This assumption seems to be reasonable for a $6 - 12$ hour outage. One can expect satellites to be self-sufficient in terms of energy consumption and maneuverability, but the infrastructure on the Earth to rely heavily on the data provided by the satellite and communication systems. A satellite may be able to operate on its own during a $6-12$ hour outage of the other infrastructure, while the remaining infrastructure is likely to fail during a $6-12$ hour outage of the satellite and communication infrastructure.

Question 3

The dependency index of infrastructure i, γ_i is defined as:

$$\gamma_i = \frac{1}{n-1} \sum_{j \neq i} a_{ij} \ (row \ summation). \tag{4}$$

The sum of the a_{ij} coefficients reveals the total direct damage on infrastructure i transmitted from the damage of each infrastructure j such that $j \neq i$. By dividing the sum by $n - 1$ we compute the index γ_i, which indicates the average damage on infrastructure i from any other infrastructure.

In a sense, this index is a measure of the dependence of an infrastructure on the operability of other infrastructure, where a high value indicates a high dependency and a low value indicates a low dependency. The larger the index, the more direct damage infrastructure i receives from the inoperability of the entire infrastructure system.

Likewise, the influence index of infrastructure j, δ_j is defined as:

$$\delta_j = \frac{1}{n-1} \sum_{i \neq j} a_{ij} \ (column \ summation). \tag{5}$$

The sum of the a_{ij} coefficients reveals the total direct damage of infrastructure j transmitted to the damage of each infrastructure i such that $i \neq j$. By dividing the sum by $n-1$ we compute the index δ_j, which indicates the average influence of infrastructure j has on any other infrastructure.

In a sense, this index is a measure of the influence an infrastructure has on the operability of other infrastructure, where a high value indicates a high influence and a low value indicates a low influence. The larger the index, the greater the direct impact the inoperability of infrastructure j has on the entire infrastructure system.

Question 4

Starting from Equation 3, we can compute the following:

$$x = Ax + f$$
$$Ix = Ax + f$$
$$(I - A)x = f$$
$$x = (I - A)^{-1}f$$

We can express the matrix $(I - A)^{-1}$ as matrix S, finally obtaining the solution in the form of:

$$x = Sf \tag{6}$$

Note that the information provided in Table 2 corresponds to matrix A. To compute the S in Equation 6 we need to follow our definition of S, $S = (I - A)^{-1}$. The matrix A must be a square matrix with coefficients between 0 and 1 and the $I - A$ matrix must invertible.

Both matrix A and matrix S are composed of i rows and j columns, where $i = j$. The elements of matrix A, a_{ij}, represent the first damage propagation step from infrastructure j on infrastructure i. The elements of matrix S, s_{ij}, represent the total damage propagation from infrastructure j on infrastructure i. This is, the total damage propagation from infrastructure j on infrastructure i that the system converges to after a series of recursive damage propagation steps.

Question 5

The matrix S is an indicator of the total impact of an external shock on the entire infrastructure system. In particular, every s_{ij} reveals the infinite propagation of the damage of an infrastructure sector j to infrastructure sector i. Every s_{ij} should be greater than or equal to the corresponding a_{ij} from matrix A. This is because s_{ij} includes the direct damage captured by a_{ij} and the subsequent inoperability propagation of failure, as explained in Question 4.

The matrix S computed after the matrix A found in Table 2 of the instructions sheet can be found in Table 1.

Table 1: Matrix S for a 6-12 hr outage.

Sector Id	1	2	3	4	5	6	7	8	9	10
1	1.004	0.237	0.325	0.510	0.038	0.029	0.016	0.027	0.053	0.319
2	0.001	1.012	0.013	0.027	0.002	0.003	0.001	0.003	0.180	0.005
3	0.003	0.124	1.005	0.126	0.003	0.006	0.003	0.003	0.027	0.009
4	0.006	0.089	0.017	1.008	0.005	0.004	0.004	0.002	0.020	0.008
5	0.005	0.061	0.013	0.027	1.001	0.006	0.008	0.008	0.019	0.022
6	0.002	0.263	0.111	0.131	0.007	1.002	0.008	0.007	0.049	0.008
7	0.004	0.118	0.035	0.110	0.008	0.004	1.001	0.004	0.029	0.011
8	0.009	0.535	0.114	0.087	0.052	0.023	0.022	1.003	0.097	0.016
9	0.002	0.036	0.011	0.009	0.005	0.000	0.002	0.005	1.006	0.006
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

From Table 1 we can observe that $s_{12} = 0.237$ is smaller than $s_{82} = 0.535$. This indicates that sector 2 has a bigger total impact on sector 8 than on sector 1 due to the propagation of

damage of sector 2 to the entire infrastructure system.

Comparing the case for the elements of matrix A, we can observe that a_{101} is 0 and s_{101} remained 0. This means that there is neither direct propagation of inoperability of sector 1 (air transportation) to sector 10 (satellite communication and navigation) during a 6-12 hr outage, nor total damage propagation.

However, a_{22} is 0 and the corresponding s_{22} is 1.012. This reveals that there is no direct damage propagation from the infrastructure on itself (based on the given assumption), yet there is a total damage on sector 2 due to the propagation of damage on the entire infrastructure system.

Question 6

As explained in Questions 4 and 5, matrix S encompasses matrix A. Matrix A captures the direct propagation of inoperability of the entire system whereas matrix S represents the infinite propagation of damage on the system. Therefore, it is easier to collect and to model the first tier of damage for every combination of ij pairs than to model the infinite propagation of damage.

From a data collection perspective, Setola et al. explain that "infrastructure owners and operators generally have a good idea of how much their infrastructures depend on the resources provided directly by other infrastructures" (2009). However, assessing further propagation of damage is more challenging because of the limited information that the owners and operators have on the "implications of higher-order (inter)dependencies" (Setola et al., 2009).

Furthermore, despite the complexity of modeling matrix S, there is a straightforward approach to find S once we have matrix A, as shown in Question 1. This further emphasizes on why it is easier to obtain the elements in matrix A compared to the ones in S.

Question 7

The overall dependency index of infrastructure i, $\overline{\gamma}_i$ is defined as:

$$\overline{\gamma}_i = \frac{1}{n-1} \sum_{j \neq i} s_{ij} \ (row \ summation). \tag{7}$$

The sum of the s_{ij} coefficients reveals the overall damage on infrastructure i transmitted from the damage of each infrastructure j ??such that $j \neq i$??, as a result of the higher-order interdependencies in the infrastructure system. By dividing the sum by $n-1$ we compute the index $overline\gamma_i$, which indicates the average overall damage on infrastructure i from any other infrastructure.

