Final Project

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April 6, 2016

1 Project Overview

This is the project

2 Infrastructure Interdependence Analysis

Question 1

Given

$$x_i = o_i + f_i = \sum_j x_{ij} + f_i \tag{1}$$

$$x_{ij} = a_{ij} x_j \tag{2}$$

we obtain:

$$x_i = \sum_j a_{ij} x_j + f_i = a_i \boldsymbol{x} + f_i,$$

where a_i is a $1 \times i$ matrix and x is an $i \times 1$ vector. Similarly, for all cases of i, we obtain the matrix equation:

$$x = Ax + f, \tag{3}$$

where x is an $i \times 1$ vector, A is a $i \times j$ matrix and f is a $i \times 1$ vector. Note that A must be a square matrix, so $j = i$; its dimensions are $i \times i$.

Question 2

Table 2 in the given instructions sheet presents matrix A, which is the matrix of influence coefficients a_{ij}. These coefficients should be understood as the fraction of inoperability transmitted by the jth infrastructure to the ith infrastructure.

The last row of matrix A corresponds to the $i = 10$ infrastructure: satellite communication and navigation. Thereby, we must understand each coefficient a_{10j} for all j to be the fraction of inoperability transmitted by the jth infrastructure to the satellite communication and navigation infrastructure (10th).

We observe that the coefficients a_{10j} for all j are 0. This means that the failure of any j infrastructure does not transmit inoperability to the satellite communication and navigation infrastructure. On the other hand, all of the coefficients a_{i10} for all $i \neq 10$ are nonzero. In other words, the operability of the satellite communication and navigation infrastructure is independent of the operability of the other infrastructure, while the operability of the other infrastructure is dependent on the operability of the satellite and communication infrastructure.

This assumption seems to be reasonable for a $6-12$ hour outage. One can expect satellites to be self-sufficient in terms of energy consumption and maneuverability, but the infrastructure on the Earth to rely heavily on the data provided by the satellite and communication systems. A satellite may be able to operate on its own during a $6-12$ hour outage of the other infrastructure, while the remaining infrastructure is likely to fail during a $6-12$ hour outage of the satellite and communication infrastructure.

Question 3

The dependency index of infrastructure i, γ_i is defined as:

$$\gamma_i = \frac{1}{n-1} \sum_{j \neq i} a_{ij} \ (row\ summation). \tag{4}$$

The sum of the a_{ij} coefficients reveals the total direct damage on infrastructure i transmitted from the damage of each infrastructure j such that $j \neq i$. By dividing the sum by $n-1$ we compute the index γ_i, which indicates the average damage on infrastructure i from any other infrastructure.

In a sense, this index is a measure of the dependence of an infrastructure on the operability of other infrastructure, where a high value indicates a high dependency and a low value indicates a low dependency.

Likewise, the influence index of infrastructure j, δ_j is defined as:

$$\delta_j = \frac{1}{n-1} \sum_{i \neq j} a_{ij} \ (column\ summation). \tag{5}$$

The sum of the a_{ij} coefficients reveals the total direct damage of infrastructure j transmitted to the damage of each infrastructure i such that $i \neq j$. By dividing the sum by $n-1$ we compute the index δ_j, which indicates the average influence of infrastructure j has on any other infrastructure.

In a sense, this index is a measure of the influence an infrastructure has on the operability of other infrastructure, where a high value indicates a high influence and a low value indicates a low influence. The larger the index of a certain infrastructure sector, the higher the criticality of this infrastructure on the infrastructure system.

- Check criticality statement

Question 4

Starting from Equation 3, we can compute the following:

$$\begin{aligned} x &= Ax + f \\ Ix &= Ax + f \\ (I - A)x &= f \\ x &= (I - A)^{-1} f \end{aligned}$$

We can express the matrix $(I - A)^{-1}$ as matrix S, finally obtaining the solution in the form of:

$$x = Sf \tag{6}$$

Note that the information provided in Table 2 corresponds to matrix A. To compute the S in Equation 6 we need to follow our definition of S, $S = (I - A)^{-1}$. The matrix A must be a square matrix with coefficients between 0 and 1 and the $I - A$ matrix must invertible.

