

Math 131: Numerical Analysis

Homework Number 1

Due: January 30, 2024, 5:00 PM

1. Mean Value Theorem (15)

For each of the following, state whether the MVT applies or not. Explain your reasoning:

1. $y = \exp(x)$ on $[0,1]$
2. $y = \tan(2\pi x)$ on $[0,2]$
3. $y = \frac{1}{|x+1|}$ on $[0,3]$
4. $y = \ln(x+1)$ on $[0, e-1]$
5. $y = 5 + |x|$ on $[-1,1]$

2. Weighted MVT for Integrals (10)

Use the Weighted Mean Value Theorem for Integrals to estimate the integral:

$$\int_0^1 \arctan(x) \cdot (1+2x) \, dx.$$

3. Taylor's Theorem (10)

Find the third Taylor polynomial $P_3(x)$ for $f(x) = \sqrt{x+1}$ about the point $x_0 = 0$. Using $P_3(x)$ approximate $\sqrt{0.5}$, $\sqrt{0.75}$, $\sqrt{1.0}$, $\sqrt{1.5}$ and compute the absolute error, e.g. $|\sqrt{0.5} - P_3(0.5)|$, for each value indicated.

4. Taylor's Theorem - Remainder Term (10)

Find the smallest value of n such that the remainder estimate $R_n(x)$ on the interval between x_0 and the indicated points, yields $|R_n| < 0.001$.

1. $f(x) = \cos(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $x_0 = 0$
2. $f(x) = \exp(-x)$ on $[-3, 3]$, $x_0 = 0$

5. JupyterLab (5)

Login in to the UC Merced JupyterLab. To access the JupyterLab hub you can go directly to: <https://ucmerced.2i2c.cloud/>. We will maintain a folder for this course under the name **Math131Sp24**, which you should be able to find under the **shared** folder. Once you've navigated to the Math131 folder look for a file by the name of **hello.ipynb**.

Open the notebooks and make sure you can run the entire notebook without errors.

Follow the instructions to code your own function. When you're done with your modifications, run the entire notebook again, save and export the notebook as a pdf file and include the output in your submission.

Some additional instructions on using the JupyterLab hub can be found at:

https://ucmerced.github.io/hpc_docs/#/jupyterhub