In a sense, we define this index as a measure of the vulnerability of an infrastructure sector within the entire infrastructure system, where a high value indicates a high vulnerability and a low value indicates a low vulnerability. The larger the index, the bigger the total damage infrastructure i receives from the inoperability of the entire infrastructure system.

Likewise, the influence index of infrastructure j, $\overline{\delta}_j$ is defined as:

$$\overline{\delta}_j = \frac{1}{n-1} \sum_{i \neq j} s_{ij} \ (column \ summation). \tag{8}$$

The sum of the s_{ij} coefficients reveals the overall damage transmitted from infrastructure j to each infrastructure i ?? such that $i \neq j$??, as a result of the higher-order interdependencies in the infrastructure system. By dividing the sum by $n-1$ we compute the index $\overline{\delta}_j$, which indicates the average total influence that infrastructure j has on any other infrastructure.

In a sense, we define this index as a measure of the criticality of an infrastructure sector within the infrastructure system, where a high value indicates a high criticality and a low value indicates a low criticality. The larger the index, the greater the total impact the inoperability of infrastructure j has on the entire infrastructure system, so the more critical this sector j is.

Question 8

By comparing the coefficients of column 2 of matrix A for the electricity sector (Table 2 of the instructions sheet), we can observe that electricity has the biggest direct influence on the sector of fuel and petroleum grid ($a_{82} = 0.500$). Moreover, it appears that the electricity sector directly depends the most on the sector of natural gas, based on the coefficients of row 2 of matrix A ($a_{29} = 0.178$).

As for the sector of natural gas, we can observe that this infrastructure has the greatest influence on the electricity sector ($a_{29} = 0.178$) while also directly depending on the electricity the most ($a_{92} = 0.030$). These coefficients reveal the significant dependency between the sectors of electricity and natural gas.

Besides the observations for each sector, we can see that the column coefficients (column 2)for the electricity sector are generally higher than the row values (row 2). This means that the electricity sector has a larger direct influence on the other sectors compared to its direct dependence on the infrastructure system. The natural gas sector has a low direct dependency, shown by having 8 of 9 row coefficients (excluding itself) be less than 0.01.In addition, the dependency and influence coefficients for the electricity sector are generally higher than the ones for natural gas, which means that the electricity sector is more critical and dependent more by the damage of other sectors. From a practical perspective, these observations can be realistic because the natural gas is a fuel source with other substitutes in the market, whereas electricity has no feasible substitutes in a 6-12 hour outage.

Question 9

The indices γ_i and δ_j have been calculated using a script and a class in MATLAB. The class includes the equations (5) and (6) provided in the instructions sheet as part of the public functions NAME. These functions are called within the script NAME to compute the indices γ_i and δ_j. **Note:** For the aforementioned script and class, please refer to the appendix. For the MATLAB code, please refer to the script entitled and the class entitled that are uploaded on compass 2g.

The indices γ_i and δ_j for every infrastructure sector are presented in the figures NUMBER AND NUMBER below.

From figure NUMBER, we observe that the air transportation sector (sector 1) has the highest dependency index ($\gamma_1 = 0.145$). This means that the air transportation sector receives the biggest direct damage from the inoperability of the entire infrastructure system. The sectors of fuel and petroleum grid and rail transportation are respectively the second and third most dependent infrastructures within the system ($\gamma_8 = 0.084$ and $\gamma_6 = 0.052$). These indices reveal that the transportation sector in general is highly dependent on other infrastructure sectors within the system and subsequently the first sector to be influenced or damaged. Other than
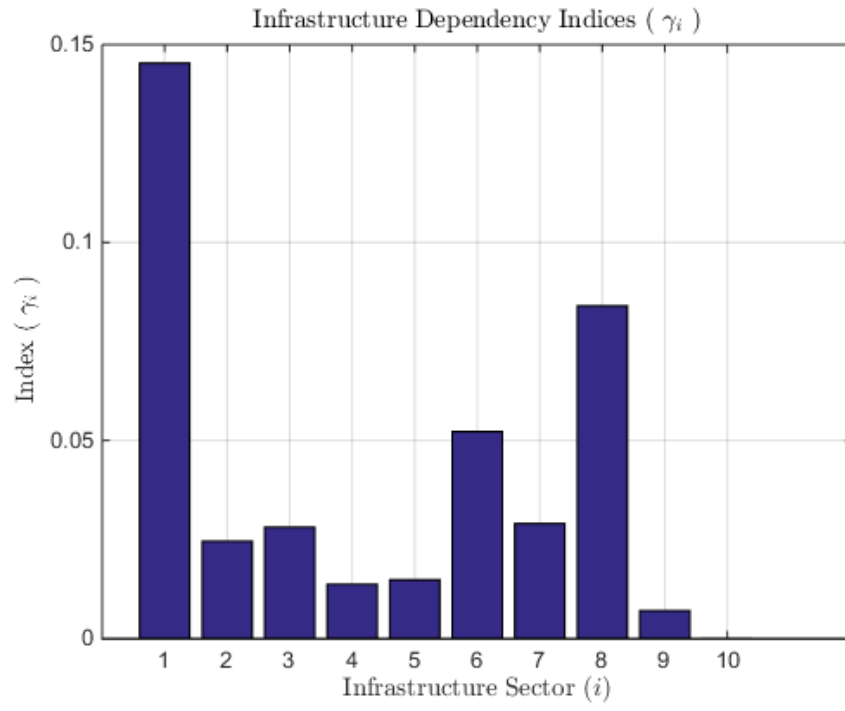
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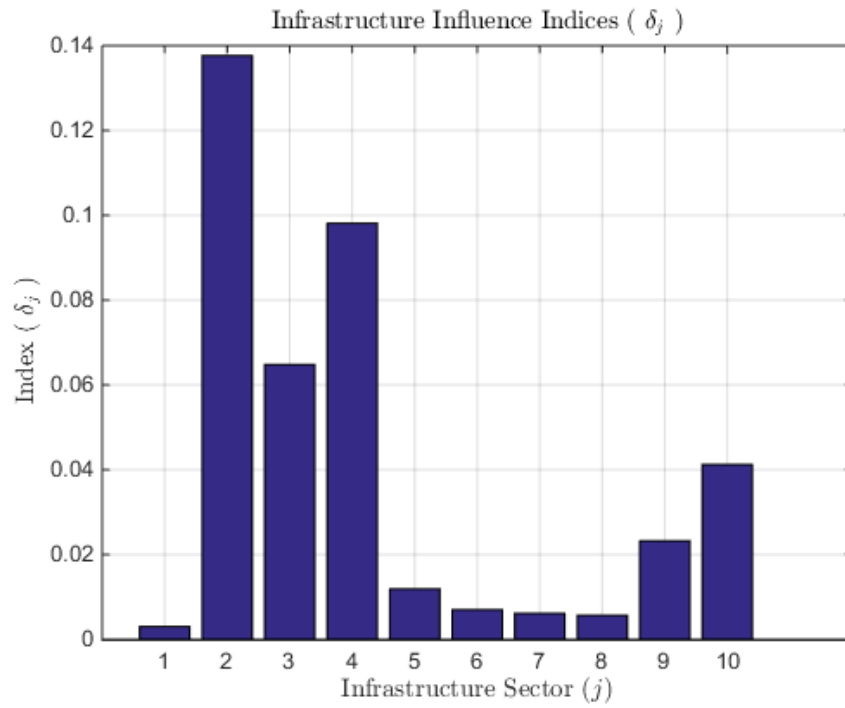Figure 1: Dependency index for all the sectors

