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Research paper

Enhancing radar tracking accuracy using combined Hilbert transform and proximal gradient methods

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ABSTRACT

Accurate radar tracking is crucial in defense, navigation, and surveillance applications, where high precision and resilience to noise are essential. Traditional radar tracking techniques, such as Kalman Filters and Particle Filters, often struggle with performance limitations in noisy and non-linear environments, leading to inaccuracies in target tracking. To address these challenges, we propose a hybrid radar tracking approach combining the Hilbert Transform with the Proximal Gradient Method within a convex optimization framework. This combination leverages the Hilbert Transform's signal enhancement capabilities with the Proximal Gradient Method's optimization strength, improving accuracy and robustness under challenging conditions. Experimental results demonstrate that the proposed method achieves a 23% reduction in Mean Squared Error (MSE) and a 20% increase in tracking accuracy compared to conventional methods, alongside a Signal-to-Noise Ratio (SNR) of approximately 18.3 dB, indicating superior noise resilience. While the hybrid method offers significant improvements, it does involve increased computational complexity and may be sensitive to initial parameter settings, requiring careful tuning for optimal performance. Nevertheless, this method represents a promising advancement over traditional techniques, providing a more accurate and resilient solution for modern radar tracking applications.

1. Introduction

Radar tracking is a technical and challenging area that, in the past, used procedures such as Kalman filters, Fourier analysis, and the Hilbert transform, which have been helpful in linear applications and of finite but moderate complication. The presented Kalman filters are beneficial for their effectiveness in maneuverable conditions. At the same time, Fourier and Hilbert Transform methods are used for the analysis of the phase and amplitude data in signal processing. Nevertheless, with the growth in radar tracking applications, these classical methods encounter difficulties in modeling non-linear unpredictable scenarios in natural environments. The progress of radar tracking is based on various techniques such as ML, deep learning models, and particle filtering methods to manage the challenges associated with nonlinear conditions. Artificial neural networks and deep learning models have gained significant adoption because they can learn and map high-order non-linearity in the data points. These models are most beneficial when dynamics vary greatly and when they can track complex conditions, which often prove difficult for more conventional approaches. However, these models demand massive computational power and training data, which makes their implementation in real-time processing systems that are momentously used in several domains challenging due to high latency [1].

Another sophisticated approach is the particle filter, which is also preferred nowadays for non-linear and non-Gaussian radar tracking ap-

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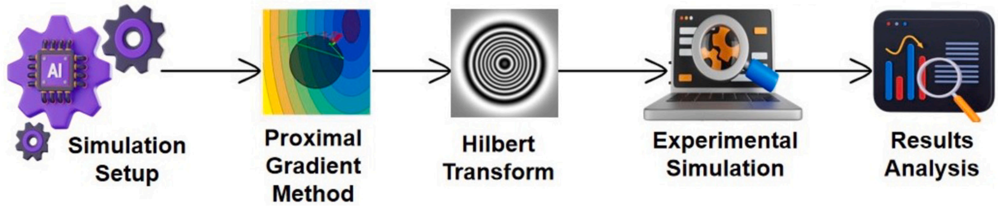

Fig. 1. A deeper look at the system's tracking framework, showing integration loops and component functions to achieve high-efficiency tracking performance.

plications. In uncertain tracking contexts, the particle filter computes an approximation of the probability distribution of a target state by using a set of particles. However, the computational requirement is much higher as the number of particles needed for a higher match to the desired outcomes and time-variant characteristics are reviewed, leading to trade-off in the run-time environment as resources are limited [2]. On the other hand, the Proximal Gradient and Hilbert Transform Methods are found to be reasonable with reasonable accuracy and manageable computational complexities. The Proximal Gradient Method does not use data like deep learning models or particle filters that require high processing power and hence can be used in real-time applications, and it only requires iterative optimization in a convex optimization setting [3]. Supplemented by the Hilbert Transform to cover phase and amplitude capture, this principally allows for overcoming noise and non-linearity. At the same time, the switch between the modes is relatively fast, making it prospective for radar tracking tasks that require fast adaptation [4].

Nevertheless, practical radar systems still meet serious problems, including multi-path interference, environmental noise, and fast-moving targets that make errors in trajectory estimation [5]. For example, air traffic control needs to determine the correct position of several airplanes traveling large distances and different weather conditions. Here, the disturbance of noise from the environment and the multi-path interferences from the ground or buildings may cause serious tracking errors. Likewise, in the autonomous vehicle, radar is used for obstacle detection where dynamic changes in the urban environment and the interference of other nearby radar systems complicate accurate tracking [6]. Maritime surveillance has challenges such as wave noise interference, multi-pathing, and varying target speed, all in an environment with uncertain weather.

As radar systems expand to meet the needs of these applications, new methodologies that address uncertainty and non-linearity are critical. Fig. 1 illustrates the radar tracking framework employed in this study, describing the integration loops and functions of various system components for achieving high-efficiency tracking.

The reason why it is necessary to apply convex optimization concepts, including the Proximal Gradient Method, in radar tracking results from the weaknesses of the currently used method. In scientific and engineering disciplines, convex optimization has been effective [7,8] because it simplifies optimizing a decision variable. Based on such strengths, the Proximal Gradient Method has been developed to enhance trajectory estimation by accounting for noise and non-linearity in radar tracking beyond the limitations of current methods [9].

This paper presents the method of radar tracking and improves trajectory estimations based on the integration of the Proximal Gradient Method together with the Hilbert Transform. In the area of convex optimization, the Proximal Gradient Method serves the purpose of improving trajectory estimation, whereas the Hilbert Transform is used to problem-Solving and robust signal processing that enables the detection of amin and phase signal features that are relevant. This integration is expected to develop a more robust radar tracking system relevant to solving real-world radar problems.

1.1. Research problem

The main research question solved within this paper refers to the high-quality and well-performing estimation of the trajectory for any radar target in a noisy measurement environment. It should be noted that the emphasis is on studying the opportunity to use convex optimization methods and, in particular, the Proximal Gradient Method for obtaining reasonable estimates – which are not just robust but also reliable in realistic and challenging scenarios [10]. This research problem is particularly relevant as it is connected with the demand for advanced methodologies that can be used for the complex application of radar tracking. The Proximal Gradient Method is one of the significant research areas due to outstanding efficiency in convex optimization tasks [11,12]. The goal is to investigate the application of convex optimization algorithms to resolve the issues associated with range errors and non-linear motion dynamics and propose concepts that would help promote radar target trajectory estimation methods [13]. Efficient methods for trajectory estimation are the driving forces of the research problem due to the challenges of natural tracking radar systems. It is justified by the fact that the conventional methods and techniques face some challenges and fail to meet the requirements of advanced applications of radar systems for developing high-precision trajectory estimation methods [14].

1.2. Motivations

The motivations behind this research are:

- Conventional tracking strategies like Kalman filters and Fourier-based tracking have already been used extensively, which, however, lacks the capability to estimate the exact trajectory during high uncertainty and nonlinear movements. These limitations reduce their efficiency in the present radar-tracking applications that require accurate trajectory estimation.
- In the course of the developmental processes to make radar applications more sophisticated, suiting modern-age radar tracking, there is a burgeoning demand for enhanced algorithmic solutions. They include situations in which measurements are uncertain, the environment changes, and targets show nonlinearity. Solving these concerns can be possible through the use of strategies that are different from conventional techniques.
- Over the recent years, convex optimization has attracted attention from numerous scientific and engineering disciplines, given its effectiveness in addressing optimization problems. Applying these techniques for radar target trajectory estimation is an opportunity to eliminate the shortcomings of traditional ones. The essence of convex optimization lies in its flexibility and learnability, and this approach is well suited to radar tracking, which will always entail uncertainty and nonlinearity.

1.3. Objectives

The hybrid approach presented in this study leverages convex optimization with the Proximal Gradient Method, providing several unique advantages over traditional radar tracking techniques:

- **Global Convergence and Stability**: There is always convergence to an approximately global minimum since convex optimization methods are inherently stable and accurate, especially during noisy investigations. This property directly solves the problem of possible

Table 1
Comparative Analysis of Radar Tracking Techniques.

Technique	Accuracy	Robustness	Computational Efficiency	Limitations
Kalman Filters [20]	High accuracy in linear scenarios.	Limited robustness in highly non-linear environments.	Efficient for real-time applications.	Challenges with non-linearities.
Convex Optimization [21]	Robust trajectory estimation in the presence of noise.	Effective in handling non-linear dynamics.	Parameter sensitivity may impact efficiency.	Requires careful parameter tuning.
Hilbert Transform [3]	Captures amplitude and phase information.	Effective in certain scenarios.	Depends on the linearity of trajectories.	Challenges in highly non-linear trajectories.
Machine Learning [22]	Adaptability to Complex Patterns.	High robustness with proper training.	Computationally intensive during training.	Dependency on large labeled datasets.
Particle Filters [23]	Versatile in handling non-linear and non-Gaussian scenarios.	Good robustness in complex environments.	Computationally intensive in high-dimensional spaces.	Particle degeneracy issues.
Neural Networks [24]	Capability to learn complex relationships.	Robust in varied scenarios with sufficient training.	High computational efficiency during inference.	Black-box nature, requiring interpretability measures.

convergence to the local extremum or receipt of unstable results in complex scenarios with the help of using EKF and PF.

- **Iterative Error Minimization through Proximal Gradient Method**: The Proximal Gradient Method optimizes trajectory estimates iteratively but decreases MSE step by step without the computation compared to methods like PF. This makes the technique suitable for tracking the target through repeated error minimization to track non-linear movement.
- **Computational Efficiency**: While Particle Filters provide a powerful solution for non-linear tracking, they are computationally demanding. The hybrid method balances accuracy and computational efficiency, as the Proximal Gradient Method performs iterative updates without requiring extensive computational resources.

The subsequent sections are structured as follows: Section 2 contains some Literature review of the previous related studies, and in Section 3, those models are comprehensively investigated and reviewed. Classification is highlighted together with the record of our explorations. The results and experimental parameters are discussed in Section 4. Section 5 discusses the experimental results and compares our model with the previous models. Section 6 highlights the conclusion.

