

**Homework 1****Due Date:** February 13th, 2026

1. Consider a standard deck of 52 cards: there are 4 suits, each with 13 consecutive values. A hand consists of a set of 5 cards.
  - (a) How many hands with all cards in the same suit are there?
  - (b) How many hands with all consecutive cards are there? Here, consecutive does not loop around (e.g., the hand with cards  $\{10, 11, 12, 13, 1\}$  is not consecutive).
  - (c) Which of the former two types of hands is more likely for a random hand?
2. Show that  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$  for any  $n \geq 1$ .
3. How many lists of  $m$  1s and  $n$  0s with exactly  $k$  consecutive runs of 1s are there? Here, a run is a maximal set of consecutive entries with the same value. Note that:
  - $\left(\!\! \begin{array}{c} n \\ k \end{array} \!\!\right)$ , i.e.  $n$  multichoose  $k$ , denotes the number of ways of choosing  $k$  elements, with repetition, from a set of size  $n$ .
  - $\left(\!\! \begin{array}{c} n \\ k \end{array} \!\!\right) = \left(\!\! \begin{array}{c} n+k-1 \\ k \end{array} \!\!\right)$  is a known identity.
4. Let  $G = (V, E)$  be a connected graph with pairwise distinct costs  $c : E \rightarrow \mathbb{R}_{>0}$ . Let  $C \subseteq E$  by any cycle in  $G$  and let  $e = \arg \max_{e \in C} c_e$ . Show that  $e$  does not belong to any minimum spanning tree of  $G$ .