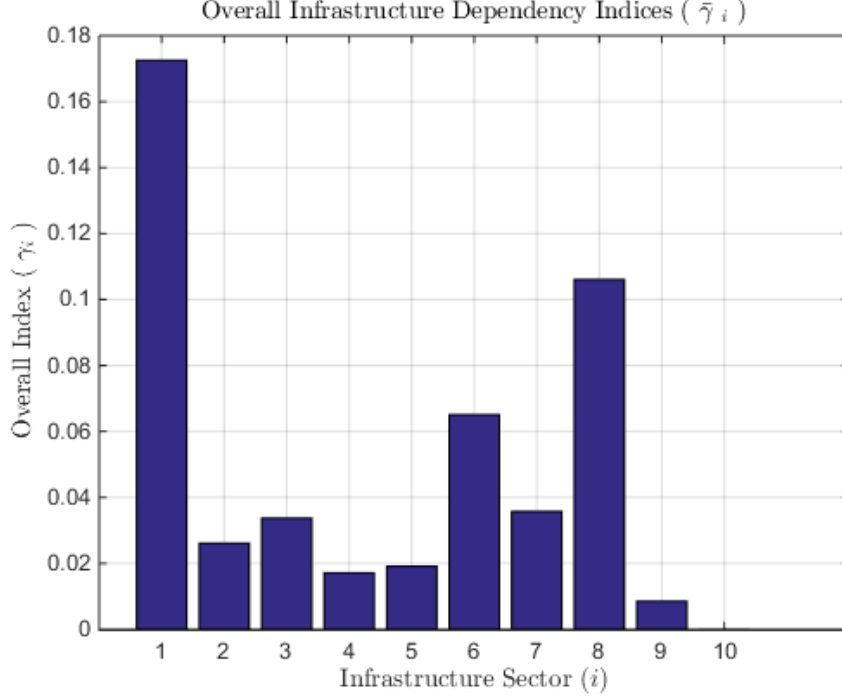Figure 2: Influence index for all the sectors

Figure 3: Overall dependency index for all the sectors

the previous three sectors, the remaining infrastructures have low direct dependency on the inoperability of other sectors as all of the remaining indices have a value below 0.03. Note that the satellite communication and navigation sector (sector 10) has an index of ($\gamma_1 0 = 0$) which indicates that sector 10 is not actually affected by the inoperability of any other infrastructure within the system.

As for the influence index, we observe that the electricity sector (sector 2) has the highest influence index ($\delta_2 = 0.138$). This means that the electricity sector transmits the biggest direct damage from its inoperability to the entire infrastructure system. The sectors of TLC wired and TLC wireless are respectively the second and third most influential infrastructures within the system ($\delta_4 = 0.098$ and $\delta_3 = 0.065$). Furthermore, the satellite communication and navigation sector has a relatively high influence index ($\delta_1 0 = 0.041$), which reveals that inoperability of sector 10 has a sizable effect on the infrastructure system. These indices reveal that the electricity and the communication sectors in general have the largest direct impact on other infrastructure sectors as a result of their inoperability. Other than the previous four sectors, the remaining infrastructures have low direct influence on the inoperability of other sectors as all of the remaining indices have a value below 0.025.

Question 10

Similar to Question 9, the indices $\overline{\gamma}_i$ and $\overline{\delta}_j$ have been calculated using a script and a class in MATLAB. **Note:** For the aforementioned script and class, please refer to the appendix. For the MATLAB code, please refer to the script entitled NAME and the class entitled NAME that are uploaded on compass 2g.

The indices $\overline{\gamma}_i$ and $\overline{\delta}_j$ for every infrastructure sector are presented in the figures NUMBER AND NUMBER below.

The comparison of the indices γ_j and $\overline{\gamma}_j$ further stresses on the difference
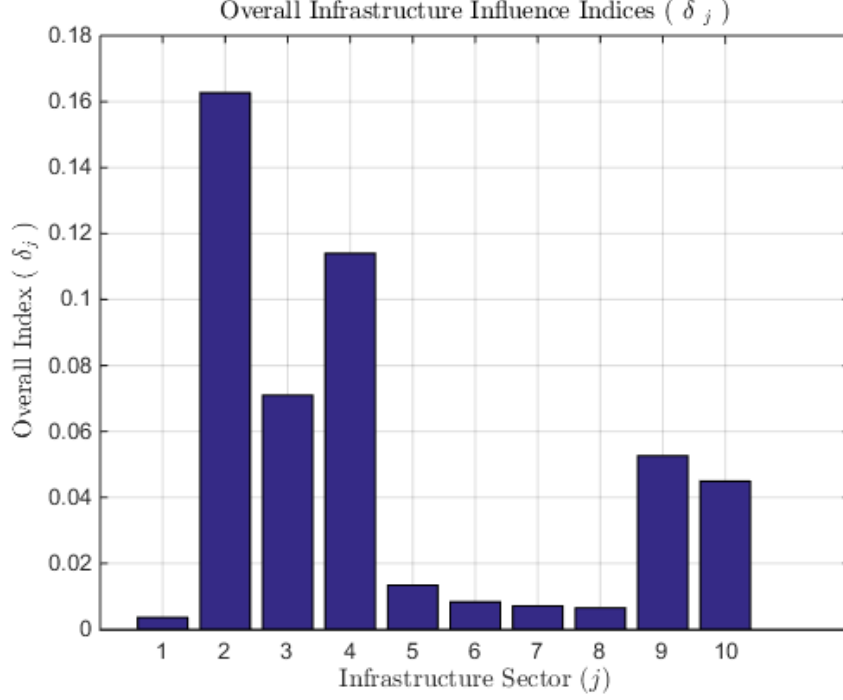
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Figure 4: Overall influence index for all the sectors

From figure NUMBER, we observe that the air transportation sector (sector 1) has the highest dependency index ($\gamma_1 = 0.145$). This means that the air transportation sector receives the biggest direct damage from the inoperability of the entire infrastructure system. The sectors of fuel and petroleum grid and rail transportation are respectively the second and third most dependent infrastructures within the system ($\gamma_8 = 0.084$ and $\gamma_6 = 0.052$). These indices reveal that the transportation sector in general is highly dependent on other infrastructure sectors within the system and subsequently the first sector to be influenced or damaged. Other than the previous three sectors, the remaining infrastructures have low direct dependency on the inoperability of other sectors as all of the remaining indices have a value below 0.03. Note that the satellite communication and navigation sector (sector 10) has an index of ($\gamma_10 = 0$) which indicates that sector 10 is not actually affected by the inoperability of any other infrastructure within the system.