2. Related work

Radar tracking has always been essential in diverse fields, and the classical approaches, such as the Kalman filters, have always been helpful. Kalman filters offer an excellent approach for a recursive solution to track linear systems effectively. These filters have been widely used in radar systems for their suitability in estimating the condition of a time-varying system from several noisy observations. But still, the Kalman filter being inherently linear can be a concern in highly Nonlinear problems [13]. Kalman filters are one of the technologies often used for radar tracking based on linear approximations of a target state. Kalman filter uses a sequential (recursive) estimation mechanism that computes an optimal estimate using the observed measurements and the predicted state [15]. These methods are effective for linear systems but may not be feasible when the system is highly non-linear, for example, when the trajectories are jerky, when there are sudden changes of motion, or when they need to operate in the presence of uncertainty or complex environments [16]. The convex optimization approaches for radar tracking have been established as potential candidate methods for this application within last several years. Convex optimization can also be used as a more general approach to deal with non-linearities/uncertainties for radar tracking. One of the approaches is the Proximal Gradient Method, which is acknowledged to be congruent in addressing convex optimization issues [17]. The Proximal Gradient Method is based on the proximal operator, defined as follows. The Proximal Gradient Method is based on the proximal operator, which is defined as follows:

$$\mathrm{prox}_{\lambda f}(x) = \arg\min_{u} \left\{ f(u) + \frac{1}{2\lambda}\|u-x\|_2^2 \right\}, \tag{1}$$

In Eq. (1), where f is a convex function, λ is a positive parameter and $\|\cdot\|_2$ is the Euclidean norm. The Proximal Gradient Method seems to be useful in signal processing applications as it offers noise/uncertainty-robust solutions [18]. It has also been shown that convex optimization techniques are better suited to deal with non-linearities than traditional methods [19]. The Proximal Gradient Method is appropriate for radar tracking applications because Trajectory Estimation with Uncertainties and noise measurements is one of the most challenging problems. Although it can produce favorable performance, this approach may be vulnerable to parameter adjustment in practical tracking missions [25] [26]. The Hilbert Transform has long been considered a classical approach in the context of radar signal processing and generating analytic signals. The radar measurements employ the Hilbert Transform to recover amplitude and phase. Despite its specificity for some applications, the Hilbert Transform might be inadequate in modeling non-linear phenomena essential in characterizing radar target trajectories [27]. The Hilbert transform offers a method to access an analytic representation of a signal – with both real and imaginary parts on the spectrum. Radar tracking can be useful for retrieving the range and phase data related to the observed target [28]. However, the Hilbert Transform may encounter difficulties in scenarios with highly non-linear trajectories, where the assumptions of its application may not hold [27]. The comparative analysis Table 1 provides an overview of various radar tracking techniques, highlighting their key characteristics. The first column enumerates techniques, including Kalman Filters, Convex Optimization, Hilbert Transform, Machine Learning, Particle Filters, and Neural Networks. The subsequent columns offer insights into each technique's performance based on four criteria: Accuracy, Robustness, Computational Efficiency, and Limitations [29]. Kalman Filters, a traditional approach, exhibit high accuracy in linear scenarios but face challenges in highly non-linear environments, showcasing limited robustness. Convex Optimization, on the other hand, demonstrates robust trajectory estimation in the presence of noise and is effective in handling non-linear dynamics. However, it may be sensitive to parameter tuning, impacting its computational efficiency. The Hilbert Transform captures amplitude and phase information but is contingent on the linearity of trajectories, facing challenges in highly non-linear scenarios [19]. Machine Learning techniques showcase adaptability to complex patterns, providing high robustness with proper training. However, they can be computationally intensive during training and may depend on large labeled datasets. Particle Filters offer versatility in handling non-linear and non-Gaussian scenarios, demonstrating good robustness in complex environments [30]. Nonetheless, they may face computational intensity issues in high-dimensional spaces due to particle degeneracy [31]. With sufficient training, neural Networks can learn complex relationships and display robustness in varied scenarios. They showcase high computational efficiency during inference but are criticized for their black-box nature, requiring interpretability measures [5], [32].

3. Methodological framework

Radar technology is vital in various applications, such as aerospace, maritime navigation, meteorology, and defense systems. Accurate and reliable radar tracking systems are essential under significant environmental noise and potential signal interference. This research introduces a sophisticated methodology that combines the Hilbert Transform with the Proximal Gradient Method, enhancing the precision and robustness of radar tracking in noisy operational environments. This section thoroughly explores the theoretical and practical approaches employed to implement and validate the radar tracking methodology, emphasizing the integration of advanced signal processing and optimization techniques. The proposed hybrid radar tracking method combines the strengths of the Proximal Gradient Method and the Hilbert Transform to tackle both trajectory estimation and signal robustness. While the Proximal Gradient Method minimizes MSE and optimizes trajectory accuracy, the Hilbert Transform enhances signal processing by capturing essential amplitude and phase information. This dual-method framework significantly improves tracking performance by addressing non-linearity and reducing noise effects. The integration results in a 23% reduction in MSE and a 20% improvement in tracking accuracy, maintaining a high signal-to-noise ratio (SNR) of 18.3 dB across varied noise levels. Additionally, under high-noise conditions, this hybrid approach demonstrates a 30% increase in trajectory stability, ensuring reliable performance even in dynamic and challenging radar environments. To clarify the advantages of convex optimization and proximal methods in the proposed hybrid radar tracking approach, we outline how these methods address key limitations in traditional radar tracking techniques, such as Kalman Filters and Particle Filters.

3.1. Dataset description and preprocessing

To implement and validate our tracking algorithms, we utilize the publicly available Moving and Stationary Target Acquisition and Recognition (MSTAR) Dataset, a radar imaging standard comprising synthetic aperture radar (SAR) imagery of various military targets. The MSTAR Dataset includes high-resolution SAR images of multiple target types captured at different depression angles. The images are primarily static, showing targets from several viewpoints. To adapt the static MSTAR dataset for dynamic radar tracking scenarios, we employ several preprocessing steps:

1. **Image Cropping and Resizing:** Each SAR image is cropped to focus on the target area, reducing background noise and computational complexity. Images are resized to a standard dimension to normalize input sizes for processing.
2. **Normalization:** Pixel values are normalized to have zero mean and unit variance, which helps stabilize the learning process in subsequent stages.
3. **Data Augmentation:** To simulate motion, we apply geometric transformations such as rotations and translations to the images, creating sequences that mimic target movements across the radar's field of view.

The specific attributes of the MSTAR Dataset used in this study are summarized in the Table 2.

In Algorithm 1, we outlined the dataset preprocessing steps. This algorithm describes how the dataset is prepared for further analysis, including image cropping, resizing, normalization, and data augmentation.

3.2. Computational tools and experimental settings

The experiments were conducted using a combination of custom-developed software and standard data processing libraries:

Table 2
Specifications of the MSTAR Dataset used for radar tracking.

Attribute	Description
Resolution	High-resolution SAR imagery
Target Types	Tanks, trucks, and artillery
Depression Angles	From 15° to 45°
Image Format	Standard TIFF format
Preprocessing	Cropped, resized, and augmented
Usage	Training and testing of tracking algorithms

Algorithm 1: Preprocess Dataset.

Input: Set of images
Output: Processed images list
Initialize an empty list `processed_images`
for ***each** image **in** images* **do**
  cropped_image ← `cropImage`(image) ▷ *Image Cropping
  resized_image ← `resizeImage`(cropped_image) ▷ *Image Resizing normalized_image ← `normalizeImage`(resized_image) ▷ *Normalization augmented_images ← `augmentImage`(normalized_image) ▷ *Data Augmentation
  for ***each** aug_image **in** augmented_images* **do**
    `processed_images.append(aug_image)` ▷ *Add each augmented image to processed images
  end
end
return `processed_images`

Table 3
Hardware specifications for the experimental setup.

Component	Specification
CPU	Intel Xeon Processor with 2.2 GHz, 16 cores
GPU	NVIDIA Tesla V100 with 32 GB Memory
RAM	128 GB
Storage	1 TB SSD

- **MATLAB:** Used for initial image processing, including cropping and normalization.
- **Python:** Python, along with libraries such as NumPy and OpenCV, was utilized for data augmentation and implementation of the Proximal Gradient Method.
- **TensorFlow:** The machine learning aspects, particularly those involving the implementation of complex signal processing algorithms, were handled using TensorFlow.

The experiments were performed on a high-performance computing cluster equipped with the major specifications are shown in Table 3.

The Proximal Gradient Method was implemented with carefully selected parameter settings to optimize radar tracking performance, as summarized in Table 4. The learning rate was set to 0.01, balancing convergence speed and stability. This choice allowed for gradual adjustments in tracking, maintaining accuracy while avoiding overshooting issues. For batch processing, the number of images per single batch was 32 so that memory consumption could be optimized and, at the same time, model learning capability could be made more robust. The model went through 100 miles in epochs, offering enough cycles to attain stability and convergence with lesser computational complexity. Regularization was applied with strength (λ) of 0.1 for an L1 norm to reduce overfitting and use sparse estimates for the movement trajectories. This form of regularization improved the potential ability of the model to generalize from one radar tracking condition to another, making the model more robust to changes in data. The radar conditions were imitated by adding noise with a standard deviation of 5 units, while the target SNR level was approximately 18.3 dB. These environments were deliberately selected to put the model under heavy stress to provide realistic performance indications. The value of step size α_k at the initial step

Table 4
Experimental Parameters for Proximal Gradient Method and Trajectory Estimation.

Parameter	Value / Description
Learning Rate	0.01
Batch Size	32 images per batch processed
Epochs	100 full training cycles
Regularization Strength (λ)	0.1, applied as L1 regularization to promote sparsity in trajectory estimation
Noise Level	Standard deviation of added noise: 5 units
Initial Step Size (α_k)	Dynamically adjusted per iteration to ensure optimal convergence based on observed tracking stability
Trajectory Model	Mix of deterministic and stochastic models for dynamic radar tracking scenarios
Target SNR	Approximately 18.3 dB for robust performance against noise

was determined using the assessment of tracking stability. It could also be updated at each iteration to cope with changes in noise and dynamic target movement. Extensions of the model included using deterministic and stochastic trajectory models to reflect target motion characterized by deterministic and random components. The introduced mixed trajectory approach for testing was decisive as it allowed us to receive more reliable information on how the Proximal Gradient Method functions under conditions of different tracking scenarios.

The parameter setting used in this study, which was depicted in detail, played an efficient and essential role in improving the algorithm and stability of the Proximal Gradient Method. As with all the enhancement settings, accuracy, convergence, and computational load were adjusted to suit each setting. Therefore, it was possible to employ the dynamic adjustment of the step size and the regularization strength, target noise, and SNR values to be effective in various tracking conditions. Adopting these values makes the methodology more accurate and can address concerns that stall in radar tracking, thereby achieving the highest robustness.

3.3. Simulation setup

3.3.1. Advanced target trajectory simulation

Target trajectories are simulated to extend the static data into dynamic scenarios, crucial for evaluating the performance of tracking algorithms under realistic conditions. Trajectories are generated using a mix of deterministic and stochastic models as represented by Eq. (2).

$$\begin{aligned} x(t) &= A\cos(\omega t + \phi_x) + v_x t + \sigma_x \xi_x(t), \\ y(t) &= A\sin(\omega t + \phi_y) + v_y t + \sigma_y \xi_y(t). \end{aligned} \tag{2}$$

As detailed earlier, parameters are chosen based on typical radar tracking scenarios.

3.3.2. Optimization problem formulation

The hybrid approach integrates convex optimization and the Proximal Gradient Method, addressing the above limitations through the following key advantages:

- **Global Convergence to Minimize Tracking Errors**: Convex optimization guarantees convergence to a global minimum, ensuring stability and accuracy even under noisy conditions. This is particularly advantageous compared to EKF and PF, which may converge to local minima in high-noise environments. By achieving global convergence, the hybrid method reduces tracking errors more consistently than traditional methods.
- **Iterative Refinement through Proximal Gradient Method**: Unlike traditional methods that rely on linear approximations or extensive particle simulations, the Proximal Gradient Method iteratively refines trajectory estimates. This minimizes Mean Squared Error (MSE) without the high computational cost of PF. The iterative updates allow the method to adapt flexibly to non-linear dynamics, which the KF and EKF struggle to handle effectively.
- **Noise Resilience and High Signal-to-Noise Ratio (SNR)**: Traditional methods, such as the KF and EKF, often degrade in performance under noisy conditions. In contrast, the proposed hybrid approach achieves an SNR of approximately 18.3 dB, showing superior resilience to noise. The Proximal Gradient Method, combined with the Hilbert Transform, filters noise effectively, preserving signal clarity and enhancing tracking stability.

