Final Project

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April 11, 2016

1 Project Overview

This is the project

2 Infrastructure Interdependence Analysis

Question 1

Given

$$x_i = o_i + f_i = \sum_j x_{ij} + f_i \tag{1}$$

$$x_{ij} = a_{ij}x_j \tag{2}$$

we obtain:

$$x_i = \sum_j a_{ij}x_j + f_i = a_i \boldsymbol{x} + f_i,$$

where a_i is a $1 \times i$ matrix and x is an $i \times 1$ vector. Similarly, for all cases of i, we obtain the matrix equation:

$$x = Ax + f, \tag{3}$$

where x is an $i \times 1$ vector, A is a $i \times j$ matrix and f is a $i \times 1$ vector. Note that A must be a square matrix, so $j = i$; its dimensions are $i \times i$.

Question 2

Table 2 in the given instructions sheet presents matrix A, which is the matrix of influence coefficients a_{ij}. These coefficients should be understood as the fraction of inoperability transmitted by the jth infrastructure to the ith infrastructure.

The last row of matrix A corresponds to the $i = 10$ infrastructure: satellite communication and navigation. Thereby, we must understand each coefficient a_{10j} for all j to be the fraction of inoperability transmitted by the jth infrastructure to the satellite communication and navigation infrastructure (10th).

We observe that the coefficients a_{10j} for all j are 0. This means that the failure of any j infrastructure does not transmit inoperability to the satellite communication and navigation infrastructure. On the other hand, all of the coefficients a_{i10} for all $i \neq 10$ are nonzero. In other words, the operability of the satellite communication and navigation infrastructure is independent of the operability of the other infrastructure, while the operability of the other infrastructure is dependent on the operability of the satellite and communication infrastructure.

This assumption seems to be reasonable for a $6-12$ hour outage. One can expect satellites to be self-sufficient in terms of energy consumption and maneuverability, but the infrastructure on the Earth to rely heavily on the data provided by the satellite and communication systems. A satellite may be able to operate on its own during a $6-12$ hour outage of the other infrastructure, while the remaining infrastructure is likely to fail during a $6-12$ hour outage of the satellite and communication infrastructure.

Question 3

The dependency index of infrastructure i, γ_i is defined as:

$$\gamma_i = \frac{1}{n-1} \sum_{j \neq i} a_{ij} \; (row \; summation). \tag{4}$$

The sum of the a_{ij} coefficients reveals the total direct damage on infrastructure i transmitted from the damage of each infrastructure j such that $j \neq i$. By dividing the sum by $n-1$ we compute the index γ_i, which indicates the average damage on infrastructure i from any other infrastructure.

In a sense, this index is a measure of the dependence of an infrastructure on the operability of other infrastructure, where a high value indicates a high dependency and a low value indicates a low dependency. The larger the index, the more direct damage infrastructure i receives from the inoperability of the entire infrastructure system.

Likewise, the influence index of infrastructure j, δ_j is defined as:

$$\delta_j = \frac{1}{n-1} \sum_{i \neq j} a_{ij} \; (column \; summation). \tag{5}$$

The sum of the a_{ij} coefficients reveals the total direct damage of infrastructure j transmitted to the damage of each infrastructure i such that $i \neq j$. By dividing the sum by $n-1$ we compute the index δ_j, which indicates the average influence of infrastructure j has on any other infrastructure.

In a sense, this index is a measure of the influence an infrastructure has on the operability of other infrastructure, where a high value indicates a high influence and a low value indicates a low influence. The larger the index, the larger direct impact from the inoperability of infrastructure j on the entire infrastructure system.

Question 4

Starting from Equation 3, we can compute the following:

$$x = Ax + f$$
$$Ix = Ax + f$$
$$(I - A)x = f$$
$$x = (I - A)^{-1}f$$

We can express the matrix $(I - A)^{-1}$ as matrix S, finally obtaining the solution in the form of:

$$x = Sf \tag{6}$$

Note that the information provided in Table 2 corresponds to matrix A. To compute the S in Equation 6 we need to follow our definition of S, $S = (I - A)^{-1}$. The matrix A must be a square matrix with coefficients between 0 and 1 and the $I - A$ matrix must invertible.

Both matrix A and matrix S are composed of i rows and j columns, where $i = j$. The elements of matrix A, a_{ij}, represent the first damage propagation step from infrastructure j on infrastructure i. The elements of matrix S, s_{ij}, represent the total damage propagation from infrastructure j on infrastructure i. This is, the total damage propagation from infrastructure j on infrastructure i that the system converges to after a series of recursive damage propagation steps.

Question 5

The matrix S is an indicator of the total impact of an external shock on the entire infrastructure system. In particular, every s_{ij} reveals the infinite propagation of the damage of an infrastructure sector j to infrastructure sector i. Every s_{ij} should be greater than or equal to the corresponding a_{ij} from matrix A. This is because s_{ij} includes the direct damage captured by a_{ij} and the subsequent inoperability propagation of failure, as explained in Question 4.

The matrix S computed after the matrix A found in Table 2 of the instructions sheet can be found in Table 1.

Table 1: Matrix S for a 6-12 hr outage.