As for the influence index, we observe that the electricity sector (sector 2) has the highest influence index ($\delta_2 = 0.138$). This means that the electricity sector transmits the biggest direct damage from its inoperability to the entire infrastructure system. The sectors of TLC wired and TLC wireless are respectively the second and third most influential infrastructures within the system ($\delta_4 = 0.098$ and $\delta_3 = 0.065$). Furthermore, the satellite communication and navigation sector has a relatively high influence index ($\delta_10 = 0.041$), which reveals that inoperability of sector 10 has a sizable effect on the infrastructure system. These indices reveal that the electricity and the communication sectors in general have the largest direct impact on other infrastructure sectors as a result of their inoperability. Other than the previous four sectors, the remaining infrastructures have low direct influence on the inoperability of other sectors as all of the remaining indices have a value below 0.025.

Add the S and bigger A and then refer to upcoming questions in with the tier stuff.

Sector 10 doesn't have an increase because it is already independent.

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Question 11

Traditionally, an electric grid is the network of "wires, substations, transformers, switches, and much more" that delivers electricity from the power-generating plants to the consumers (Energy.gov). On the other hand, a smart electric grid indicates that the conventional electric grid has become computerized. A grid becomes smart when "two-way digital communication and computer processing" have been introduced in the system. These incorporated technologies allow the electricity utilities to control their power generation by benefiting from better demand monitoring and management technologies (Energy.gov).

Despite the apparent benefits of having a smart grid, such as increased efficiency, better energy security, etc... the smart technologies would modify the dependencies of the electricity sector with other sectors in the infrastructure system. To identify the changes in the dependencies in the infrastructure system, we assessed whether a 6-12 hour outage of the electricity sector would increase or decrease the propagation of inoperability on every other sector in the system.

According to the website (WEBSITE), a smart grid will help integrate more large-scale renewable energy resources into the exisitng grid, as well as achieve a more efficient electricity generation and transmission. In our infrastructure system, a better integration of renewable energy resources translates into requiring less fossil fuels such fuel and petroleum, in addition to natural gas. This means that the electricity sector will be less influenced or dependent by the fuel and gas sector and by the natural gas sector (sectors 8 and 9), assuming that a smart grid would be fully functional. So in a 6-12 hour outage of sectors 8 and 9, the smart grid can ramp up the production of power from renewable sources in case there was no inventory of fossil fuel resources in the concerned plants.

Moreover,given that a smart grid fundamentally relies on the communications sector to achieve a more efficient power generation, the electricity sector will have an increased dependency on the TLC wireless sector, TLC wired, and the satellite communication and navigation sector (sectors 3,4, and 10). So the electricity sectors will be more influenced by any damage of these aforementioned sectors.

Furthermore, for the remaining sectors in the system, we considered that the dependencies with the electricity sector will not change for a 6-12 hour outage, whether this grid was either traditional or smart. For instance, the water management sector will be similarly influenced by a power outage of the electricity sector for 6-12 hours, and the electricity sector, smart or not, will be equally affected by a potential outage of the water sector for 6-12 hours.

To sum up, the actual coefficients of matrix A that will be change are:

- Increase in the coefficients a_{23}, a_{24} and a_{210}

- Decrease in the coefficients a_{28} and a_{29}

- No change in the remaining coefficients

Question 12
- TODO: Matlab code to add/decrease 10

Question 13

The snow storm in this problem describes an external shock (f) that impacts some elements of the infrastructure system. The effect causes damage to propagate throughout the infrastructure system. The resulting damage is calculated through Equation 6 with the results shown in the table below.

Table 2: Damage from a snow storm

Sector Id	1	2	3	4	5	6	7	8	9	10
Overall Damage (%)	38.96	10.30	1.64	1.29	1.08	52.81	1.51	6.83	0.45	0

The most damaged infrastructure was the rail transportation due to the external shock taking out 50% of the rail network, resulting in 52.81% damage to the sector. The least affected sector was the Satellite Communication and Navigation sector with no damage caused, followed by the Natural Gas sector which had 0.45% damage. Additional damage done to the air transportation sector, electricity sector, and rail transportation sector due to the propagation of damage through the infrastructure system resulted in further degradation of 3.96% for air transportation, 0.30% for electricity, and 2.81% for rail transportation. When only considering the damage propagated throughout the system, the most damaged infrastructure would be the fuel and petroleum grid at 6.83% degradation. The least damaged system does not change from the satellite communication and navigation sector but the second least damaged infrastructure would be the electricity sector at 0.30% rather than the natural gas sector at 0.45%.

Question 14

Matrix A is not an appropriate influence matrix to analyze the effects of a heavy snow storm. This is because matrix A contains many limitations inherent within the matrix itself.

One of the limitations within the matrix A is that the values in the matrix are for an outage of 6-12 hours and there is no guarantee that the snow storm will last within that time range. Though in the Setola et. al paper, there were additional matrices to account for the different lengths of outages; the matrix A given does not account for these differences

Another limitation of matrix A consist of how the data to create the values in the matrix were collected. The information was gathered through questionnaires given to experts within each sector asking about how an outage of another sector would influence the operability of their sector (Setola, 2009). There are no means to know the number of experts that were questioned and to objectively assess each answer since every response to the questionnaire was subjective and was given one value ranging from 0 to 0.5 depending on the response of the expert. There is also the inherent assumption that the experts know the direct influence each of the other sectors has on their sector.

Though the matrix A attempts to account for uncertainty from the answers to the questionnaire using fuzzy numbers, each expert had no idea on how their answers would be translated to a numerical value. Furthermore, the fuzzy numbers that was used to capture the uncertainty within the answers consisted of additional uncertainties about the reliability of the expert, and the confidence level of the expert which again assigns values to subjective answers ($+/-0.005$ to $+/-0.02$) (Setola, 2009).

Question 15

Question 4 presented the analytical solution $x = Sf$ for the overall propagation of damage of the infrastructure system. This solution was obtained using properties of linear algebra.