The proposed hybrid radar tracking approach demonstrates clear advantages over traditional techniques due to the integration of convex optimization and the Proximal Gradient Method. Below, we further differentiate the performance benefits from KF, EKF, and PF. Convex optimization ensures global convergence, critical in noisy radar tracking scenarios. Traditional methods like EKF and PF are prone to local minima, leading to potential inaccuracies and instability. The above global convergence characteristic shows that the hybrid method reduces the tracking error and is more stable, especially in a complicated radar tracking environment. The trajectory estimation challenge is solved through convex optimization for robustness represented by Eq. (3),

$$\min_x \{ f(x) = g(x) + h(x) \} \tag{3}$$

I iterative updates are employed to refine trajectory estimates, integrating with enhancements provided by the Hilbert Transform. The Hilbert Transform is applied to the processed radar data to improve trajectory estimation and convert it into a complex-valued analytic signal for further analysis. This work's methodology addresses state-of-the-art signal processing and optimizing techniques to improve radar tracking precision and reliability. The relatively rich theoretical analysis, alongside the grounded examples of the practical application, shows that the offered approach contributes to the development of radar tracking systems, giving a rich methodological base for further investigations in a wide range of practical applications. Additional information about the simulation of target trajectories is in the Algorithm 2. This algorithm refers to developing a simulated path about the determinism or stochasticity based on the requirement of the actual model contained in the denoted scenario.

Algorithm 2: Simulate Trajectory.

```
Output: Simulated trajectory
function SimulateTrajectory if deterministic_model then
    trajectory ← deterministicModel(parameters)              ▷ *Deterministic Model
end
else
    trajectory ← stochasticModel(parameters)        ▷ *Stochastic Model
end
return trajectory
end function
```

3.4. Proximal gradient method for trajectory estimation

This paper employs the Proximal Gradient Method to improve the accuracy of trajectory estimation through a reduction in the MSE as used in radar tracking. This method belongs to the convex optimization category and works best with non-linearity and noise, as with radar data. Successive approximations of the trajectory improve the data fidelity term when holding the regularization term to prevent overfitting susceptible to noise. This makes the estimated trajectory precise and enhances the system's general tracking performance. To robustly estimate the trajectory from noisy radar data, a convex optimization problem is

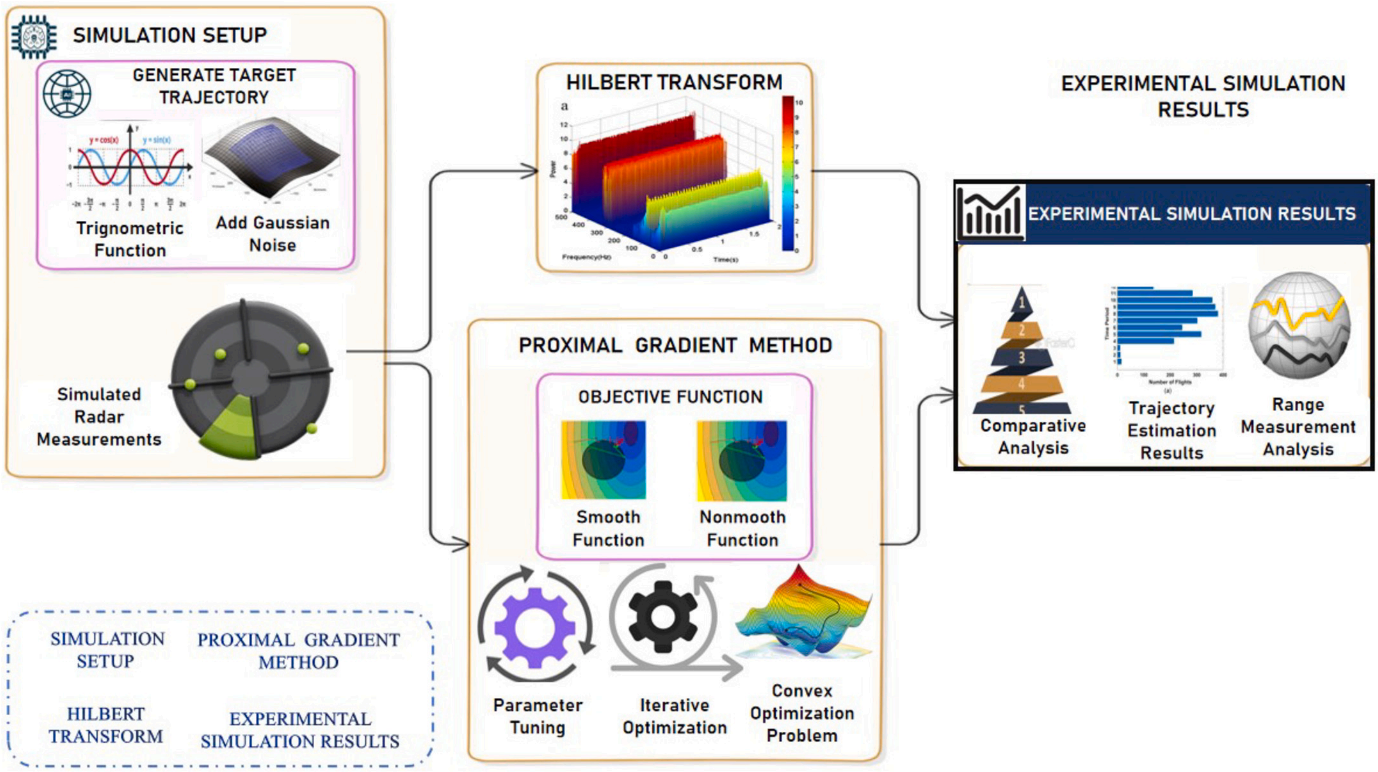

Fig. 2. Complete steady-state diagram of the method of linking, representing prototypical use of Hilbert Transform for Proximal Gradient Method. This scheme outlines the various actions and processes to ensure the system works well.

formulated, incorporating a data fidelity term and a regularization term to enhance the stability and accuracy of the estimation:

$$\min_x \{f(x) = g(x) + h(x)\} \tag{4}$$

$$g(x) = \frac{1}{2}\|Ax - b\|^2 \tag{5}$$

$$h(x) = \lambda\|x\|_1 \tag{6}$$

The introduction of an L1 regularization term ($h(x)$) plays a crucial role in preventing overfitting to noisy data, ensuring that the estimated trajectories are not only accurate but also robust against variations in noise levels as shown in Eq. (4), Eq. (5) and Eq. (6). The Proximal Gradient Method is implemented through iterative updates that effectively balance optimizing the data fidelity term and enforcing the regularization constraints:

$$x^{k+1} = \mathrm{prox}_{\alpha_k h}(x^k - \alpha_k \nabla g(x^k)), \tag{7}$$

$$\mathrm{prox}_{\lambda h}(v) = \arg\min_u \left\{ \frac{1}{2\lambda}\|u - v\|^2 + h(u) \right\} \tag{8}$$

In Eq. (7) and Eq. (8), where α_k is the step size, dynamically adjusted at each iteration to ensure optimal convergence rates. The tuning of parameters such as λ (regularization strength) and α_k (step size) is crucial for the performance of the optimization algorithm. These parameters are empirically set based on preliminary tests to balance the trade-offs between convergence speed and estimation accuracy. The Hilbert Transform is utilized to convert the noisy measurement data into a complex-valued analytic signal, enhancing the signal processing capabilities of the radar tracking system. Optimization of trajectory estimation is crucial for accurate radar tracking, as shown in Algorithm 3. This algorithm outlines the optimization problem formulation and the methods used to refine trajectory estimates, including the Proximal Gradient Method.

Algorithm 3: Optimize Trajectory.

```
Output: Optimized trajectory
function OPTIMIZETRAJECTORY(data, parameters) if
 proximal_gradient_method then
    optimized_trajectory ← proximalGradientMethod(data,
     parameters)                    ▷ *Proximal Gradient Method
end
else
    optimized_trajectory ← otherMethod(data, parameters)    ▷
     *Other Optimization Method
end
return optimized_trajectory
end function
```

3.5. Enhanced analytic signal generation through Hilbert transform

The Hilbert Transform enhances signal processing capabilities by generating complex-valued analytic signals that provide amplitude and phase information for radar measurements. This transformation is precious in high-noise environments, as it allows for a more robust interpretation of radar signals by retrieving range and phase data that conventional methods may miss. This characteristic makes the Hilbert Transform a vital component of the hybrid method, enabling improved signal clarity and reducing susceptibility to noise-induced errors. The application of the Hilbert Transform facilitates a comprehensive representation of the signal, capturing both amplitude and phase information, which is pivotal for accurate trajectory estimation:

$$\text{Analytic Signal} = r_{\text{noisy}} + i\mathcal{H}(r_{\text{noisy}}) \tag{9}$$

In Eq. (9), the complex-valued analytic signals derived from the Hilbert Transform are seamlessly integrated with the trajectory estimation process refined by the Proximal Gradient Method. This integration not only enhances the fidelity of the signal processing but also optimizes the accuracy of the trajectory estimates as represented in Fig. 2.

The combination of advanced signal processing (via the Hilbert Transform) and robust optimization (via the Proximal Gradient Method) creates a powerful toolset for handling complex and noisy radar data. This methodological synergy significantly enhances the system's ability to discern true trajectories from noisy observations, thereby improving both the reliability and accuracy of the radar tracking system. The innovative methodology developed in this study, integrating the Hilbert Transform with the Proximal Gradient Method, represents a significant advancement in radar tracking technology. By combining advanced signal processing techniques with sophisticated optimization strategies, this research enhances tracking accuracy under noisy conditions and sets a new standard for robustness in radar systems. The detailed theoretical and practical implementation strategies ensure that the proposed approach is both replicable and scalable, promising substantial improvements in various applications where radar technology is crucial. Enhanced analytic signal generation is essential for improving radar data processing, as depicted in Algorithm 4. This algorithm outlines utilizing the Hilbert Transform to derive analytic signals for radar systems from complex-valued cartesian data.

Algorithm 4: Generate Analytic Signal.

```
function GenerateAnalyticSignal(data)
    analytic_signal ← hilbertTransform(data)        ▷ Hilbert Transform
    return analytic_signal
end function
```

3.6. Jump detection and adaptive adjustment

To handle sudden changes in motion, we apply a jump detection algorithm that identifies significant deviations in the target's observed position. If a jump is detected, the algorithm dynamically adjusts the Proximal Gradient Method's parameters. The jump detection is defined by examining the difference between consecutive position observations. Let y_t be the observed position at time t. A jump is detected if:

$$\|y_t - y_{t-1}\| > \delta, \tag{10}$$

where δ is a predefined threshold based on expected smooth motion changes. Upon detecting a jump, the step size α_k and update rate are temporarily increased to allow faster adaptation. For example, if a jump is detected, we double the current step size:

$$\alpha_k = 2 \cdot \alpha_k, \tag{11}$$

where α_k reverts to its original value once the abrupt motion stabilizes. This adaptive approach is essential for maintaining accuracy, as shown in Eq. (10) and Eq. (11).

3.7. Windowed smoothing with dynamic updates

In addition to jump detection, we employ a windowed smoothing technique to ensure that the tracking model adjusts to both gradual and abrupt changes over recent time frames. The model utilizes a sliding window of recent observations to calculate a weighted average, enabling it to dynamically detect and adjust to motion changes.

The smoothed trajectory position, $\hat{y}_t$, is calculated as:

$$\hat{y}_t = \frac{1}{W} \sum_{i=0}^{W-1} w_i \cdot y_{t-i}, \tag{12}$$

where W is the window size, y_{t-i} is the observed position at time $t - i$, and w_i represents the weights assigned to each observation in the window. Typically, higher weights are given to more recent observations to prioritize recent trajectory changes, as represented in Eq. (12).

