

# Authentication Using Wavelets

## Part II

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Joint Mathematics Meetings, 2010

# Outline

1 Project Beginnings

2 Project Details

3 Summary

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# Wavelet Workshop

In August 2009, I attended the

## Discrete Wavelet Module-Writing Workshop

Workshop Leaders:

- Patrick Van Fleet, University of St. Thomas
- Catherine Beneteau, University of South Florida
- Caroline Haddad, SUNY Geneseo
- David Ruch, Metropolitan State College of Denver.

Supported by: MAA, PREP and NSF

# Our Team and Module

## Our Team:

- John Merkel
- Jill Guerra
- Caroline Haddad
- Rachel Weir

**Our Module:** Use wavelets to classify handwriting as forgery or authentic.

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Based primarily on Lyu, Rockmore, Farid: *A digital technique for art authentication*

- Obtain several fingerprint samples for several people
- Convert to digital files (take a picture)
- Apply wavelet transform to picture
- Collect statistical data to form *signature vector* in  $\mathbb{R}^{72}$
- Categorize fingerprints based on distance between their signature vectors

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# The pictures

Pictures are all  $256 \times 256$  pixels.



Figure: Gryc1.png

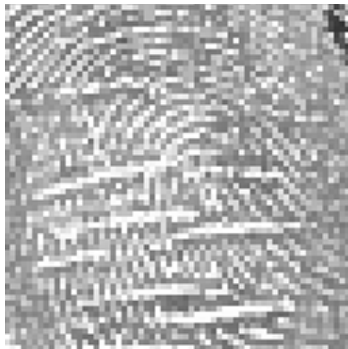


Figure: Gryc2.png

# Wavelet Transform

## 4 Iterations of Daubechies(4)



Figure: Temba1.png

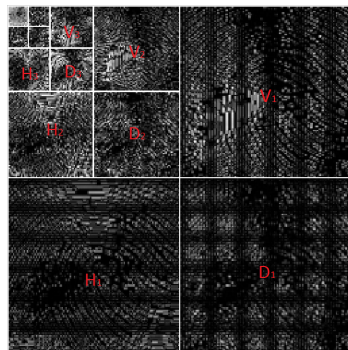
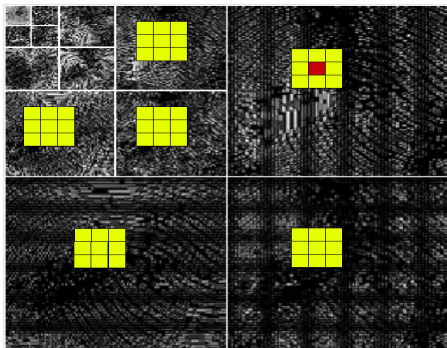


Figure: Transformed Temba1.png

# Predictive Neighbors

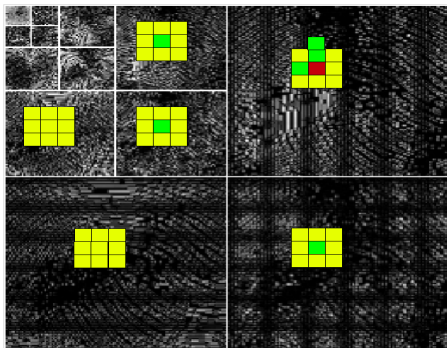
Which **yellow** pixels are best at predicting the value of the **red** pixel?





# Predictive Neighbors

Which **yellow** pixels are best at predicting the value of the **red** pixel?



# Create Linear Predictor

Buccigrossi gives the following as the best neighboring pixels to use for a linear predictor:

