Math 131: Numerical Analysis
Homework Number 6
Due: April 17, 2024 10:00 PM

Special Instructions:

- Two of the equations you will need are in your book (Equation 4.27, page 199, and Equation 4.31, page 201). You can also find them in Lecture Slides. You can use either as your reference, but please remember to be careful about how h is defined.
- If instructed to "write/implement a code" or "write/implement an algorithm", you should interpret this as meaning that you need to produce a code in Jupyter/python notebook.
- Unless explicitly stated, you may not use any system/python packages or functions that implement one of the algorithms for this assignment. Standard mathematical functions (e.g. pow, abs, log, sin, exp, etc.) are allowed.
- You should include all the code source you implemented as one python notebook when turning in your assignment. Your code must run as is (and provide the correct results) to receive full credit.
- PDF files of your code, screenshots, etc. will not be graded.
- All code should be properly documented and include (**at a minimum**) 1) a summary of what the code is doing, 2) a description of all parameters that are used, and 3) a description of the output.
- The notebook itself should also be properly documented. Points will be **heavily** deducted for any notebook and/or code that is not properly documented.

1. Quadrature Basics (40 points)

This part will consider the following integral:

$$I(f) = \int_{-1}^{1} 1 + e^{2x} \sin 3x \; dx \tag{1.1}$$

(a) Implement a python function to compute the integral of a given function using the Trapezoid Rule. Apply your function to compute an approximation to the integral (1.1).

(b) Implement a python function to compute the integral of a given function using Simpson's Rule. Apply your function to compute an approximation to the integral (1.1)

(c) Repeat (a) and (b), but break the computations into 4 equal parts such that:

$$I(f) = \int_{-1}^{-0.5} 1+e^{2x} \sin 3x \; dx + \int_{-0.5}^{0} 1+e^{2x} \sin 3x \; dx + \int_{0}^{0.5} 1+e^{2x} \sin 3x \; dx + \int_{0.5}^{1} 1+e^{2x} \sin 3x \; dx$$

(d) Analyze all your results and compare and contrast the two quadrature formulas as well as breaking the problem into smaller parts.

2. Newton-Cotes (Theory): (60 points)

This part will consider the following integral:

$$I(f) = \int_{1}^{3} \frac{dx}{x}. \tag{2.2}$$

(a) Derive the **closed Newton-Cotes** formula on 4 equally spaced nodes (also known as Simpson's Three-Eighths rule, Equation 4.27). Apply the derived formula to approximate $I(f)$ in (2.2). You may code this or compute it by hand.

(b) Determine the bound on the error of the approximation (**Hint:** since you know the function you are integrating you can find the bounds on all of its derivatives on the interval $[1, 3]$.

(c) Derive the **open Newton-Cotes** formula on three equally spaced nodes (Equation 4.31). Use the formula to approximate $I(f)$ and determine the bound on the error of the approximation. You may code this or compute it by hand.

3. Bonus: (20 points)

A colleague has come to you for advice on a quadrature formula to use for one of their problems. Given a choice between the four formulas (Trapezoid, Simpson's, Closed Newton-Cotes (3/8 Rule), and open Newton-Cotes make a recommendation on which one to use. Make any reasonable assumptions on the problem you like - some things to consider might be problem size, smoothness of the integrand, knowledge of derivatives, etc. Justify your assumptions and recommendations.