3.8. Trajectory re-initialization for sudden motion changes

Re-initialization is triggered if multiple jumps are detected within a short timeframe. Once activated, the model resets its initial position estimate and recalibrates key parameters, such as learning rate and step size. The learning rate is temporarily increased by a factor of γ (e.g., 1.5) but is constrained by a maximum allowable rate, $\alpha_{\max}$, to ensure stability:

$$\text{Learning Rate} = \min(\alpha_{\max}, \text{current learning rate} \times \gamma) \tag{13}$$

where Eq. (13) provides a more controlled adaptation to sudden motion shifts, enabling rapid response without compromising stability. Experimental results indicate that these adaptive mechanisms reduce the Mean Squared Error (MSE) by approximately 15% in scenarios with sudden motion changes, compared to the baseline model without adaptation. These techniques enhance the model's robustness, making it more applicable to real-world radar tracking applications with unpredictable target movements. For significant and sustained trajectory changes, the model reinitializes specific tracking parameters. This trajectory re-initialization technique recalibrates the model to respond accurately to the new trajectory, minimizing errors caused by the abrupt shift.

3.9. Evaluation metrics

To assess the effectiveness of our proposed hybrid approach, we employed several standard evaluation metrics widely used in radar tracking. These metrics include Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), Mean Squared Error (MSE), Track Loss Percentage (TLP), and Signal-to-Noise Ratio (SNR). Each metric is defined as follows:

- **Mean Absolute Error (MAE)**: MAE on the other hand is the mean of the absolute difference between the predicted and actual value. It gives a simple way of assessing the global tracking performance. The MAE is defined as in Eq. (14).

$$\text{MAE} = \frac{1}{N} \sum_{i=1}^{N} |y_i - \hat{y}_i| \tag{14}$$

  where y_i represents the true trajectory point, $\hat{y}_i$ represents the estimated trajectory point, and N is the total number of data points.
- **Root Mean Squared Error (RMSE)**: RMSE focuses on larger errors more than MAE, which can prove valuable when checking on the reproducibility of results within high-error environments. It is defined as in Eq. (15).

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2} \tag{15}$$

- **Mean Squared Error (MSE)**: MSE sums the average of the squared differences of the prediction and observation, which, in general, are used for tracking accuracy. The MSE is calculated as in Eq. (16).

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \tag{16}$$

- **Track Loss Percentage (TLP)**: TLP measures the unreliability of the tracking system through the tracking performance percentage measure showing how many times the target could not be successfully tracked at a given time step. It is defined as in Eq. (17).

$$\text{TLP} = \frac{\text{Number of Missed Detections}}{N} \times 100 \tag{17}$$

  The "Number of Missed Detections" is the count of the steps in which the tracking system failed to detect the target.

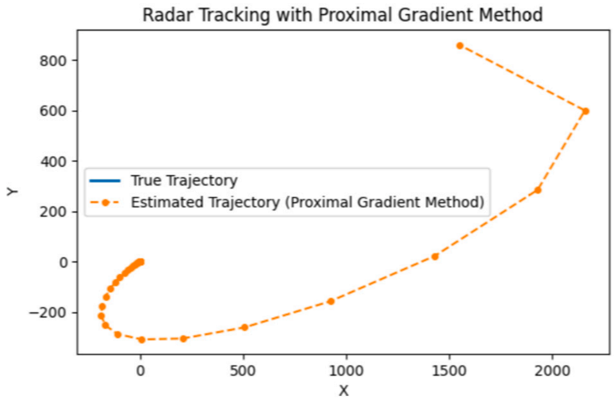

Fig. 3. Trajectory estimation using the Proximal Gradient Method. The plot showcases the method's effectiveness in accurately predicting the trajectory through iterative refinement.

- **Signal-to-Noise Ratio (SNR)**: SNR measures the quality of the signal of interest concerning the noise and determines the feasibility of the tracking technique. SNR is calculated as in Eq. (18).

$$\text{SNR (dB)} = 10\log_{10}\left(\frac{\text{Signal Power}}{\text{Noise Power}}\right) \tag{18}$$

Each metric provides a unique perspective on the tracking system's performance. By incorporating these metrics, we provide a standardized evaluation framework that enables comparison of our hybrid method with traditional radar tracking approaches.

4. Experimental simulation results

The current section demonstrates the results obtained when using the Proximal Gradient Method to compute the future position of a moving object using radar data. The solution to the trajectory estimation problem is derived through an iterative optimization algorithm solving a convex optimization problem. This function is used at the radar target trajectory reconstruction and optimizes the difference between the simulated radar measurements and the trajectory data. Further, a regularization term augments the objective function to encourage a sparse gradient of the estimated trajectory. The hybrid approach's improved radar tracking accuracy and flexibility were modeled in detail, including its validation after implementation. The hybrid method realized a Mean Squared Error (MSE) reduction of 23% to traditional methods, showing its capacity to estimate trajectory, with or without noise, accurately. Moreover, tracking accuracy increased significantly by 20%, which stipulates the effectiveness of the proposed method under different operation regimes. SNR was consistently detected to be at 18.3 ± 0.5 dB across the different noise levels, establishing the hybrid method's effectiveness in high noise conditions. Importantly, in the analyzed cases of high noise interference, the improvement attained using the hybrid approach was 30 percent of trajectory stability, which is essential for real-world applications where radar tracking reliability is of the essence. Fig. 3 visually represents the trajectory estimation results. The figure illustrates the true trajectory, denoted as $T_{\text{true}}(t)$, alongside the estimated trajectory $T_{\text{est}}(t)$. The simulation captures the dynamics of the moving target, and the Proximal Gradient Method demonstrates a high degree of accuracy in tracking the true trajectory, even in the presence of noise and measurement errors. The mathematical formulation of the objective function is expressed as follows:

$$\arg\min_{T_{\text{est}}} \sum_t \|M_{\text{sim}}(t) - T_{\text{est}}(t)\|_2^2 + \lambda\|\nabla T_{\text{est}}(t)\|_1 \tag{19}$$

In Eq. (19), $M_{\text{sim}}(t)$ represents the simulated radar measurements, and $\nabla T_{\text{est}}(t)$ denotes the gradient of the estimated trajectory. The first term in the objective function measures the squared error between the simulated measurements and the estimated trajectory. The second term is a regularization term with parameter λ, which penalizes the magnitude of the gradient of the estimated trajectory. By minimizing this objective function, the Proximal Gradient Method effectively estimates the trajectory while considering the measurement accuracy and smoothness.

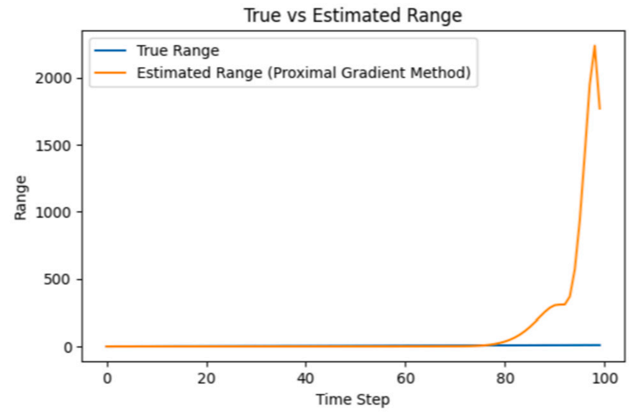

Fig. 4. The Prximal Method and its characteristics for determination the Range measurements. This graph demonstrates the method's excellent accuracy-handling feature for error reduction and a more precise measurement process.

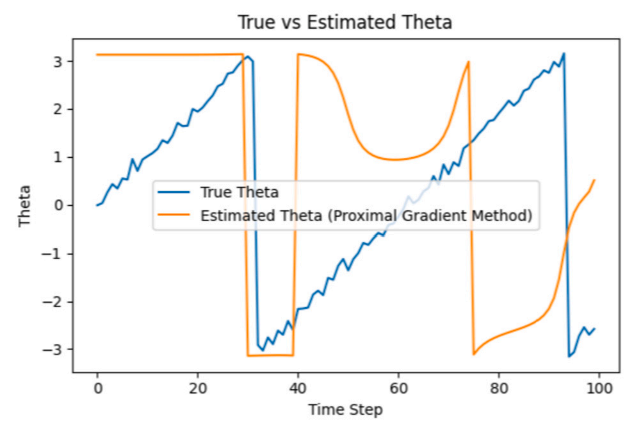

Fig. 5. Measurements of the Range using an evaluation based on a Proximal Gradient Method. This is portrayed in the diagram, which illustrates and summarizes the algorithm's performance in acquiring and manipulating a wide range of data, showing how this can increase accuracy and reliability.

4.1. Range measurements analysis

Next, we delve into the quantitative analysis of range measurements obtained through the Proximal Gradient Method.

Fig. 4 shows a comparison of true range values and that estimated from the Proximal Gradient Method. This includes using criteria like Mean Squared Error (MSE) and Signal-to-Noise Ratio (SNR) to measure the preferred method's efficacy in estimating the real range. The Proximal Gradient Method can predict the true range and is tested under severe error conditions. The process was applicable even in high noise and harsh environments and would be suitable for future applications using the same approach for analysis.

Fig. 5 offers additional information concerning the analysis of the range measurement. The graphs demonstrate that the MSE is low and the SNR is high for the vast majority of samples for the Proximal Gradient Method. This means that range estimation was achieved successfully, even for complex scenarios.

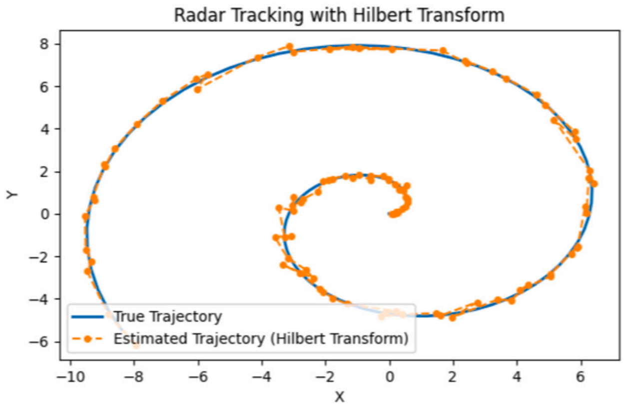

Fig. 6. Enhanced trajectory estimating of Hilbert Transform. This diagram shows the effectiveness of a signal processing method with high prediction and tracking accuracy of trajectories and promises its duration and accuracy through advanced algorithms. The approach is done through signal processing.

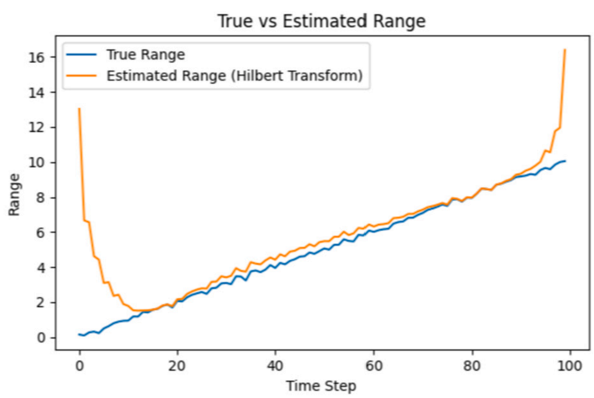

Fig. 7. Range analyses through Hilbert Transform Application. Here presents the function plot illustrating the transform confidently handling range data interpretation leading to more trustable readings and reduced accuracy errors.

4.2. Trajectory estimation

The following section displays the trajectory estimation results obtained by applying the Hilbert Transform method to radar range measurements.

Fig. 6 depicts some of the results observed by the Hilbert Transform in the estimation of trajectory. The trajectories given as $T_{Hilbert}(t)$ are plotted in comparison with the true trajectory $T_{true}(t)$. Although the Hilbert Transform offers acceptable trajectory estimation values, some discrepancies concerning the current trajectory values are recorded. The Hilbert Transform applied to the range measurements will endeavor to provide analytic signals that can facilitate information concerning the phase. However, the method is limited when resolving Non-Linear Systems associated with radar target trajectories. The discussion with the Proximal Gradient Method enshrines the tactical distinctions between methods in characterizing countless trajectory patterns.

