Can we really model a dripping and no-dripping condition? How?

- Multiphase Models available and applicable to this multiphase topology:
  - ✓ Multiphase Segregated Flow Model (Eulerian Eulerian)
  - ✓ Eulerian Multiphase Mixture Model
  - √ Volume of Fluid (VOF)

### Eulerian Multiphase Mixture Model

**Volume Fraction Governing Equation** 

$$\frac{\partial}{\partial t} \int_{V} \alpha_{i} \, dV + \int_{A} \alpha_{i} \mathbf{v}_{m} \cdot d\mathbf{a} = \int_{V} \left( S_{u_{i}} - \frac{\alpha_{i}}{\rho_{i}} \frac{D\rho_{i}}{Dt} \right) dV + \underbrace{\int_{A} \frac{\mu_{i}}{\sigma_{i} \rho_{m}} \nabla \alpha_{i} \cdot d\mathbf{a}}_{\text{Turbulent Term}} - \underbrace{\int_{V} \frac{1}{\rho_{i}} \nabla \cdot \left( \alpha_{i} \rho_{i} \mathbf{v}_{d, i} \right) dV}_{\text{Slip Velocity Term}}$$

Continuity

$$\frac{\partial}{\partial t} \int_{V} \rho_{m} \, dV + \int_{A} \rho_{m} \mathbf{v}_{m} \cdot d\mathbf{a} = \int_{V} S_{u} \, dV$$

Momentum

$$\frac{\partial}{\partial t} \int_{V} \rho_{m} \mathbf{v}_{m} \ dV + \int_{A} \rho_{m} \mathbf{v}_{m} \otimes \mathbf{v}_{m} \cdot d\mathbf{a} = -\int_{A} p \mathbf{I} \cdot d\mathbf{a} + \int_{A} \mathbf{T}_{m} \cdot d\mathbf{a} + \int_{V} \mathbf{f}_{b} \ dV + \int_{V} \mathbf{s}_{u} \ dV - \underbrace{\sum_{i} \int_{A} \alpha_{i} \rho_{i} \mathbf{v}_{d,i} \otimes \mathbf{v}_{d,i} \cdot d\mathbf{a}}_{\text{Slip Velocity Term}}$$

### Volume of Fluid (VOF)

**Volume Fraction Governing Equation** 

$$\frac{\partial}{\partial t} \int_{V} \alpha_{i} dV + \oint_{A} \alpha_{i} \mathbf{v} \cdot d\mathbf{a} = \int_{V} \left( S_{\alpha_{i}} - \frac{\alpha_{i}}{\rho_{i}} \frac{D\rho_{i}}{Dt} \right) dV - \int_{V} \frac{1}{\rho_{i}} \nabla \cdot \left( \alpha_{i} \rho_{i} \mathbf{v}_{d,i} \right) dV$$

Continuity

$$rac{\partial}{\partial t} \left( \int\limits_V 
ho dV 
ight) + \oint_A 
ho {f v} \cdot d{f a} = \int\limits_V S dV$$

Momentum

$$\frac{\partial}{\partial t} \left( \int_{V} \rho \mathbf{v} dV \right) + \oint_{A} \rho \mathbf{v} \otimes \mathbf{v} \cdot d\mathbf{a} = -\oint_{A} p \mathbf{I} \cdot d\mathbf{a} + \oint_{A} \mathbf{T} \cdot d\mathbf{a} + \int_{V} \rho \mathbf{g} dV + \int_{V} \mathbf{f}_{b} dV - \sum_{i} \int_{A} \alpha_{i} \rho_{i} \mathbf{v}_{d,i} \otimes \mathbf{v}_{d,i} \cdot d\mathbf{a}$$

### **Volume Fraction Governing Equation**

Volume Fraction Governing Equation 
$$\frac{\partial}{\partial t} \int_{V} \alpha_{i} \, dV + \int_{A} \alpha_{i} \mathbf{v}_{m} \cdot d\mathbf{a} = \int_{V} \left( S_{u_{i}} - \frac{\alpha_{i}}{\rho_{i}} \frac{D\rho_{i}}{Dt} \right) dV + \underbrace{\int_{A} \frac{\mu_{i}}{\sigma_{i} \rho_{m}} \nabla \alpha_{i} \cdot d\mathbf{a}}_{\text{Turbulent Term}} - \underbrace{\int_{V} \frac{1}{\rho_{i}} \nabla \cdot (\alpha_{i} \rho_{i} \mathbf{v}_{d,i}) dV}_{\text{Slip Velocity Term}}$$

$$\frac{\partial}{\partial t} \int_{V} \alpha_{i} dV + \oint_{A} \alpha_{i} \mathbf{v} \cdot d\mathbf{a} = \int_{V} \left( S_{\alpha_{i}} - \frac{\alpha_{i}}{\rho_{i}} \frac{D\rho_{i}}{Dt} \right) dV - \int_{V} \frac{1}{\rho_{i}} \nabla \cdot (\alpha_{i} \rho_{i} \mathbf{v}_{d,i}) dV$$

$$Continuit$$

$$rac{\partial}{\partial t} \, \int_{V} lpha_{i} dV + \oint_{A} lpha_{i} {f v} \cdot d{f a} = \int_{V} \left( S_{lpha_{i}} - rac{lpha_{i}}{
ho_{i}} rac{D
ho_{i}}{Dt} 
ight) dV - \int_{V} rac{1}{
ho_{i}} \, 
abla \cdot \left( lpha_{i} 
ho_{i} {f v}_{d,i} 
ight) dV$$