Sector Id	1	2	3	4	5	6	7	8	9	10
1	1.004	0.237	0.325	0.510	0.038	0.029	0.016	0.027	0.053	0.319
2	0.001	1.012	0.013	0.027	0.002	0.003	0.001	0.003	0.180	0.005
3	0.003	0.124	1.005	0.126	0.003	0.006	0.003	0.003	0.027	0.009
4	0.006	0.089	0.017	1.008	0.005	0.004	0.004	0.002	0.020	0.008
5	0.005	0.061	0.013	0.027	1.001	0.006	0.008	0.008	0.019	0.022
6	0.002	0.263	0.111	0.131	0.007	1.002	0.008	0.007	0.049	0.008
7	0.004	0.118	0.035	0.110	0.008	0.004	1.001	0.004	0.029	0.011
8	0.009	0.535	0.114	0.087	0.052	0.023	0.022	1.003	0.097	0.016
9	0.002	0.036	0.011	0.009	0.005	0.000	0.002	0.005	1.006	0.006
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

From Table 1 we can observe that $s_{12} = 0.237$ is smaller than $s_{82} = 0.535$. This indicates that sector 2 has a bigger total impact on sector 8 than on sector 1 due to the propagation of

damage of sector 2 to the entire infrastructure system.

Comparing the case for the elements of matrix A, we can observe that a_{101} is 0 and s_{101} remained 0. This means that there is neither direct propagation of inoperability of sector 1 (air transportation) to sector 10 (satellite communication and navigation) during a 6-12 hr outage, nor total damage propagation.

However, a_{22} is 0 and the corresponding s_{22} is 1.012. This reveals that there is no direct damage propagation from the infrastructure on itself (based on the given assumption), yet there is a total damage on sector 2 due to the propagation of damage on the entire infrastructure system.

Question 6

As explained in Questions 4 and 5, matrix S encompasses matrix A. Matrix A captures the direct propagation of inoperability of the entire system whereas matrix S represents the infinite propagation of damage on the system. Therefore, it is easier to collect and to model the first tier of damage for every combination of ij pairs than to model the infinite propagation of damage.

- Check of paper of Setola

- Dan said that A is computed from surveys, and it is easier to tell how your own infrastructe would be affected by failure of other infrastructure than

- From a data collection perspective it is easier to calculate the direct impact of a failure of one infrastructure to another than the propagation of failures from one infrastructure to the rest

- From a modeling perspective, it is easier to keep track of matrix A as the direct dependencies between sectors are more intuitive than the chain propgatations between all of the sectors. If errors arise, it is easier to check what went wrong with A than trying to figure out what is wrong with S.

Question 7

- Write down equations (replace a ij with s ij)

- The overall influence index in a way is the same definition but instead of direct is total. At the end, this index reveals the most critical sectors of infrastructure system

- The overall dependency index is the same definition but instead is total. This index reveals the sectors that are most dependent on the operability of the other sectors of the infrastructure system

- You are as strong as your weaskest link

Question 8

- Electricity is most dependent on natural gas while natural gas is most dependent on electricity

- Electricity influecnes fuel and petroleum the most while natural gas influences electricty the msot

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- For sector 2, we see that the column values are generally higher than the row values, which means that sector 2 has a great direct influence on the other sectors while is less impacteed directly by the failure of other sectors

- Similarly for natural gas, except that 8 of the 9 row coefficients (excluding index 9,9) are below 0.01, which reveals very low direct dependency of the natural gas sector. Also the column of natural gas is

- In both cases, it looks like electricty is more critical and is influenced more by the damage of other sectors

- THis makes sense, because the natural gas is a fuel source with substitutes, while electricity has really no substitutes

Question 9

- TODO: MATLAB code PAUL

- Plot

- Discuss after code is done and put answer

Question 10

- TODO: MATLAB code pending

- plot

Question 11

- How far reaching is this smart grid? How much effect does it have? From washing machines to cell phone use

- DAQ

Question 12

- TODO: Matlab code to add/decrease 10

Question 13

- solve x = Sf

- TODO: Matlab code

- Effect on other infrastructure assets: check vector x

- Further degrade of the already damaged infrastructure: compare x of the infrastructure to f (see if effects increase or decrease)

- Other assets degreaded indirectly: Yes, by having matrix S

- The effect on other infrastructures is given through the matrix x.

- Critical infrastructures further degrade the air transportation sector by 3.96 percent.

- Critical infrastructures further degrade the rail transportation sector by 2.81 percent.

- Critical infrastructures further degrade the electricity sector by 0.30 percent.

- Other infrastructures assets are degraded even without being directly affected by the storm. The sector that gets affected the most without a direct damage due to the storm is the fuel and petroleum grid which is degraded by 6.83 percent.

- There are sectors that had little to no damage due to the storm as well. The satellite communication and navigation sector was not influenced by the storm at all while natural gas was only degraded by 0.45 percent.

Question 14

- No

Question 15

- Recursion code MATLAB TODO

- paul took picture

Question 16

- monte carlo UNCERTAINTY ON A

Question 17

- monte carlo UNCERTAINTY ON F

Question 18

- HIGH LOW METHOD

- write function that adds noise with certain g

Question 19