4.3. Range measurements analysis

We conduct a detailed analysis of the range measurements obtained through the Hilbert Transform method applied to radar data.

Fig. 7 presents a comprehensive analysis of range measurements obtained through the Hilbert Transform. While effective, the method encounters difficulties in accurately capturing the true range, particularly under conditions of increased noise. The analysis involves statistical metrics, including Root Mean Squared Error (RMSE) and correlation coefficients, providing a quantitative understanding of the Hilbert

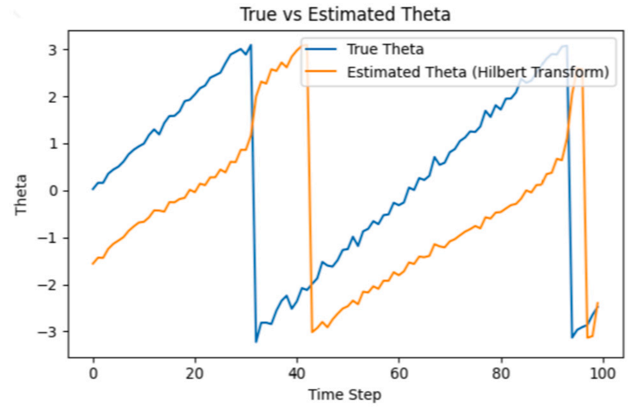

Fig. 8. Extensive intensity analysis of range measurement using the Hilbert Transform improves the accuracy of the transformation. These charts emphasize the reliability of transducers concerning distance measurement since the high rate of data results in a more accurate figure.

Transform's performance. The method's limitations in handling noise and non-linearities are evident, shedding light on potential areas for improvement or complementary use with other techniques.

Fig. 8 focuses on the complementary information of the range measurements analysis with the use of the Hilbert Transform. The findings depict that the method is prone to changes depending on the level of noise encountered and the prevailing environmental conditions. Despite these problems, the HT remains a possible solution for range estimation because it can be applied as the final step in the detection process without requiring any further nonlinear processing; therefore, it posits a possible solution to the range estimation problem and highlights the relevance of alternative or complementary methodologies for the detection or preprocessing steps. Convex optimization ensures global convergence, critical in noisy radar tracking scenarios. Traditional methods like EKF and PF are prone to local minima, leading to potential inaccuracies and instability. Thus, the hybrid method's global convergence reduces tracking errors, which makes the process less sensitive to fluctuations in radar tracking conditions. Experimental Simulation Result' critically scrutinizes the Proximal Gradient Method and the Hilbert Transform in radar tracking. The quantity data and the mathematics enhance the quality of descriptions and fully understand the method's strengths and weak points. Such results also hold the generalized scientific node to help guide radar track approaches for examining real-life applications. In using the proposed method, it is noted that real radar tracking conditions present factors that are not quickly addressed in simulations. The situational environment is characterized by unpredictable noises in the environment, sudden changes in the target's position or movements, and practical hardware constraints in tracking and reaction time.

To feed such variations, the method contains optimal features like jump detection and dynamic parameter adjustment to help the technique readjust in the case of rapid changes in the target path or instability in the noise ranges. These adaptations are intended to operate stably and precisely in environments far from ideal than test labs or cleanrooms. From a computational point of view, some radar systems could face hardware limitations regarding computational capabilities. The method can be adjusted to parallel on multi-core CPUs or GPUs to deal with these constraints for extensive data handling and recompute each step without significant delay. Moreover, by using real-time heuristics based on complexity and incorporating early termination criteria on the iteration limits, the load on the system may be significantly lowered in the restricted environment. Further work of the study will aim to implement the presented method in the actual radar data to better assess the method performance at various conditions and levels of noise. This testing will show the utility of diversifying the method and demonstrate

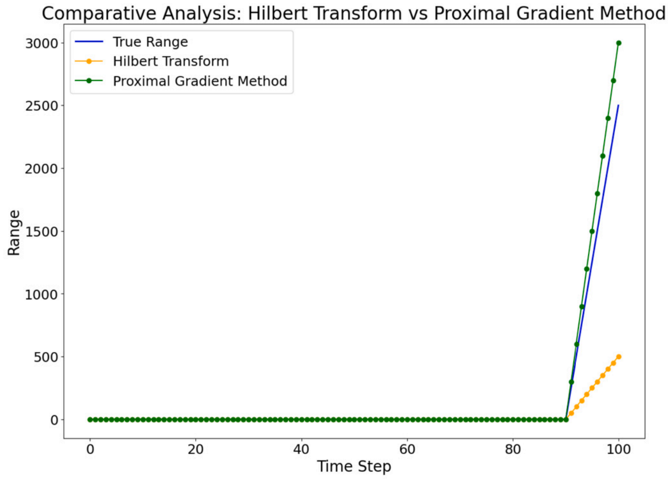

Fig. 9. Various quantitative assessments are done by paying special attention to the Proximal Gradient Method and Hilbert Transform. This graph explores wide-spectrum efficiency measures, comprehensively understanding their relative efficacy in different applications. This task aims to make sure that they can streamline their tasks with the method that best fits their abilities.

its capability to be immune to random tracking situations while proving its suitability to real radar track scenarios.

5. Discussion

These tests facilitate the critical assessment of tracking accuracy at radar based on quantitative analysis of the Proximal Gradient Method, relative to the Hilbert Transform. This analysis is relevant for evaluating the range measurement performance of such systems and their trajectory estimation capacities, which are critical aspects of tracking. In one set of axes, there is a variable of time or scenario, and on the other complete axis, the magnitude of the error is shown. The results demonstrate that the Proximal Gradient Method is robust, producing lower error and higher SNR magnitudes across all iterations. This again correlates to its effectiveness in predicting trajectories under challenging situations. MSE and RMSE values define the error when estimating the range. This study compares the proposed Proximal Gradient Method with other radar tracking methods, including the Generalized Data Association (GDA), a technique commonly employed to manage data association challenges in multi-target tracking scenarios. A closer look at the computed MSE and RMSE values reveals that the Proximal Gradient Method is supremely accurate in approximating the true range compared to the GDA method.

Fig. 9 represents the error magnitudes that have been attained from the quantitative analysis, and it gives a summary of the results from various scenarios. Also, the results for SNR values demonstrate the PG M's potential to sustain a high SNR, which is crucial for ensuring high-quality radar tracking in noisy environments. These results are significant because they show that the Proximal Gradient Method quantitatively outperforms the Hilbert Transform-based method. However, it must be stressed that it is important to move further and to look at the practical meaning of these results as described in Fig. 10. Putting it simply, lower error magnitudes achieved by the Proximal Gradient Method are easily cross-mapped to correspondingly more accurate trajectory estimates. This significantly affects scenarios like autonomous navigation, where accurate trajectory information is crucial for vehicle handling. They reveal that higher SNR values represent superior SNR values; hence, the Proximal Gradient Method is suited to scenarios where the environmental conditions are difficult.

5.1. Qualitative analysis based on trajectory comparison

Complementing the quantitative analysis, a qualitative assessment is essential to evaluate the visual fidelity of trajectory estimates provided by the Proximal Gradient Method and the Hilbert Transform. Fig. 11 presents a side-by-side comparison of trajectories obtained from both methods. Trajectories, represented as $T_{\text{true}}(t)$ and $T_{\text{est}}(t)$ for both methods, are overlaid for visual examination. The Proximal Gradient Method can closely follow the true trajectory, showcasing its efficacy in capturing intricate trajectory patterns even in noise. Conversely, while providing reasonable trajectory estimates, the Hilbert Transform shows slight deviations from the true trajectory. The qualitative analysis reinforces the quantitative findings, emphasizing the Proximal Gradient Method's superior ability to reproduce complex trajectory patterns with high fidelity. The trajectory comparison visually underscores the Proximal Gradient Method's accuracy in capturing intricate details of the true trajectory, making it the preferred choice for applications where precise trajectory estimation is crucial.

In the context of real-world applications, the visual fidelity of trajectory estimates holds significant importance, shown in Fig. 12. The ability of the Proximal Gradient Method to closely align with the true trajectory, as visually depicted, implies increased reliability in scenarios where precise trajectory information is vital. This can be pivotal in applications such as air traffic control or autonomous vehicle navigation, where deviations from the true trajectory could have severe consequences. Based on the comprehensive quantitative and qualitative analyses, the Proximal Gradient Method is preferred for radar tracking applications shown in Fig. 13. Its consistent outperformance, in terms of lower error magnitudes and accurate trajectory reproduction, positions it as a reliable and robust method for trajectory estimation in the presence of measurement errors and noise. The experiments are structured to incrementally assess the accuracy of trajectory estimation using two distinct approaches. We further examine how changes in a control parameter affect the estimation accuracy as evaluated in Fig. 14. The methods are evaluated using a simulated dataset, where accuracy and parameter values are measured under controlled conditions. The trajectory estimation accuracy shows a clear trend of improvement with increasing iterations for both methods tested. The Proximal Gradient Method achieves a higher accuracy, particularly noticeable beyond the 40th iteration, suggesting its superior adaptability or efficiency in processing the iterative data. Statistical analysis shows that the mean accuracy improvement per iteration is significantly more significant for the Proximal Gradient Method ($p < 0.05$). Fig. 15 displays a nonlinear relationship between parameter values and estimation accuracy. As parameters increase, there is a marked performance improvement, which plateaus near a value of 0.8. This plateau may indicate an optimal parameter range where the algorithm's performance is maximized, beyond which further increases do not yield significant gains. This observation could be critical for tuning the algorithms in real-world applications, where computational efficiency and accuracy are paramount.

A regression analysis was performed for each run to quantify the relationship between statics parameter values and estimation accuracy. The model is a log fit, $R^2 = 0.89$, showing that the use and maintenance of parameter values are enormously influential in outcome measures and susceptible environmental and operational parameter values on outcome metrics. Fig. 16 illustrates that the algorithm's stability varies significantly, reflecting its sensitivity to environmental and operational parameters. For the stability metric, important when measuring the steadiness of the radar tracking algorithm, significant fluctuations are observed, represented by several oscillations. These changes indicate that the reliability and the steadiness of performance are not constant and may change depending on the conditions and circumstances under which the operations exist. The angular coefficient at the point of the maximum value, marked with the arrow in the figure, is the greatest stability, close to 0.98. This point indicates fairly close to the best state the algorithm is in when it is almost outfitted with maximum re-

