Tecnológico de Monterrey
Escuela de Negocios

CD2007
CASE STUDIES

Case 1

Set up a problem in R to maximize

$$x1 + 9\,x2 + x3$$

subject to

$$x1 + 2\,x2 + 3\,x3 <= 9$$

$$3\,x1 + 2\,x2 + 2\,x3 <= 15.$$

Solution

Defintions

```
f.obj <- c(1, 9, 1)
f.con <- matrix (c(1, 2, 3, 3, 2, 2), nrow=2, byrow=TRUE)
f.dir <- c("<=", "<=")
f.rhs <- c(9, 15)
#
```

Output

```
lp ("max", f.obj, f.con, f.dir, f.rhs)
## Not run: Success: the objective function is 40.5
lp ("max", f.obj, f.con, f.dir, f.rhs)$solution
```

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Case 2

A manufacturing manager is in charge of minimizing the total costs (raw materials, labor and storage costs) of the following four months. In Table 3.1 can be found the cost of raw materials of one unit of final product, the demand of final product and the working hours available for each month.

Labor costs are of 12 e per hour, and only worked hours are payed. Each unit of final product needs 30 minutes of labor.

Storage costs are equal to 2 e for each unit stored at the end of the month. Any unit produced at a given month can be used to cover the demand of the same month, or be stored to cover the demand of months to come. At the beginning of month 1 there is no stock, and there are no minimum stock requirements for any month.

Month	1	2	3	4
Unit cost (€)	6	8	10	12
Demand (units)	100	200	150	400
Working hours available	200	200	150	150

Table 3.1: Information for the production plan

Tasks & Solution

- Define the decision variables (provide a brief definition of each set of defined variables), objective function and constraints of a linear programming model that minimizes total production costs.

The variables used in to define the model are defined for $i = 1, \ldots, 4$:

- Variables q_i representing the quantity produced in month i
- Variables s_i representing the stock at the end of month i

The constraints d_i ensure that the demand is covered and constraints u_i should be added to make q_i no larger that its required upper bound.

$$\text{MAX } z = \sum_{i=1}^{4} (12q_i + 2s_i)$$

d1) $q_1 - s_1 = 100$

d2) $s_1 + q_2 - s_2 = 200$

d3) $s_2 + q_3 - s_3 = 150$

d4) $s_3 + q_4 - s_4 = 400$

u1) $q_1 \leq 400$

u2) $q_2 \leq 400$

u3) $q_3 \leq 300$

u4) $q_4 \leq 300$

$s_i \geq 0$

- Modify the model of the previous section if a **fixed cost** of 1,000 EUR has to be considered for each month that there is production. This cost is assumed only if there is production different from zero in that month.

For this version of the model, four binary variables b_i are added, which equal one if there is production in month i, and zero otherwise. A set of constraints of the kind $q_i \leq Mb_i$ have been defined, although the constraints of upper bound can be also used, for instance making $q_1 \leq 400b_1$:

$$\text{MAX } z = \sum_{i=1}^{4} (12q_i + 2s_i + 1000b_i)$$

$$\text{d1) } q_1 - s_1 = 100$$

$$\text{d2) } s_1 + q_2 - s_2 = 200$$

$$\text{d3) } s_2 + q_3 - s_3 = 150$$

$$\text{d4) } s_3 + q_4 - s_4 = 400$$

$$\text{u1) } q_1 \leq 400b_1$$

$$\text{u2) } q_2 \leq 400b_2$$

$$\text{u3) } q_3 \leq 300b_3$$

$$\text{u4) } q_4 \leq 300b_4$$

$$s_i \geq 0,\ b_i \text{ binary}$$