- Check if A is invertible then I-A is invertible
- We should make sure that all of the items in S are not negative since s is computed as 1 over 1 - a

Question 5

- Each element of S is an indicator of the indirect impact on the entire infrastructure system due to the propagation of failure from infrastrucure sector j to infrastructure sector i
- Give explanatory example (MAYBE)

Table 1: Add caption

Sector Id	1	2	3	4	5	6	7	8	9	10
1	1.004	0.237	0.325	0.510	0.038	0.029	0.016	0.027	0.053	0.319
2	0.001	1.012	0.013	0.027	0.002	0.003	0.001	0.003	0.180	0.005
3	0.003	0.124	1.005	0.126	0.003	0.006	0.003	0.003	0.027	0.009
4	0.006	0.089	0.017	1.008	0.005	0.004	0.004	0.002	0.020	0.008
5	0.005	0.061	0.013	0.027	1.001	0.006	0.008	0.008	0.019	0.022
6	0.002	0.263	0.111	0.131	0.007	1.002	0.008	0.007	0.049	0.008
7	0.004	0.118	0.035	0.110	0.008	0.004	1.001	0.004	0.029	0.011
8	0.009	0.535	0.114	0.087	0.052	0.023	0.022	1.003	0.097	0.016
9	0.002	0.036	0.011	0.009	0.005	0.000	0.002	0.005	1.006	0.006
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

The matrix S is an indicator of the total impact of an external shock on the entire infrastructure system. In particular, every s_{ij} reveals the infinite propagation of the damage of an infrastructure sector j to infrastructure sector i. So, every s_{ij} should greater than or equal to the corresponding a_{ij}, because s_{ij} includes the direct damage which is represented by a_{ij}. For example, s_{12},0.237, is smaller than s_{82}, 0.535, from table NAME!!. This indicates that sector 2 has a bigger total impact on sector 8 than sector 1 due to the propagation of failure of sector 2 to the entire infrastructure system. In another case, a_{101} was 0 and s_{101} remained 0. This

means that there is neither direct propagation of inoperability of sector 1 (air transportation) to sector 10 (satellite communication and navigation) during a 6-12 hr outage, nor total damage propagation. However, a_{22} is 0 and the corresponding s_{22} is 1.012. This reveals that even though there was no direct damage propagation of the failure of sector on itself (based on the given assumption), there is a total damage on the infrastructure itself due to the propagation of damage on the infrastructure system.

Question 6

- Check of paper of Setola
- Dan said that A is computed from surveys, and it is easier to tell how your own infrastructe would be affected by failure of other infrastructure than
- From a data collection perspective it is easier to calculate the direct impact of a failure of one infrastructure to another than the propagation of failures from one infrastructure to the rest
- From a modeling perspective, it is easier to keep track of matrix A as the direct dependencies between sectors are more intuitive than the chain propgatations between all of the sectors. If errors arise, it is easier to check what went wrong with A than trying to figure out what is wrong with S.

Question 7

- Write down equations (replace a ij with s ij)
- The overall influence index in a way is the same definition but instead of direct is total. At the end, this index reveals the most critical sectors of infrastructure system
- The overall dependency index is the same definition but instead is total. This index reveals the sectors that are most dependent on the operability of the other sectors of the infrastructure system
- You are as strong as your weaskest link

Question 8

- Electricity is most dependent on natural gas while natural gas is most dependent on electricity
- Electricity influecnes fuel and petroleum the most while natural gas influences electricty the msot
- For sector 2, we see that the column values are generally higher than the row values, which means that sector 2 has a great direct influence on the other sectors while is less impacteed directly by the failure of other sectors
- Similarly for natural gas, except that 8 of the 9 row coefficients (excluding index 9,9) are below 0.01, which reveals very low direct dependency of the natural gas sector. Also the column of natural gas is
- In both cases, it looks like electricty is more critical and is influenced more by the damage of other sectors

- THis makes sense, because the natural gas is a fuel source with substitutes, while electricity has really no substitutes

Question 9

- TODO: MATLAB code PAUL
- Plot
- Discuss after code is done and put answer

Question 10

- TODO: MATLAB code pending
- plot

Question 11

- How far reaching is this smart grid? How much effect does it have? From washing machines to cell phone use
- DAQ

Question 12

- TODO: Matlab code to add/decrease 10

Question 13

- solve x = Sf
- TODO: Matlab code
- Effect on other infrastructure assets: check vector x
- Further degrade of the already damaged infrastructure: compare x of the infrastructure to f (see if effects increase or decrease)
- Other assets degreaded indirectly: Yes, by having matrix S

Question 14

- No

Question 15

- Recursion code MATLAB TODO
- paul took picture

Question 16

- monte carlo UNCERTAINTY ON A

Question 17

- monte carlo UNCERTAINTY ON F

Question 18

- HIGH LOW METHOD
- write function that adds noise with certain g

Question 19