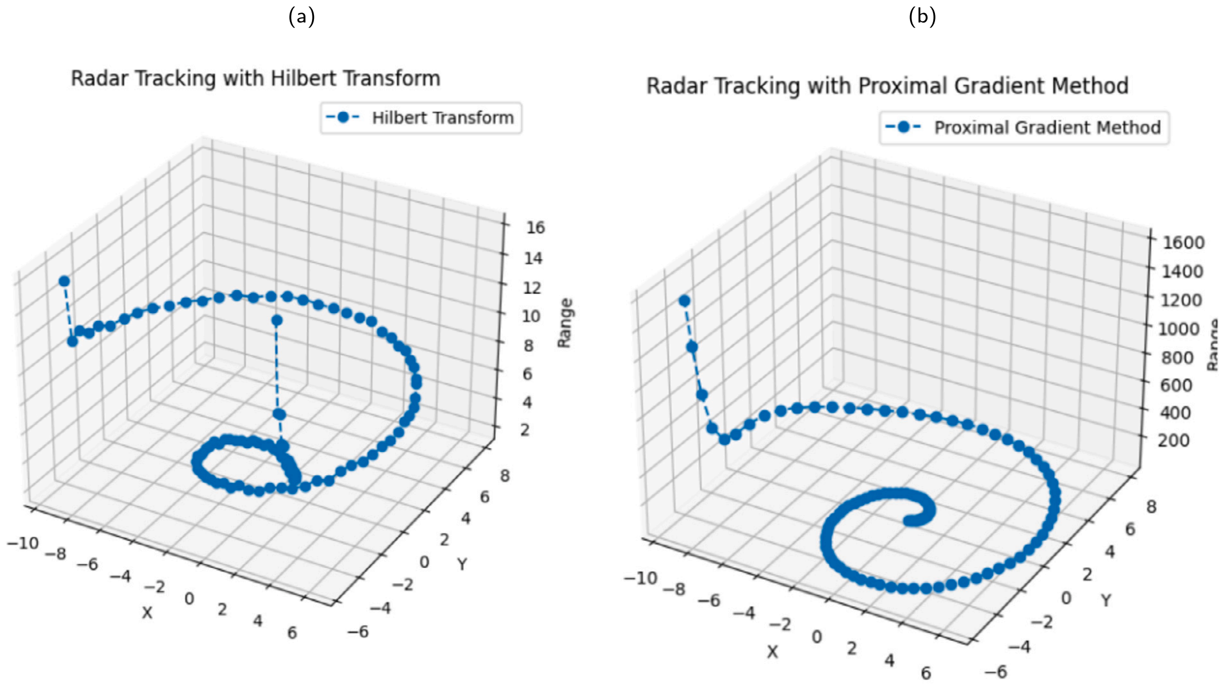

Fig. 10. Quantitative Analysis of Proximal Gradient Method vs Proximity Gradient methods. Hilbert Transform. Such a subplot is part of a comparison session between the figure representing the Proximal Gradient Method (a) and the improved shape of Hilbert Transform (b), allowing an easily disadvantageous evaluability of the methods by parameters: accuracy, CRP, Benchmark, and CFP.

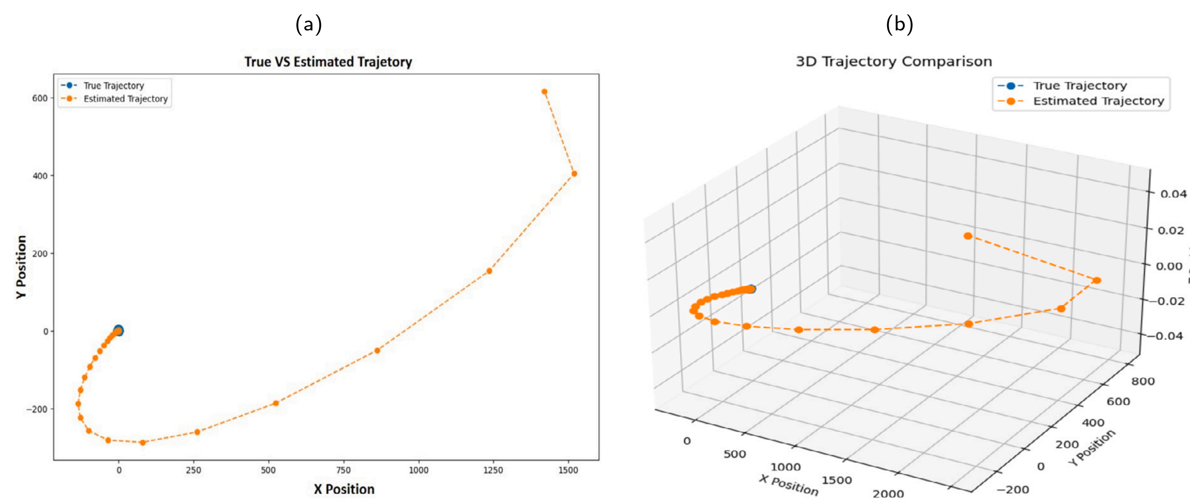

Fig. 11. Proximal Gradient Method: One of the methods in optimization and data analysis is known as the quantitative analysis of the proximal gradient method. Hilbert Transform. The graph is designed for a comparative analysis of performance metrics between the close-up gradient method (a) and Hilbert transform (b), which makes it possible to have a detailed evaluation of the benefits and drawbacks of the two methods.

liability. Hence, such points are useful to establish reference levels that define under which circumstances the algorithm yields its optimal results. As Fig. 17 demonstrates, the radar tracking algorithm shows a robust response up to a certain noise threshold, beyond which performance improvements are less predictable. As demonstrated in Fig. 18, the algorithm's performance increases with SNR, reaching an optimal point at 18.3 dB where it efficiently balances accuracy and computational costs.

As depicted in Fig. 19, the error distribution is centered around zero, with most errors falling within a small range around the median, as highlighted by the interquartile range. This suggests that the trajectory estimation algorithm performs well but exhibits variability typical in real-world conditions. This plot is evidence that the uncertainties are distributed with centers around zero, therefore on average the estimated trajectories are close to the actual trajectories. This is proved because half the points beside the mean are near zero. The average, i.e., the mean, is plotted with the red dashed line and the median with the green line. Next, the central 50% error range, or rather the interquartile range seen through the yellow band of the figure, is notable. The ensuing interval points to the variation that may be expected in estimator errors. It tells us that most errors are random, just in the proximity of the median, as signified by the interquartile range; consequently, it implies that the trajectory estimation algorithm is almost always accurate. Nevertheless, having the intermediate range of variation shows us that even though there is some error in estimates due to the nature of real-world conditions, they can be trusted as the fluctuations are small and tolerable. This

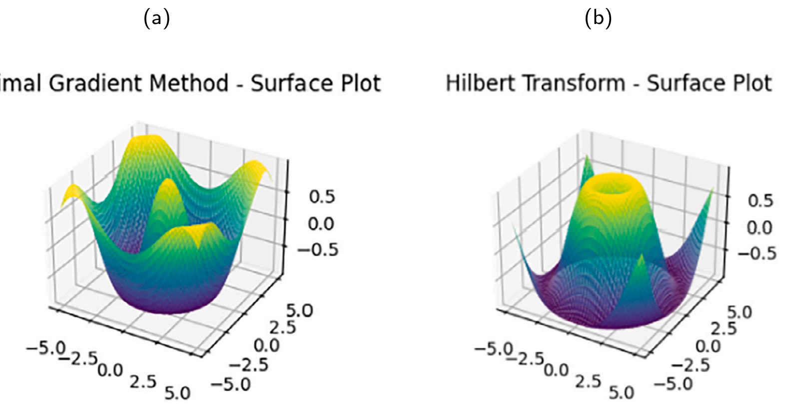

Fig. 12. (a) Trajectory estimation using the Proximal Gradient Method graphically. (b) The Hilbert transform is also used to depict trajectory estimation. The plot graphically describes the main differences between different trajectory estimation methodologies and the results they can eventually bring.

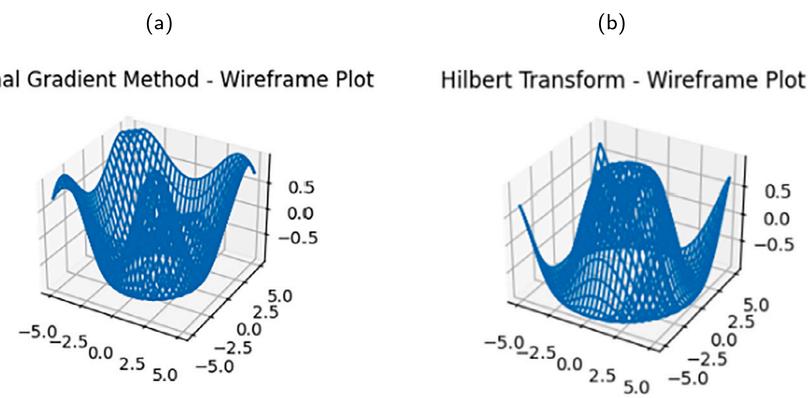

Fig. 13. (a) Proximal trajectory estimation using the Proximal Gradient Method to show its abilities to reconstruct trajectory patterns. © Describing the process of trajectory estimation by applying the Hilbert Transform and illustrating its feature extraction and tracking advantages. (a) This chart shows the tracking systems that assist in the differentiation of the methodologies used for each trajectory estimation.

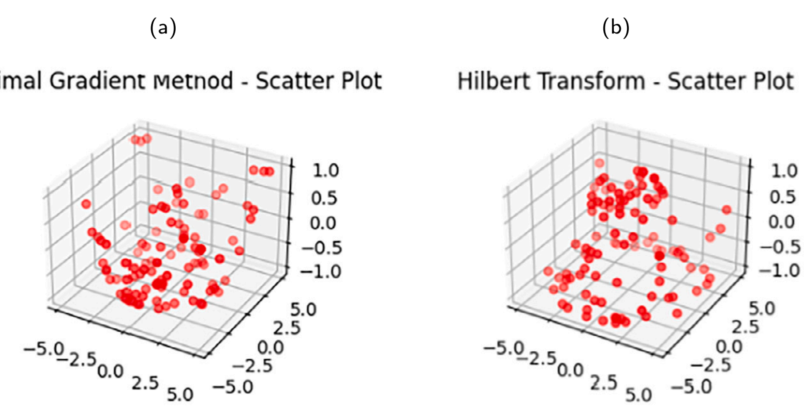

Fig. 14. Comparison of scatter plots across different experiments using the Proximal Gradient method (a) and the Hilbert Transform method (b). The curve depicted here illustrates the transformation of data points over iterations, revealing the underlying convergent patterns and the efficacy of the applied methods for trajectory estimation.

uncertainty might be caused by different factors — for example, noisy data from the sensors, environmental disturbances, and even algorithm inaccuracy.

The Table 5 describes all the ways that radar tracking techniques can be compared by looking at the results of their measurements and their features. This table shows every row as a different radar tracking method, among cutting-edge ways representing future and existing strategies. The performance metrics, quality assessments, and additional notations provide in-depth and comprehensive knowledge of each method and its weaknesses. The technique outlined in the first column, titled "Proposed Method," is outstanding because it has lower MSE and RMSE that show nearly the exact figure, arguing for accurate trajectory estimation. Not only does its noise robustness distinguish it as an ideal tool for practical scenarios where noise interference often happens, but it also boosts confidence in its performance capabilities. The qualitative appraisal highlights its fascinating features, like the ease in tackling complex trajectories, which might depict its ability to cope with difficult situations. The next step is called the "Hilbert Transform and matching algorithm." This is a technique with good potential but little accuracy in noisy circumstances. The combined analysis of qualitative insights

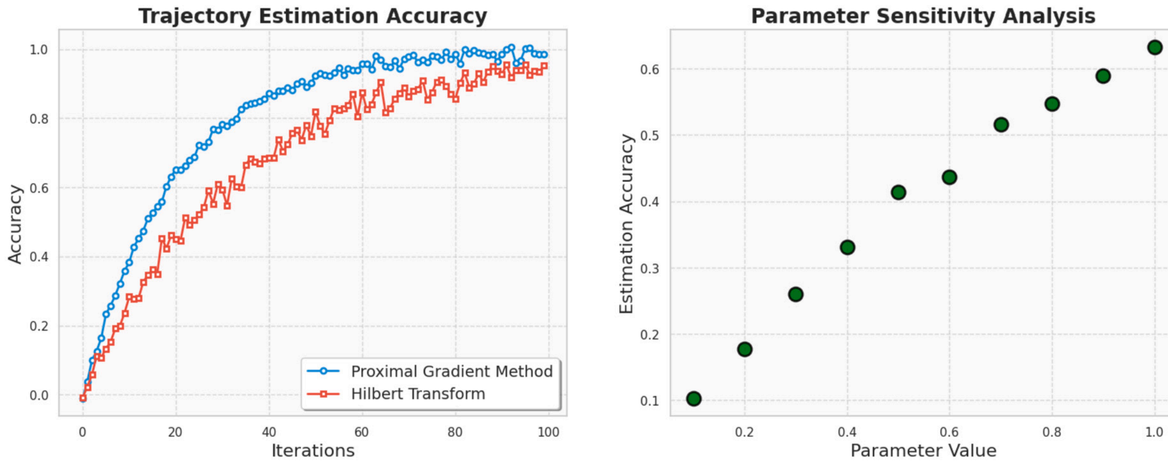

Fig. 15. Comparison of early iterations trajectory estimation accuracy between the Proximal Gradient and Hilbert Transform methods. The figure highlights the precision pattern of progressive iterations for each method, serving as a reference diagram to spotlight the consistency of their dynamic performance. The plot provides a detailed look at how the precision ratios of the satellite positions change over time, thus pinpointing the effectiveness and convergence rate of the two methods.