This solution can be supported by studying the step-by-step propagation of damage of an infrastructure system. The damage propagation starts from zero-damage state, continues by absorbing the external impact f, and recursively computes additional damage derived from the preceding step (the additional damage from the previous damage). After a number of damage propagation steps, the state of the system should match that found using the equation presented in Question 4.

A recursive function was implemented in MATLAB for this purpose. The source code can be found in the Appendix and in the attached files, along with the documentation that gives instructions for running it. Figure 5 presents 10 tiers of damage propagation, starting from the 0_{th} tier (initial conditions with no damage) to the 10_{th} tier. Similarly, Figure 6 presents the incremental damage on each sector after each damage propagation step.

Note that the damage propagation steps presented in Figure 5 converge to the same values found in Question 13 after a k number of steps. This is further explained by the damage increments after a certain number of propagation steps approaching zero, as seen in Figure 6. The system damage at $k = 0$ is zero, but approaches to the solution from $x = Sf$ after a number of tiers, which may vary from sector to sector. That is, some sectors may converge to faster than others. Initially the damage increments are generally large, but approach zero after the solution from $x = Sf$ is approached. This is not a coincidence, it is purely a property of linear algebra: $(I - A)^{-1}f = Sf$ is the damage propagation at an infinite number of propagation steps.
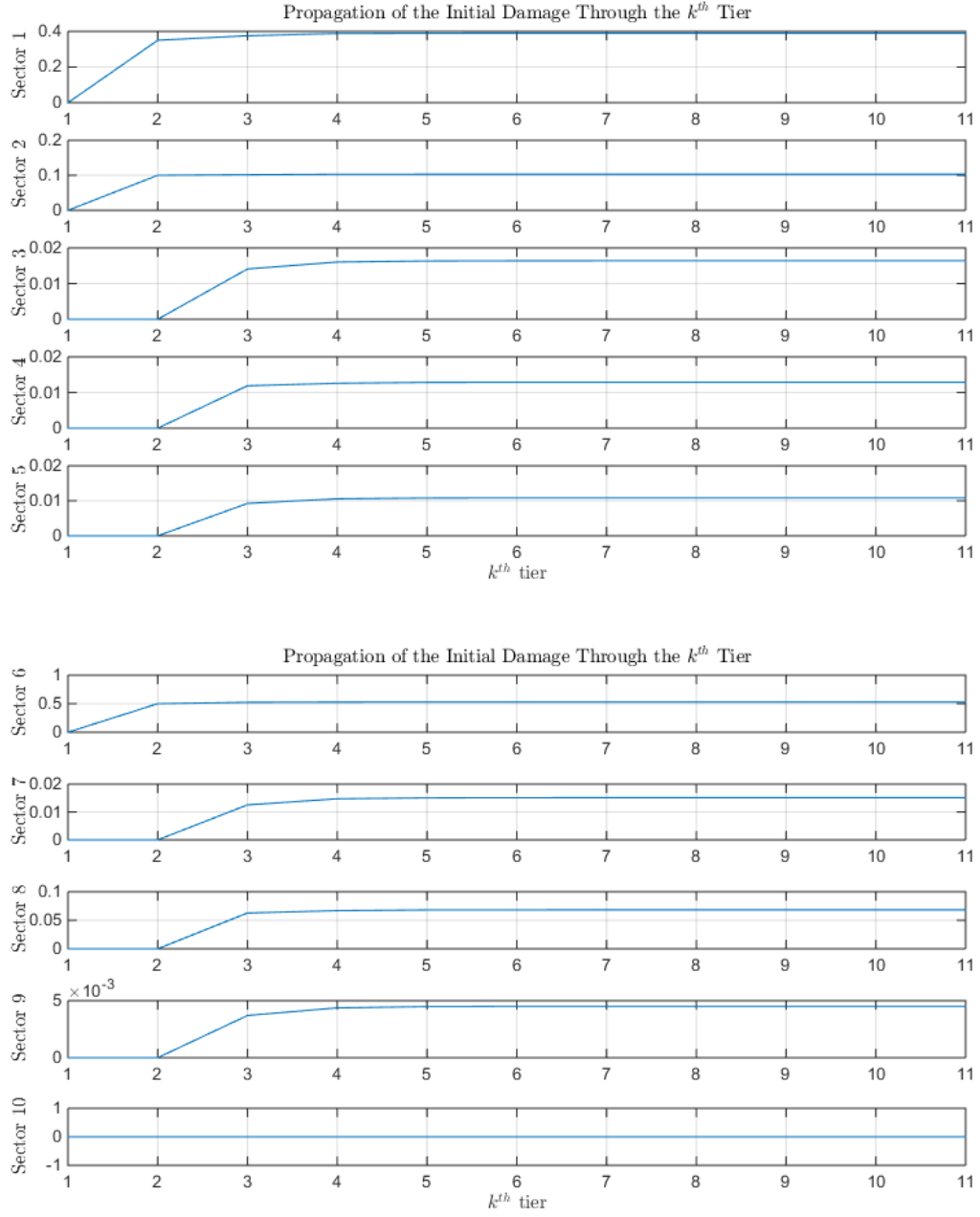
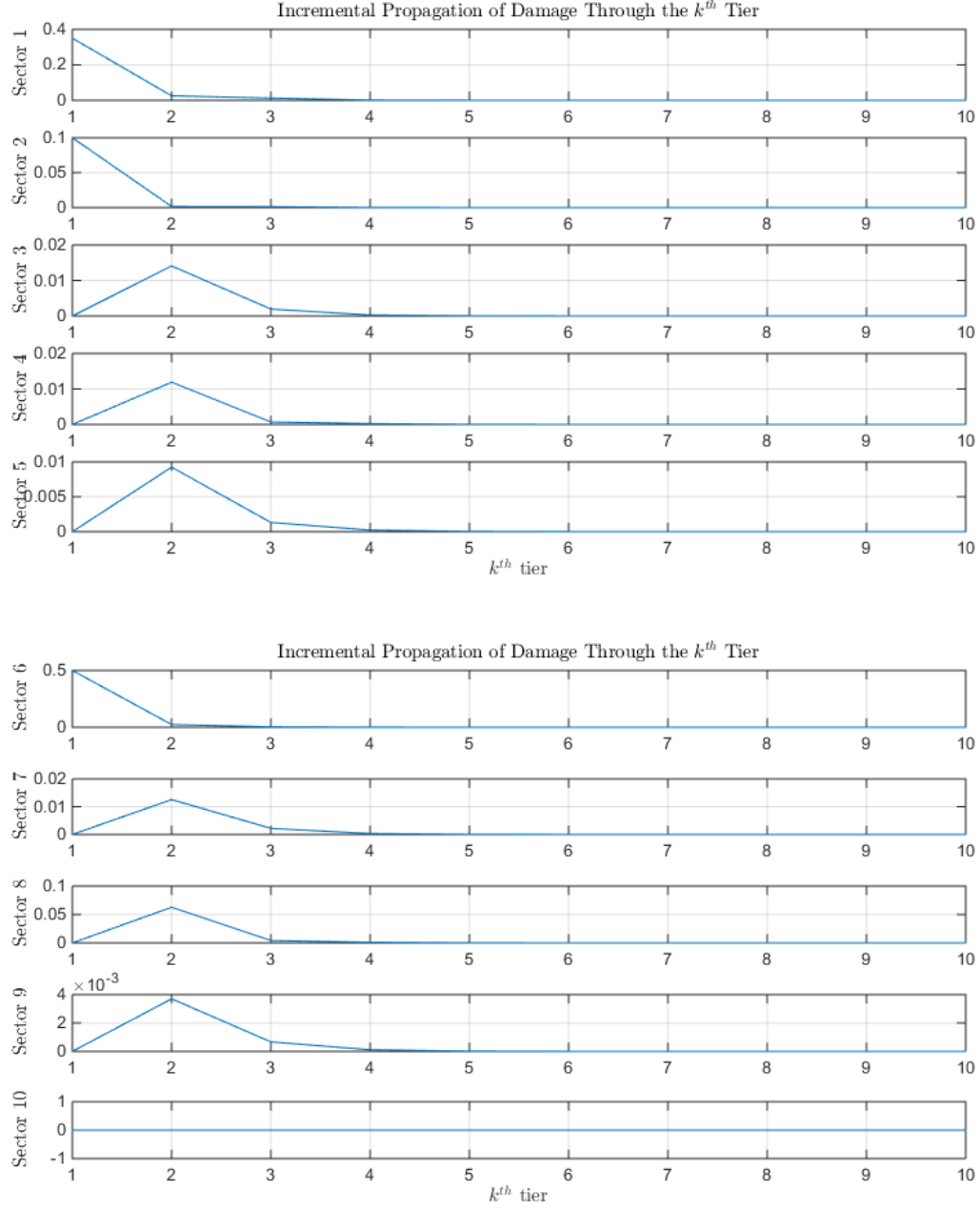Figure 5: Ten tiers of damage propagation for sectors 1 through 10.

