

Problem Set 1**Due Date:** February 13, 2026

1. Consider a standard deck of 52 cards: there are 4 suits, each with 13 consecutive values. A hand consists of a set of 5 cards.
 - (a) How many hands with all cards in the same suit are there?
 - (b) How many hands with all consecutive cards are there? Here, consecutive does not loop around (e.g., the hand with cards $\{10, 11, 12, 13, 1\}$ is not consecutive).
 - (c) Which of the former two types of hands is more likely for a random hand?
2. Show that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ for any $n \geq 1$.
3. How many lists of m 1s and n 0s with exactly k consecutive runs of 1s are there? Here, a run is a maximal set of consecutive entries with the same value. Note that:
 - $\binom{n}{k}$, i.e. n multichoose k , denotes the number of ways of choosing k elements, with repetition, from a set of size n .
 - $\binom{n}{k} = \binom{n+k-1}{k}$ is a known identity.
4. Let $G = (V, E)$ be a connected graph with pairwise distinct costs $c : E \rightarrow \mathbb{R}_{>0}$. Let $C \subseteq E$ by any cycle in G and let $e \in \arg \max_{e \in C} c_e$. Show that e does not belong to any minimum spanning tree of G .