

Pairs Trading Model: Key Formulas and Definitions

QUANTT Team — Stage 1: Pairs Trading Algorithm

1. Hedge Ratio (β)

The hedge ratio defines how much of one stock offsets the movement of the other. It is estimated using **Ordinary Least Squares (OLS)** regression between two price series:

$$P_{\text{KO},t} = \alpha + \beta P_{\text{PEP},t} + \varepsilon_t$$

Where:

- $P_{\text{KO},t}$ = Price of Coca-Cola at time t
 - $P_{\text{PEP},t}$ = Price of Pepsi at time t
 - β = Hedge ratio (regression slope)
 - ε_t = Residual term
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2. Spread

The spread measures the relative mispricing between the two assets:

$$\text{Spread}_t = P_{\text{KO},t} - \beta P_{\text{PEP},t}$$

A high or low spread indicates potential divergence from the mean.

3. Rolling Mean of the Spread

The rolling mean represents the equilibrium or “fair value” level of the spread over a chosen lookback window N :

$$\bar{S}_t = \frac{1}{N} \sum_{i=t-N+1}^t S_i$$

Where S_i is the spread at time i .

4. Rolling Standard Deviation of the Spread

The rolling standard deviation measures how volatile or dispersed the spread is over the same window:

$$\sigma_t = \sqrt{\frac{1}{N-1} \sum_{i=t-N+1}^t (S_i - \bar{S}_t)^2}$$

5. Z-Score (Standardized Spread)

The Z-score quantifies how extreme the current spread is compared to its historical average:

$$Z_t = \frac{S_t - \bar{S}_t}{\sigma_t}$$

Interpretation:

$$\begin{cases} Z_t > +2 & \Rightarrow \text{Spread high} \rightarrow \text{Short KO, Long PEP} \\ Z_t < -2 & \Rightarrow \text{Spread low} \rightarrow \text{Long KO, Short PEP} \\ |Z_t| < 0.5 & \Rightarrow \text{Exit positions} \end{cases}$$

6. Portfolio Daily Return (PnL)

The daily return of the portfolio is given by:

$$r_t = w_{\text{KO},t-1}R_{\text{KO},t} + w_{\text{PEP},t-1}R_{\text{PEP},t}$$

Where:

- $w_{\text{KO},t-1}, w_{\text{PEP},t-1}$ = position weights (positive for long, negative for short)
 - $R_{\text{KO},t}, R_{\text{PEP},t}$ = daily percentage returns of KO and PEP
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7. Cumulative Equity Curve

The cumulative equity (growth of the portfolio) over time is:

$$E_t = \prod_{i=1}^t (1 + r_i)$$

8. Performance Metrics

(a) Sharpe Ratio

Measures risk-adjusted return:

$$\text{Sharpe} = \frac{\text{mean}(r_t)}{\text{std}(r_t)} \times \sqrt{252}$$

(252 trading days per year)

(b) Total Return

Overall return over the backtest period:

$$\text{Total Return} = (E_T - 1) \times 100\%$$

(c) Maximum Drawdown

Measures the largest decline from a peak before recovery:

$$\text{Max Drawdown} = \max_t \left(\frac{\text{Peak}_t - E_t}{\text{Peak}_t} \right)$$

(d) Profit Factor (optional)

Ratio of total profits to total losses:

$$\text{Profit Factor} = \frac{\sum(\text{Profits})}{|\sum(\text{Losses})|}$$

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9. Summary of Key Variables

Symbol	Meaning	Units / Notes
$P_{\text{KO},t}$	Coca-Cola price	Price in USD
$P_{\text{PEP},t}$	Pepsi price	Price in USD
β	Hedge ratio	Dimensionless
S_t	Spread at time t	Price difference
\bar{S}_t	Rolling mean of spread	Price difference
σ_t	Rolling standard deviation	Price difference
Z_t	Z-score of spread	Standard deviations
r_t	Daily portfolio return	Percent
E_t	Cumulative equity	Percent

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