Figure 6: Ten tiers of incremental damage propagation for sectors 1 through 10.

Question 16

One thousand replications of Monte Carlo simulation were performed for modeling uncertainty in matrix A. Specifically, noise was added to the elements of matrix A using a uniform distribution that ranged from -15% to -15% of each value. That is, each element a_{ij} of matrix A was updated using

$$a_{ij} = a_{ij} \cdot (100\% - 15\%) + U(0,1) \cdot a_{ij} \cdot (30\%). \tag{9}$$

The results of the simulation can be found as histograms of 100 bins in Figure 11. Important statistical descriptors can be found in Figure 12 as box plots.
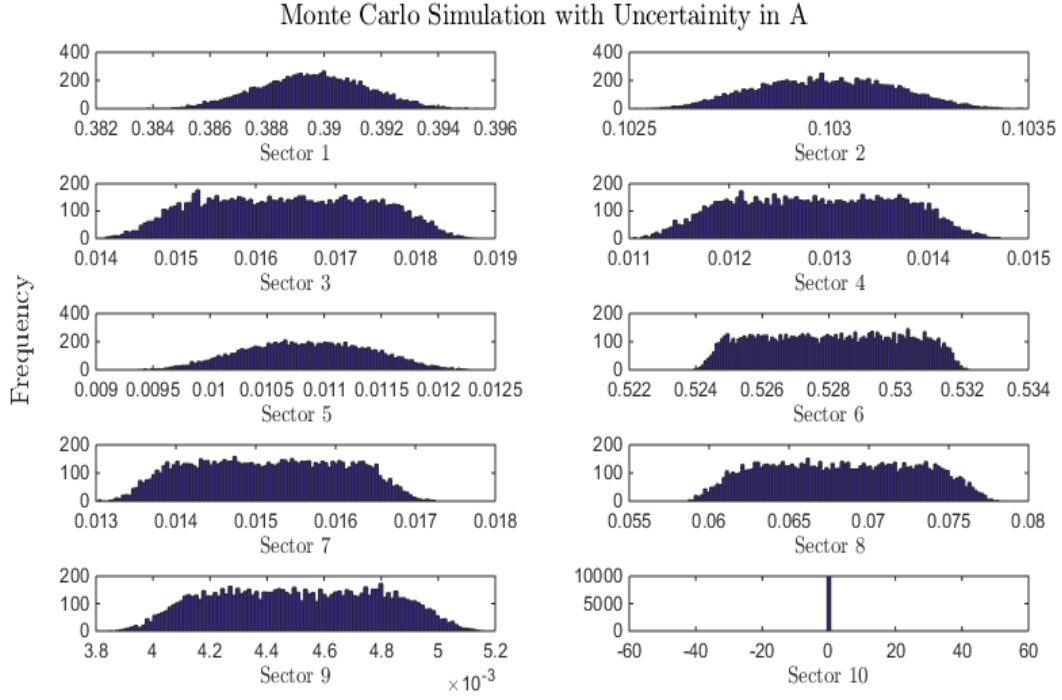
Figure 7: Histograms of Monte Carlo simulation on overall damage propagation for each sector with uncertainity in A.

From Figure 12 we observe that it is not trivial to determine whether the overall damage propagation follows a uniform distribution, a normal distribution, or some other distribution. This is expected, as the influence of the uncertainty in each sector may influence a given sector may vary.

Question 17

One thousand replications of Monte Carlo simulation were performed for modeling uncertainty in vector f. Specifically, noise was added to the elements of matrix f using a uniform distribution that ranged from -15% to -15% of each value using Equation 9.