Mixture Model

VOF

### **Continuity**

$$rac{\partial}{\partial t} \left( \int\limits_{V} 
ho dV 
ight) + \oint_{A} 
ho {f v} \cdot d{f a} = \int\limits_{V} S dV \qquad \qquad rac{\partial}{\partial t} \, \int_{V} 
ho_{m} \, dV + \int_{A} 
ho_{m} {f v}_{m} \cdot d{f a} = \int_{V} S_{u} \, \, dV$$

#### **Momentum**

$$\frac{\partial}{\partial t} \int_{V} \rho_{m} \mathbf{v}_{m} \; dV + \int_{A} \rho_{m} \mathbf{v}_{m} \otimes \mathbf{v}_{m} \cdot d\mathbf{a} = -\int_{A} p \mathbf{I} \cdot d\mathbf{a} + \int_{A} \mathbf{T}_{m} \cdot d\mathbf{a} + \int_{V} \mathbf{f}_{b} \; dV + \int_{V} \mathbf{s}_{u} \; dV - \underbrace{\sum_{i} \int_{A} \alpha_{i} \rho_{i} \mathbf{v}_{d,i} \otimes \mathbf{v}_{d,i} \cdot d\mathbf{a}}_{\text{Slip Velocity Term}}$$

Mixture Model

$$\frac{\partial}{\partial t} \left( \int_{V} \rho \mathbf{v} dV \right) + \oint_{A} \rho \mathbf{v} \otimes \mathbf{v} \cdot d\mathbf{a} = -\oint_{A} p \mathbf{I} \cdot d\mathbf{a} + \oint_{A} \mathbf{T} \cdot d\mathbf{a} + \int_{V} \rho \mathbf{g} dV + \int_{V} \mathbf{f}_{b} dV - \sum_{i} \int_{A} \alpha_{i} \rho_{i} \mathbf{v}_{d,i} \otimes \mathbf{v}_{d,i} \cdot d\mathbf{a} \qquad \underline{\mathsf{VOF}}$$

Differences between both formulations

- How the convective term is resolved
- Source terms

### Source term

Surface tension "The surface tension force is a tensile force tangential to the interface separating two fluids. It works to keep the fluid molecules at the free surface in contact with the rest of the fluid. The interfacial surface force is modeled as a volumetric force." StarCCM+ documentation

#### **Momentum**

$$\frac{\partial}{\partial t} \int_{V} \rho_{m} \mathbf{v}_{m} \ dV + \int_{A} \rho_{m} \mathbf{v}_{m} \otimes \mathbf{v}_{m} \cdot d\mathbf{a} = -\int_{A} p \mathbf{I} \cdot d\mathbf{a} + \int_{A} \mathbf{T}_{m} \cdot d\mathbf{a} + \int_{V} \mathbf{f}_{b} \ dV + \int_{V} \mathbf{s}_{u} \ dV - \underbrace{\sum_{i} \int_{A} \alpha_{i} \rho_{i} \mathbf{v}_{d,i} \otimes \mathbf{v}_{d,i} \cdot d\mathbf{a}}_{\text{Slip Velocity Term}}$$

Mixture Model

$$\frac{\partial}{\partial t} \left( \int_{V} \rho \mathbf{v} dV \right) + \oint_{A} \rho \mathbf{v} \otimes \mathbf{v} \cdot d\mathbf{a} = -\oint_{A} p \mathbf{I} \cdot d\mathbf{a} + \oint_{A} \mathbf{T} \cdot d\mathbf{a} + \int_{V} \rho \mathbf{g} dV + \int_{V} \mathbf{f}_{b} dV - \sum_{i} \int_{A} \alpha_{i} \rho_{i} \mathbf{v}_{d,i} \otimes \mathbf{v}_{d,i} \cdot d\mathbf{a}$$

$$+ \int_{V} \left( -\sigma \nabla \cdot \left( \frac{\nabla \alpha_{i}}{|\alpha_{i}|} \right) \nabla \alpha_{i} + (\nabla \sigma)_{t} |\nabla \alpha_{i}| \right) dV$$

$$+ \int_{V} \left( -\sigma \nabla \cdot \left( \frac{\nabla \alpha_{i}}{|\alpha_{i}|} \right) \nabla \alpha_{i} + (\nabla \sigma)_{t} |\nabla \alpha_{i}| \right) dV$$