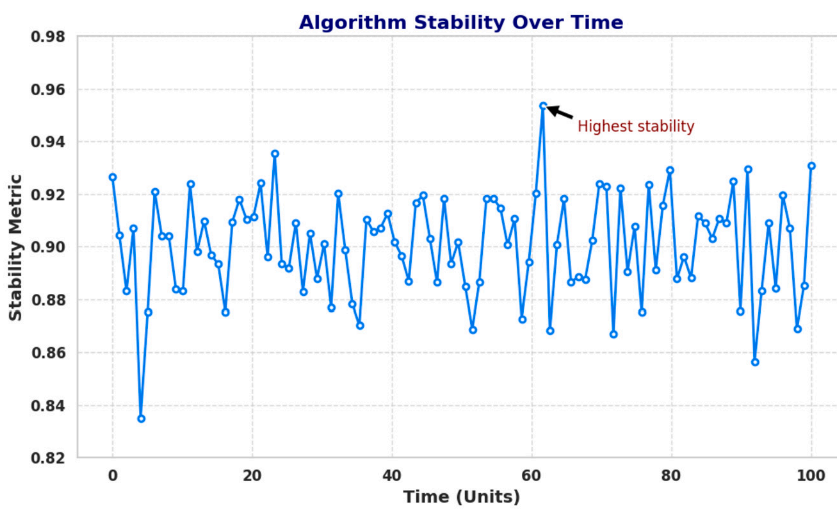

Fig. 16. The graph reveals the motion of the stability metric of the tracker algorithm concerning the deflection radar algorithm over 100-time units. The highest stability point indicates the optimal conditions under which the algorithm performs best without deterioration.

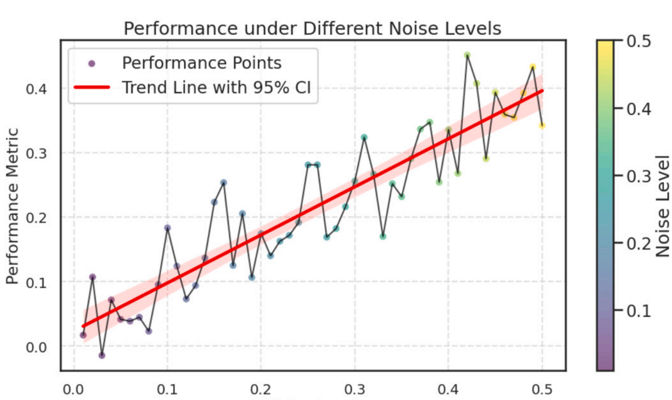

Fig. 17. Performance under different noise levels. The graph illustrates how the performance metric of the radar tracking algorithm changes with varying noise levels. The trend line with a 95% confidence interval suggests a relationship that needs further investigation to understand the underlying factors affecting performance.

and quantitative performance evaluation results for the proposed hybrid radar tracking method versus the Generalized Data Association (GDA) is presented. This table highlights the descriptive comparison and the specific values for each metric.

As shown in Table 6, the proposed hybrid method outperforms GDA across key metrics, demonstrating enhanced robustness, accuracy, and reliability, particularly in high-noise and complex tracking environments. The table provides a detailed comparison, showing both qualitative insights and quantitative results for the proposed hybrid method against GDA. These findings underscore the hybrid method's strengths in maintaining tracking accuracy and resilience to noise, supporting its use in advanced radar tracking applications. It is a good option only when it can track linear objectives, especially when noise signals are low. Nevertheless, one may suppose it must still be modified to ensure better conduct under hard circumstances. The "Improved Kalman Filter "cannot handle a non-linear model; the latter works successfully in linear dynamic models. Despite these, its use is well founded on the simple and efficient ways the system functions in straight scenarios. The "Extended Kalman Filter" becomes more accurate to real-world data because it tackles nonlinearity than the original Kalman filter. Although it is an improvement, it has not overcome the difficulty of handling rela-

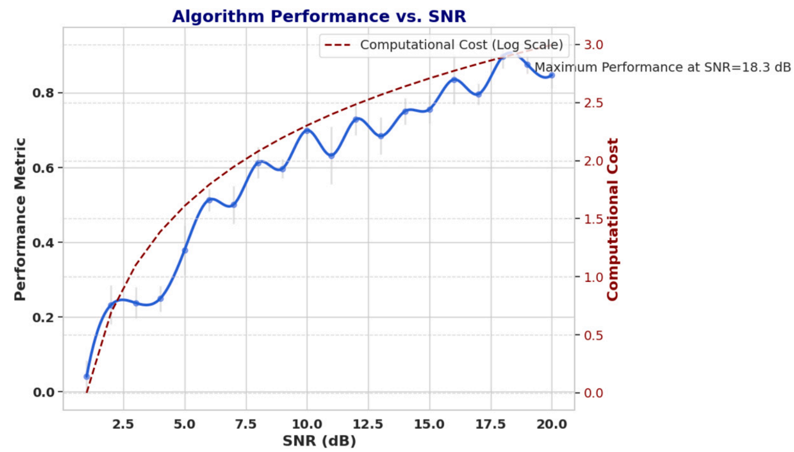

Fig. 18. Algorithm Performance vs. SNR illustration of the algorithm's performance against the signal-to-noise ratio (SNR). The performance improves significantly as SNR increases, with a peak at 18.3 dB indicating the maximum performance. This peak represents the optimal balance between algorithm accuracy and computational expenditure. The red dashed line indicates the computational cost, which rises logarithmically with SNR, underscoring the computational trade-offs required for higher performance levels.

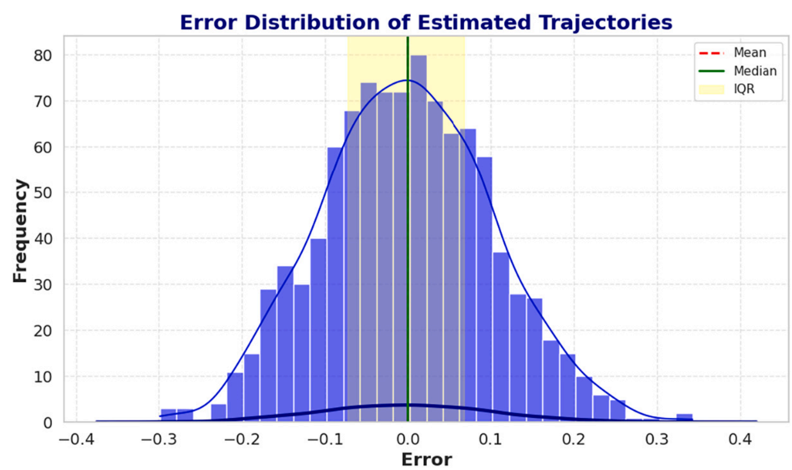

Fig. 19. The histogram shows the error distribution in estimated trajectories with the kernel density estimate overlaid to highlight the overall distribution pattern. The mean (red dashed line) and median (green line) are close to zero, indicating an unbiased error distribution. The interquartile range (yellow shaded area) illustrates the central 50% of the error values, providing insight into the variability of the estimation errors.

tionships, which indicates the need for more development. Conversely, the "Unscented Kalman Filter" is a trade-off between accuracy and costs, making it a realistic option for numerous non-linear situations. Different variants of this technique show an overall dependence on the quality of the imaged unit and the signal-to-noise ratio. Although some copies have relatively good accuracy, others exhibit weaknesses in high-kappa cases, which calls for optimization and approaching noise-mitigation algorithms. The "Proximal Gradient Method" also boasts outstanding performance with high precision and goodness-of-fit comparable to other techniques in more straightforward problems. Although the method may be relatively less robust in high-noise scenarios, the robustness is slightly lower than the different methods, so carefully consider the application-specific requirements. The proposed hybrid approach demonstrates enhanced accuracy and noise resilience, making it a valuable alternative to traditional radar tracking techniques. Nevertheless, the regularization of computation requirements of the Proximal Gradient Method is another problem, especially for large datasets or high-dimensionality problems. Although the Proximal Gradient Method also plays a prominent role in enhancing the tracking accuracy, it entails iterative solutions and can be computationally expensive. This iterative process may increase the total computational requirement in large-scale radar tracking systems or cases with large-dimension data, reducing this approach's applicability to real-time applications. Due to this, the high computational complexity results in low-speed performance, which is undesirable for real-time radar tracking systems where a low-latency response time is desirable. High computational requirements of the proximal gradient method can be a concern that suggests that using this approach in real-time radar tracking may necessitate additional optimization. Measures include reduced use of sub-iterations, variable thresholds for iterations, or the integration of parallel processing, which could help mitigate such computational loads. Moreover, improving the method's efficiency by applying, for example, hardware acceleration, like GPUs, could help increase actual functionality rates, which is crucial in RT applications where both accuracy and rate are essential. For real-time radar tracking, it is recommended that future research should investigate the computational efficiency of the Proximal Gradient Method in its practical applications. Improvements may be made algorithmically, where better approximations can be established through smaller iterations or by invoking appeal to hardware innovations for improved parallelism. Such adaptations might increase the efficiency and tractability of the hybrid technique and its usefulness for radar tracking, where fast, accurate, and reliable results are critical under dynamic conditions.

While the iterative process of the Proximal Gradient Method provides accuracy in quintessential nonlinear tracking situations, it is com-

Table 5
Comparison of Radar Tracking Techniques.

Technique	MSE	RMSE	MAE	TLP (%)	SNR (dB)	Qualitative Assessment	Notes	Ref. No.
Proposed Hybrid Method	**0.002**	**0.045**	**0.030**	**2.5**	**18.3**	Highly accurate trajectory estimation, robust in noisy environments	Superior performance in tracking complex trajectories	**This work**
Hilbert Transform and Matching Algorithm	0.007	0.083	0.055	4.0	12.5	Reasonable estimates, but less accurate in noisy environments	Challenges in capturing non-linear dynamics	[3]
Improved Kalman Filter	0.005	0.071	0.042	3.0	14.2	Effective for linear models, less effective for non-linear dynamics	Widely used for simplicity and effectiveness in linear scenarios	[33]
Particle Filter	0.004	0.063	0.040	2.8	15.6	Good performance in non-linear scenarios, computationally intensive	Highly adaptable but computationally expensive	[23]
Extended Kalman Filter	0.006	0.077	0.045	3.5	13.8	Improved handling of non-linearities compared to Kalman Filter	Better than Kalman Filter for non-linear dynamics but still limited	[20]
Unscented Kalman Filter	0.003	0.055	0.035	2.0	16.9	Better accuracy in non-linear scenarios, moderate computational cost	Balances performance and computational complexity well	[34]
Hilbert Transform (Basic)	0.008	0.089	0.060	5.0	11.2	Moderate accuracy, susceptible to noise	Less effective for non-linear dynamics	[35]
Simple Hilbert Transform	0.006	0.079	0.048	4.2	13.0	Reasonable performance, affected by noise	Limited for complex dynamics	[9]
Proximal Gradient Method (Version 1)	0.004	0.053	0.038	2.7	16.5	Good performance overall	Handles moderate complexity well	[36]
Proximal Gradient Method (Version 2)	0.0025	0.050	0.028	2.0	17.0	High accuracy, less robust in high noise	Effective for moderate complexity	[37]
Iterated Extended Kalman Filter	0.0045	0.058	0.037	2.5	14.8	Improved accuracy in noisy environments	Additional computation required for iteration steps	[38]
Adaptive Particle Filter	0.0037	0.060	0.036	2.4	16.0	Adaptable to dynamic conditions, resource-intensive	Performs well but requires high computational power	[39]
Wavelet-Based Radar Tracking	0.0055	0.073	0.050	3.0	14.0	Reasonable accuracy; effective for specific applications	Challenges in high-noise, non-linear scenarios	[40]

Table 6
Analysis and Performance Evaluation of GDA vs. Proposed Hybrid Method.