The results of the simulation can be found as histograms (of 100 bins) in Figure **??** and box plots in Figure **??**.
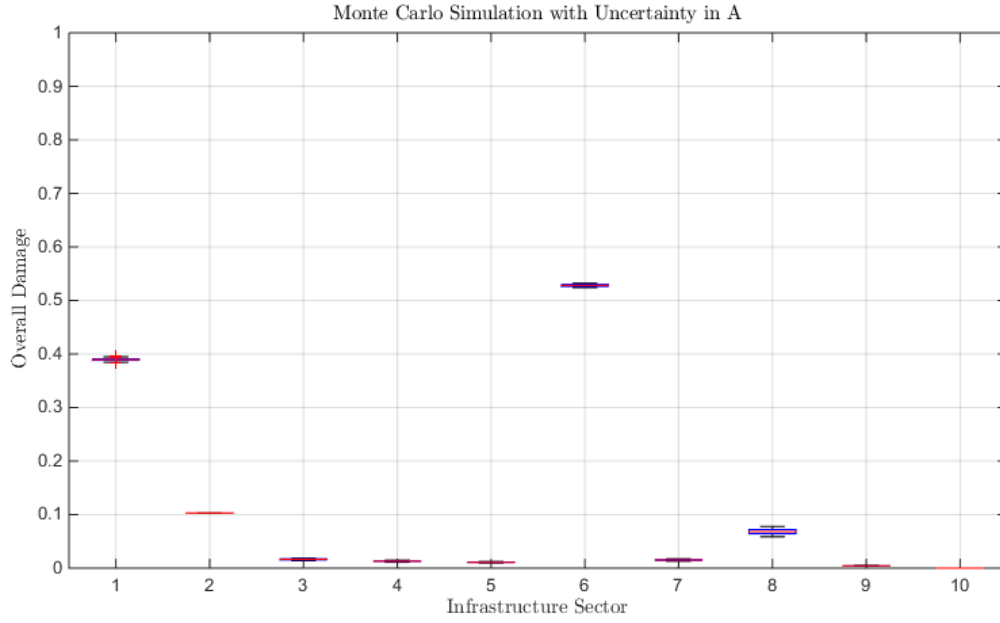
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Figure 8: Box plots of Monte Carlo simulation on overall damage propagation for each sector with uncertainty in A.
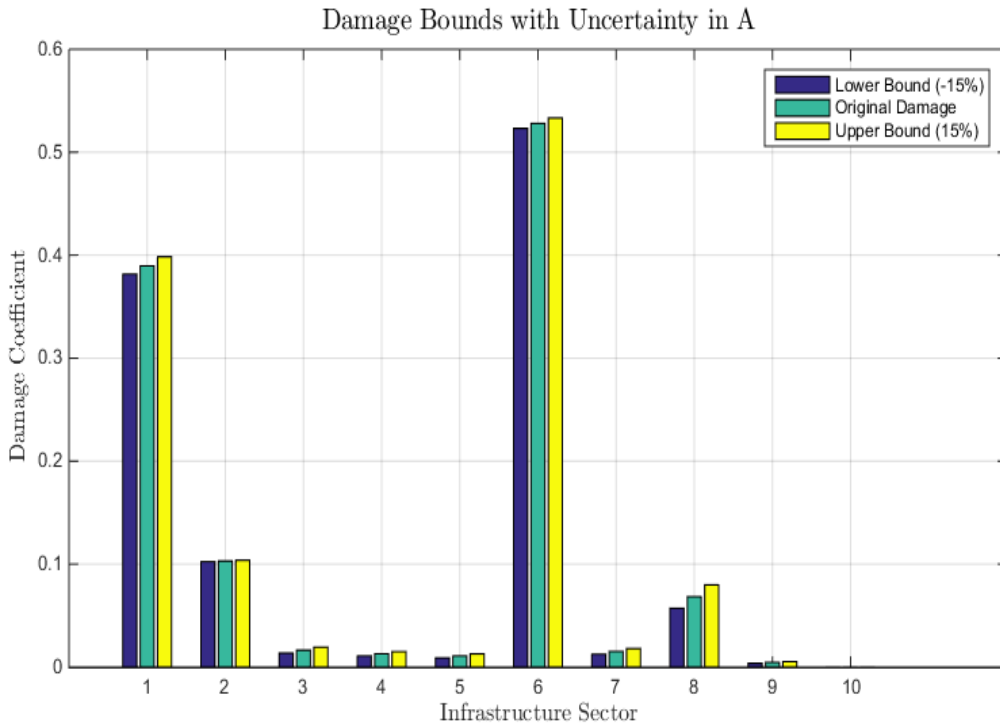

Figure 9: Bounds on overall system damage with a -15% to 15% uniformly distributed uncertainty in A.
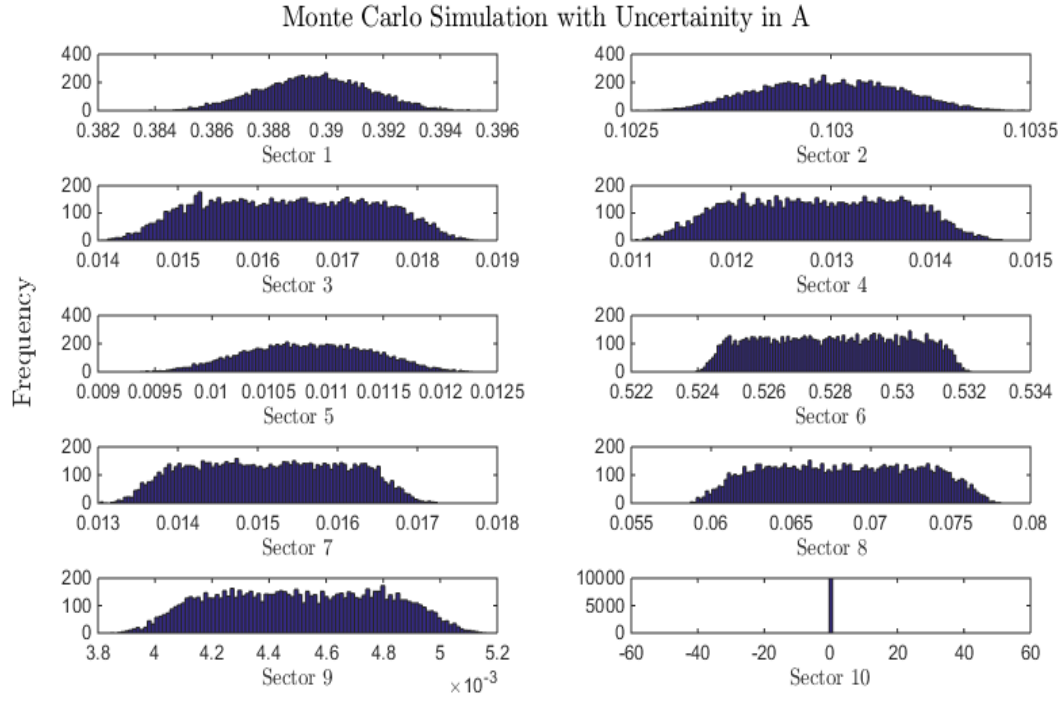
Figure 11: Histograms of Monte Carlo simulation on overall damage propagation for each sector with uncertainty in A.
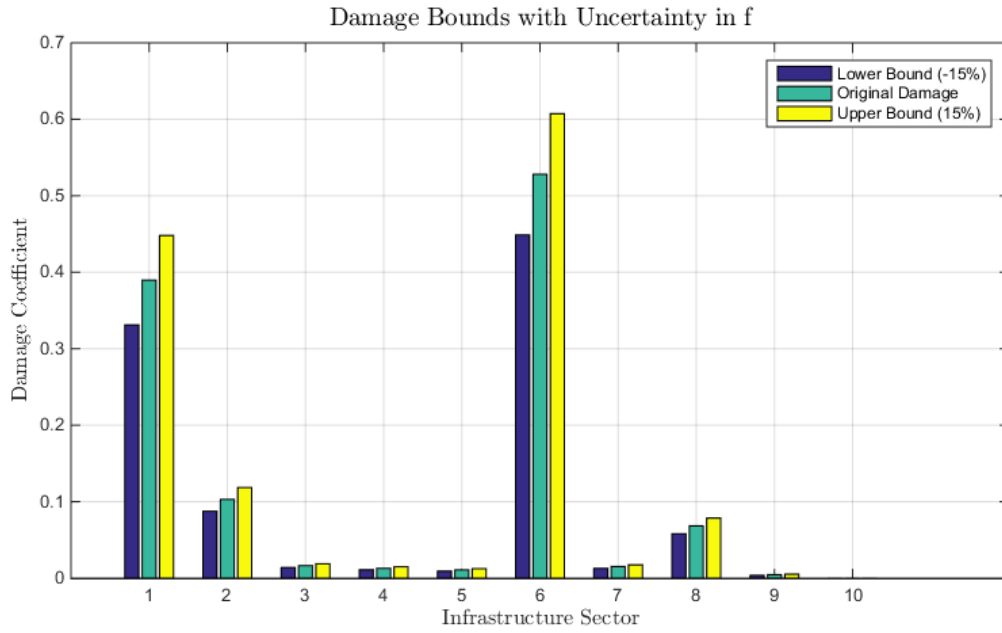

