Homework I, Fall 2014

Be sure to include all the material that shows your work and to present your results in a way that is easy to understand. Remember: ignorance and inability to communicate are observationally equivalent. Go as far as you can in each exercise.

1. Github

Set up a GitHub repo for this class. You should upload all the codes and results of this homework to the repo. Write the documentation required for a third party to look at your code and results.

2. Integration

Compute:

$$\int_0^T e^{-\rho t} u(1 - e^{-\lambda t}) dt$$

for $T = 100$, $\rho = 0.04$, $\lambda = 0.02$, and $u(\cdot) = -e^{-c}$ using quadrature (midpoint, Trapezoid, and Simpson rule) and a Monte Carlo. Compare both the performance of methods.

3. Optimization: basic problem

Use the Newton-Raphson, BFGS, steepest descent, and conjugate descent method to solve:

$$\min_{x,y} 100(y - x^2)^2 + (1 - x)^2$$

Compare the performance of each method.

4. Computing Pareto efficient allocations

Consider an endowment economy with m different goods and n agents. Each agent $j = 1, ..., n$ has an endowment $e_j^i > 0$ for every $i = 1, ..., m$ and a utility function of the form:

$$u^i(x) = \sum_{j=1}^{m} \alpha_j \frac{x_j^{1+\omega_j^i}}{1 + \omega_j^i}$$

where $\alpha_j > 0 > \omega_j^i$ are agent-specific parameters.

Given some social weights, $\lambda_j > 0$, solve for the social planner's problem for $m = n = 3$ using your favorite optimization method. Try different values of α_j, ω_j^i and $\lambda_j > 0$. For example, you can compute first the case where all the agents have the same parameters and social weights and later a case where there is a fair degree of heterogeneity.

How does the method perform? How does heterogeneity in the agent-specific parameters affect the results? Can you handle the case $m = n = 10$?

5. Computing Equilibrium allocations

Using the same model as in the previous exercise, can you find the equilibrium prices p^i? Hint: Solve for the first-order conditions of each agent, aggregate the excess demands, and solve the resulting system of nonlinear equations.

6. Value Function Iteration

All the next exercises will be based on the following model. There is a representative household with preferences over private consumption, c_t, government consumption g_t, and labor l_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} 0.97^t \left(\log c_t + 0.2 \log g_t - \frac{l_t^2}{2} \right)$$

The household consumes, saves, works, and pays labor taxes, with a budget constraint (notation is self-explanatory):

$$c_t + i_t = (1 - \tau_t)\, w_t l_t + r_t k_t$$

where the tax rate follows a Markov chain that takes values in:

$$\tau_t \in \{0.2, 0.25, 0.3\}$$

with transition matrix:

$$\begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.05 & 0.9 & 0.05 \\ 0 & 0.1 & 0.9 \end{pmatrix}$$

There is a production function:

$$c_t + i_t + g_t = e^{z_t} k_t^{0.33} l_t^{0.67}$$

with a law of motion for capital with investment adjustment costs:

$$k_{t+1} = 0.9 k_t + \left[1 - 0.05 \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t$$

and a technology level z_t that follows a Markov chain that takes values in:

$$z_t \in \{-0.0673, -0.0336, 0, 0.0336, 0.0673\}$$

with transition matrix:

$$\begin{pmatrix} 0.9727 & 0.0273 & 0 & 0 & 0 \\ 0.0041 & 0.9806 & 0.0153 & 0 & 0 \\ 0 & 0.0082 & 0.9836 & 0.0082 & 0 \\ 0 & 0 & 0.0153 & 0.9806 & 0.0041 \\ 0 & 0 & 0 & 0.0273 & 0.9727 \end{pmatrix}$$

Finally, there is a government that uses taxes to pay for government consumption with a balanced budget period by period:

$$g_t = \tau_t w_t l_t$$

Please answer the following exercises.

6.1. Social Planner

Find the associated social planner's problem to this model and write it recursively.

6.2. Steady State

Compute the deterministic steady state of the model when $\tau_{ss} = 0.25$ and $z_{ss} = 0$.

6.3. Value Function Iteration with a Fixed Grid

Fix a grid of 250 points of capital, centered around k_{ss} with a coverage of $\pm 30\%$ of k_{ss} and equally spaced and a grid of 50 points on lagged investment, centered around i_{ss} with a coverage of $\pm 50\%$ of k_{ss} and equally spaced. Iterate on the value function of the household using linear interpolation until the change in the sup norm between two iterations is less than 10^{-6}. Compute the policy function. Describe the responses of the economy to a technology shock and a tax shock.

6.4. Value Function Iteration with an Endogenous Grid

Repeat the previous exercise with an endogenous grid.

6.5. Comparison of Grids

Compare the solutions in 2) and 3) in terms of 1) accuracy, 2) computing time, and 3) complexity of implementation. Provide evidence to support your claims.

6.6. Switching between Policy and Value Function Iteration

Recompute your solution to 2) switching between policy and value function iteration, i.e., skipping the max operator in the Bellman equation nine out of each ten times. Compare accuracy and computing time between the simple grid scheme implemented in 2) and the results from the accelerator scheme. Provide evidence to support your claims.

6.7. Multigrid

Implement a multigrid scheme (Chow and Tsitsiklis, 1991) for a Value function iteration, with the grid centered around k_{ss} with a coverage of $\pm 30\%$ of k_{ss} and equally spaced (you can keep the grid of investment fixed).

You will have 100 capital grid points in the first grid, 500 capital grid points in the second, and 5000 capital grid points in the third.

Compare accuracy and computing time between the simple grid scheme implemented in 2) and the results from the multigrid scheme. Provide evidence to support your claims.

6.8. Stochastic Grid

Implement a stochastic grid scheme (Rust, 1997) for a Value function iteration, with 500 vertex points with a coverage of $\pm 25\%$ of k_{ss} (you can keep the grid of investment fixed). Compare accuracy and computing time between the simple grid scheme implemented in 2) and the results from the multigrid scheme. Provide evidence to support your claims.