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Week 13 - Report

Since the last report, I have made major progress, specifically regarding changes in the Proximal Gradient Descent Jupyter notebook. Until last time, I had only been testing a few matrices of different dimensions and printed out a data frame for each using different step-size (α) values. From the results, I analyzed that most of the time the algorithm quit because not enough progress was being made but even when we converged, we had no clue whether or not the final function value that was calculated was correct.

As a result, we decided to repolish the code by splitting everything up into their own respective functions, such as setting up parameters for the problem and printing out the data returned from the Proximal Gradient Descent function. More importantly, we were interested in resolving the issue of knowing whether the correct function value was computed when the algorithm "converged." In order to figure this out, we decided to design two test problems in which we knew the answer to the objective function, $f(x) = \min\{\|Ax - b\|_1 + 2\|x\|_1\}$. The dimensions of the matrix A that we decided to test on were $5x5, 10x10, 20x20,$ and $40x40$, where we only ran them for a maximum of 200 iterations. We used an initial x vector of all 0's.

- **Test Problem 1:** If we assume the true solution of x to be a vector of all ones, then we expect $f_{min} = 2\|x\|_1 = 2n$, where n is the number of columns in the matrix A.
    - Based on our data frame results, we successfully saw that the last function value for each dimension was close to $2n$. In the cases that we did not converge, the last function value was close to $2n$ but it was due to algorithm reaching the maximum number of iterations.
- **Test Problem 2:** If we assume the true solution of x to be a vector of all 0's except let the first entry be a 1, then we expect $f_{min} = 2\|x\|_1 = 2$.

- Based on our data frame results, for the first two dimensions $5x5$ and $10x10$, we did not converge due to reaching the maximum number of iterations, however, the last returned function values were very close to 2. On the contrary, for the other two dimensions $20x20$ and $40x40$, we converged but the last function values returned were 3.6 and 3.1. After analyzing the data, the reason that they "converged" was due to the norm of their step sizes being under the $1e-4$ tolerance. In other words, we converged because we weren't making enough progress quickly enough.

Furthermore, we have began transitioning into simulating a real world application, such as image restoration. We are using existing code from https://proximity-operator.net/tutorial.html. I had encountered issues on Jupyter since in order to run the code, you need to pip install their Python library but I was unable to install anything due to working inside the UC Merced system. With the help of Professor Meza, I downloaded Anaconda so that I could run my Jupyter notebooks locally instead and that worked great. I was able to run their notebook but there were some bugs that I solved with ChatGPT but there are still some minor bugs, such as images displaying multiple times (occasionally). Nevertheless, I am now able to run the code using my own image.

Lastly, I have quickly read the article "Proximal Splitting Methods in Signal Processing*", written by Patrick L. Combettes and Jean-Christophe Pesquet, and summarized the main overview regarding the information provided in the abstract, introduction, and conclusion. The document to this summary can be found at ✅ Overview of "Proximal Splitting Methods in Signal Processing*".