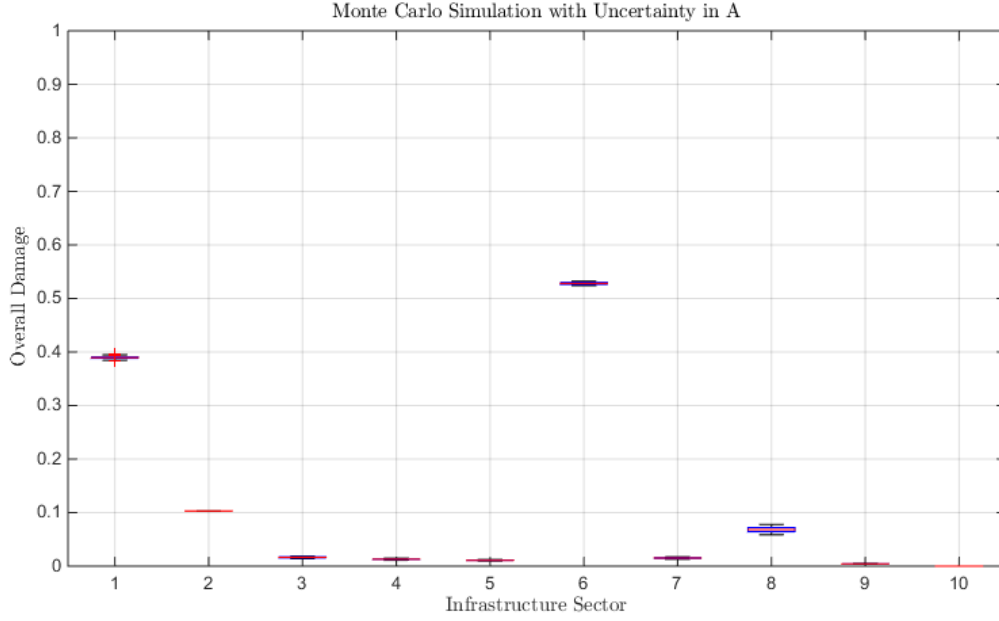Figure 10: Bounds on overall system damage with a -15% to 15% uniformly distributed uncertainty in f.

Figure 12: Box plots of Monte Carlo simulation on overall damage propagation for each sector with uncertainty in A.

Question 18

Question 19

Electricity Utilities

Question 20

There are three sets that compose the decision variables, which are:

$$x_{it}, \forall i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \forall t \in \{1, 2, 3, 4, 5, 6\}$$
$$y_i, \forall i \in \{5, 6, 7, 8\}$$
$$z_k, \forall k \in \{1, 2, 3\}$$

where x_{it} is the energy (MW) produced by power plant i during load block t, y_i is the design capacity of new power plant i (MW), and z_k is the implementation rate of demand side management program k, such that $z_k \in [0, 1]$. This amounts to 61 decision variables.

- The decision variable vector NAMELESS will be a column vector with dimensions 61 x 1. OR the decision variable vector lives in the space of R^{61}

Question 21

The expression for the objective function which minimizes the total global warming potential is:

$$\min \sum_{i=1}^{9} \sum_{t=1}^{6} g_i x_{it} n_t$$

where g_i is the global potential (kg CO_2 eq/MWh) from plant i, and n_t is the number of hours in load block t (MW).

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Note that this expression will result in the minimum global warming potential in the units of kg of CO_2 eq, instead of tons of CO_2 eq. The unit conversion does not affect the solution from the optimization problem. We will convert the units of the solution accordingly.

Question 22

The expression for the objective function which minimizes the total global warming potential is:

$$\min \sum_{i=1}^{9} \sum_{t=1}^{6} \bar{c}_i{}^v x_{it} n_t + \sum_{i=1}^{9} \bar{c}_i{}^c y_i \cdot 10^3 + \sum_{k=1}^{3} \sum_{t=1}^{6} \bar{c}_k{}^d z_k s_{kt}^{max} n_t$$

where,

- $\bar{c}_i{}^v$ is the expected variable cost associated with operating plant i (\$/MWh)

- $\bar{c}_i{}^c$ is the expected annualized capital cost of plant i (\$/kW). Note that we multiplied the values of the design capacities of new power plants, y_i by a factor of 10^3 to convert the units from MW to kW

- $\bar{c}_k{}^d$ is the expected cost of implementing demand side management program k (\$/MWh)

- s_{kt}^{max} is the maximum energy savings available in load block t from demand side management program k (MW)

Question 23

$$\sum_{i=1}^{9} x_{it} + \sum_{k=1}^{3} z_k s_{kt}^{max} \geq l_t, \forall t \in \{1, 2, 3, 4, 5, 6\}$$

There are six constraints