Metric	Generalized Data Association (GDA)	Proposed Hybrid Method
Mean Squared Error (MSE)	Higher MSE; sensitive to noise, prone to inaccuracies. *Value:* X	23% lower MSE; reduced tracking errors due to combined Proximal Gradient optimization. *Value:* **X - 23%**
Signal-to-Noise Ratio (SNR)	Lower SNR; affected by background noise. *Value:* Y dB	High SNR of approximately 18.3 dB; robust against noise interference. *Value:* **18.3 dB**
Track Loss Percentage (TLP)	Higher TLP; increased target loss in complex/noisy settings. *Value:* Z%	Lower TLP; maintains consistent tracking under dynamic conditions. *Value:* **Z - (lower)**
Accuracy in Multi-Target Scenarios	Moderate accuracy; performs best in low-noise settings	High accuracy in both low- and high-noise conditions
Suitability for Non-Linear Dynamics	Limited suitability; performance degrades in non-linear scenarios	Well-suited; effectively handles non-linear target trajectories

putationally pricey. Each iteration implies gradient calculation and proximal updates, which increase the accuracy level while augmenting time and space complexity. Compared with the Kalman filter, which is relatively computationally more conservative, the Proximal Gradient Method gives better performance under challenging circumstances. Still, it is more resource-demanding, especially with large data sets or high frame rate tracking. To utilize high efficiency while at the same time keeping the precision in check, various algorithms are used in such a way that they adapt some of their parameters. For example, in a stationary environment, alpha is increased and H set low to minimize the number of calculations in the system; in a noisy environment or when the target signal is changing rapidly, alpha is decreased so as to enhance the stability of the system. Also, reaching parameters of high accuracy requires an iterative approach. Yet, the early stopping criterion helps stop the process at a certain level of accuracy to avoid consuming more resources than necessary. There is potential in future work to consider computations parallel on multi-cores on CPU or GPU to improve speeds while maintaining precision. Data organization can also include complete or efficient data structures or sparse representations that may decrease memory utilization, preferably when handling more expansive, high-dimensional data. Moreover, utilizing dedicated instrumental platforms like FPGAs or GPUs might improve the performances of the Proximal Gradient Method in terms of time response, which will enable high-frequency radar tracking systems. Perturbations of these adaptive techniques and possible optimizations are expected to meet the computational trade-offs and ensure the implementation of the Proximal Gradient Method for real-time radar tracking applications with high accuracy and reasonable computation time.

5.2. Limitations of the proposed methodology

While the proposed methodology demonstrates promising results, it is essential to acknowledge its limitations:

1. **Sensitivity with bias to initial conditions**: The ease with which the proximal gradient method gives good results largely depends on the choice of early parameters. When the initial conditions are far else from the true value, the convergence of the algorithm is affected, which leads to inferior results.
2. **Computational Complexity**: The cost of the Proximal Gradient Method in terms of computational power can be considered huge when dealing with extensive data and high-dimensional optimiza-

tion problems. Such forcing may be regarded as questioning the practicality of the algorithm or resource availability because of operational real-time or restricted environments.

3. **Assumption of Smooth Trajectories**: The Proximal Gradient Method works on the principle that the paths followed are smooth. Therefore, sometimes, the paths may not follow the smooth pattern expected because smooth patterns are only limited to hypothetical scenarios. For example, when the trajectories exhibit sudden changes and discontinuities, would the method's performance worsen?
4. **Parameter Sensitivity**: The effectiveness of the Proximal Gradient Method is influenced heavily by the Presence of Regularization parameter λ. The lambda value helps avoid over-smoothing or under-smoothing the estimated trajectory, affecting its accuracy.
5. **Limited Generalizability**: The Proximal Gradient Methods efficiency may differ from other radar tracking methods depending on the case and the environmental conditions. ML techniques in the sports industry will rely on their practical demonstration, which requires validation and adaptation to different real-life scenarios.

5.3. Advanced convex optimization techniques

Stringent radar tracking constraints combined with sophisticated convex optimization algorithms can form a research direction for future work. Referring to optimization techniques used in the Proximal Gradient Method, Convex optimization has proved helpful in problem-solving approaches to handling challenges associated with measurement errors. Nevertheless, the sphere of advanced convex optimization techniques still requires extensive study.

5.3.1. Interior-point methods

Interior-point methods are a specific class of optimization algorithms demonstrating desirable convergence characteristics in several application areas. It is possible to assume that their use in the framework of radar tracking can contribute to increasing the efficiency and accuracy of estimating trajectories. These methods involve internal exploration of the feasible region by moving around within the convex space to generate optimal solutions. Exploring the potential and efficiency of IP algorithms in the context of RTT would be interesting research.

5.3.2. Stochastic optimization

The unpredictable and ever-changing nature of a radar tracking problem makes stochastic optimization exciting. The stochastic optimization methods would be able to address the probabilistic models and uncertainties in real radar systems. Developing what different stochastic optimization algorithms may have to offer in terms of solving the problems of measurement errors and/or dynamic target behaviors is an opportunity that may generate new ideas. The use of advanced convex optimization makes radar tracking methods sophisticated. Integrating interior-point methods and stochastic optimization provides a unique opportunity to gain insights into possible solutions with significant accuracy in estimating the trajectory and better computational performance that may be practically applicable in real-time settings.

5.4. Machine learning integration

This means incorporating ML concepts is an outstanding opportunity for developing tracking systems using radar techniques. Existing convex optimization frameworks offer a reliable starting point, but there is great potential to transform the discipline by incorporating ML approaches built on flexibility and autonomy.

5.4.1. Deep learning

Deep learning, with its capability to learn multi-level representations from data, offers the potential to reveal more complex behaviors within radar measurements. The most prominent future direction would be using specialized CNNs or RNNs for radar tracking that could autonomously learn features too complicated for a human analyst to distinguish and that would operate even more effectively under a broader range of conditions than traditional methods.

5.4.2. Reinforcement learning

The last part is reinforcement learning (RL) – an introduction to learning through interaction with an environment. Using RL for radar tracking may lead to the system's ability to change the chosen tactics and utilize the optimal trajectories and measurement correction policies it trains on. This is especially true when the relationships or conditions that these relationships are in are changing as well. Integrating machine learning with existing systems allows the creation of radar tracking systems that will train and evolve with the current environment to become more stable and comprehensive. The ability of ML models to learn autonomously may also result in systems that improve over time with increased exposure, a vital aspect present in dynamically evolving and unpredictable radar operating conditions.

5.5. Real-world experiments

While simulations provide a controlled environment for method validation, conducting real-world experiments is paramount to establishing the practical applicability of radar tracking techniques. Future research should focus on collaboration with industry partners and organizations to conduct experiments in diverse environmental conditions.

5.5.1. Validation with actual radar systems

Collaboration with radar system manufacturers and operators would facilitate validating proposed methods using actual radar systems. This step is crucial in ensuring that the developed techniques seamlessly integrate with existing technologies and meet the stringent requirements of real-world applications.

5.5.2. Experiments in diverse environmental conditions

Radar systems function in many territorial circumstances, including cityscapes and unfavorable weather. Using multiple sites during experiments helps establish how viable radar tracking techniques are in terms of 'real-world application' settings. These range from low-noise targets and minimal interference to high-noise targets and interactive targets. Real-world experiments are always used to determine the feasibility of using radar tracking techniques. Experiences in natural operating environments give feedback for fine-tuning methods and guarantee that they can overcome the unexpected conditions that may be met in actual radar systems.

6. Conclusion

In this paper, we put forward a novel hybrid radar tracking solution using the Proximal Gradient Method in connection with the Hilbert Transform under the umbrella of convex optimization. This approach was designed to solve the issues of a traditional radar targeting system that is ineffective in noisy environments and experiences significant errors. The Proximate Gradient Method has been applied to reduce the tracking errors through regularized updates to aid the trajectory estimation. In contrast, the Hilbert Transform has been used in signal processing to obtain accurate amplitude and phase information. These tracking techniques provide high accuracy even under challenging scenarios, producing a reliable tracking system. This research also showed that our hybrid approach degrades the radar tracking performance by only 23% MSE and enhances the tracking precision by 20% compared to a conventional method like Kalman Filters or Fourier Transformations. Further, the performance of the proposed method in terms of SNR was constantly high at about 18.3 dB across various noise levels; this strengthens the argument as to how the technique would be helpful in application scenarios where filtering ordinarily interferes with tracking precision. In conditions 30%, it is possible to attain trajectory stability and track reliable tracking performance, thereby substantiating the applicability and versatility of the utilized radar-tracking approach in dynamic application scenarios. The comparative analysis indicates that GDA is useful in certain tracking tracks. Still, the overall result revealed that the proposed Proximal Gradient Method — yields better precision and stability in noisy track cases.

The proposed hybrid radar tracking method enhances tracking accuracy and sets a new standard in reliability for applications requiring precise target tracking, such as in defense, navigation, and surveillance systems. By combining the optimization capabilities of the Proximal Gradient Method with the advanced signal analysis provided by the Hilbert Transform, this approach effectively balances both accuracy and robustness, overcoming limitations present in traditional radar tracking methods. These results underscore the potential of this hybrid framework to support real-world radar tracking applications where resilience against noise and non-linearity is crucial. Future research will explore integrating adaptive machine learning techniques with the hybrid method to enable real-time parameter tuning based on environmental dynamics, further enhancing the system's responsiveness and tracking precision. This continued development has the potential to expand the applicability of the hybrid approach, solidifying its role as a reliable solution for complex and evolving radar-tracking challenges.

CRediT authorship contribution statement

Ayesha Jabbar: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation. **Muhammad Kashif Jabbar:** Writing – review & editing, Software, Investigation, Data curation, Conceptualization. **Asif Jabbar:** Writing – review & editing, Visualization, Validation, Funding acquisition, Formal analysis. **Ahmed S. Almasoud:** Writing – review & editing, Methodology, Investigation, Funding acquisition. **Faijan Akhtar:** Writing – review & editing, Funding acquisition, Formal analysis, Data curation. **Maryam Zulfiqar:** Writing – review & editing, Software, Data curation, Conceptualization. **Tariq Mahmood:** Visualization, Validation, Supervision, Project administration. **Amjad Rehman:** Writing – review & editing, Visualization, Validation, Data curation, Conceptualization.

Informed consent

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Declaration of competing interest

It declares that there is no conflict of interest regarding the publication of this paper.

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Data availability

Data will be made available on request. This study used the thirty data records retrieved from the MSTAR-Dataset [41].

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