$$\begin{aligned}
 L_V^k(r, c) &= w_1 V_k(r, c-1) + w_2 V_k(r-1, c) + w_3 V_{k+1}(\lfloor r/2 \rfloor, \lfloor c/2 \rfloor) \\
 &\quad + w_4 D_k(r, c) + w_5 V_k(r-2, c) + w_6 D_{k+1}(\lfloor r/2 \rfloor, \lfloor c/2 \rfloor) \\
 L_D^k(r, c) &= w_1 D_k(r-1, c) + w_2 D_k(r, c-1) + w_3 D_{k+1}(\lfloor r/2 \rfloor, \lfloor c/2 \rfloor) \\
 &\quad + w_4 H_k(r, c) + w_5 V_k(r, c) + w_6 D_k(r, c-2) \\
 L_H^k(r, c) &= w_1 H_k(r, c-1) + w_2 H_k(r-1, c) + w_3 H_{k+1}(\lfloor r/2 \rfloor, \lfloor c/2 \rfloor) \\
 &\quad + w_4 D_k(r, c) + w_5 V_k(r, c-2) + w_6 D_{k+1}(\lfloor r/2 \rfloor, \lfloor c/2 \rfloor)
 \end{aligned}$$

But how to find weights?

# Determining Weights - Definitions

Define the *weight vector* by  $\mathbf{w} = (w_1, \dots, w_6)^T$ .

Let  $\mathbf{v} = \text{flatten}(V_1)$ .

Let  $Q$  be the matrix with 6 columns of neighboring pixel values.

Then we can write our linear predictor for  $V_1$  as

$$L_V^1 = Q\mathbf{w}.$$

Define the *error vector* by  $\mathbf{e} = \mathbf{v} - L_V^1 = \mathbf{v} - Q\mathbf{w}$ .

# Determining Weights - Formula

Weights for the linear predictor that minimize the *error function*

$$E(\mathbf{w}) = \mathbf{e} \cdot \mathbf{e} = (\mathbf{v} - L_V^1)^2 = (\mathbf{v} - Q\mathbf{w})^2$$

are given by

$$\mathbf{w} = (Q^T Q)^{-1} Q^T \mathbf{v}.$$

Repeat process for 9 subbands  $V_1, V_2, V_3, H_1, H_2, H_3, D_1, D_2, D_3$

# Collecting Stats for Signature Vector

For each of the 9 subbands calculate

- 1 Mean of weights
- 2 Variance of weights
- 3 Skewness of weights
- 4 Kurtosis of weights
- 5 Mean of  $\mathbf{e}$
- 6 Variance of  $\mathbf{e}$
- 7 Skewness of  $\mathbf{e}$
- 8 Kurtosis of  $\mathbf{e}$

Each fingerprint has 72 statistical parameters. These form the signature vector in  $\mathbb{R}^{72}$

# Mutual Distance Table

		Chancellor			Green			Gryc			Mathews			Merkel			Temba		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	1	0.00	3.82	6.43	13.82	14.15	14.39	12.41	12.41	12.13	14.43	12.87	13.23	11.63	12.76	12.36	15.35	13.70	17.60
Chancellor	2	3.82	0.00	6.77	13.26	13.36	13.59	12.29	11.93	11.76	13.59	12.03	12.65	11.64	12.00	11.99	14.41	12.69	17.81
	3	6.43	6.77	0.00	13.37	14.28	14.62	13.45	12.62	13.32	14.44	12.77	14.13	13.48	13.42	13.35	15.57	13.68	18.73
	4	13.82	13.26	13.37	0.00	7.14	7.99	10.67	10.79	10.78	9.57	8.50	9.67	11.33	9.69	10.14	8.84	7.64	16.06
Green	5	14.15	13.36	14.28	7.14	0.00	5.05	10.58	10.00	9.64	9.64	9.41	9.21	10.52	9.82	9.14	7.55	8.40	15.92
	6	14.39	13.59	14.62	7.99	5.05	0.00	11.11	11.40	10.29	10.07	10.22	10.44	11.46	10.82	10.51	7.98	9.18	16.54
	7	12.41	12.29	13.45	10.67	10.58	11.11	0.00	11.82	10.62	11.73	11.18	10.76	12.34	12.56	10.85	11.57	11.51	13.48
Gryc	8	12.41	11.93	12.62	10.79	10.00	11.40	11.82	0.00	10.16	12.99	11.16	12.45	9.41	8.82	10.13	12.47	10.29	18.48
	9	12.13	11.76	13.32	10.78	9.64	10.29	10.62	10.16	0.00	11.67	11.75	9.77	9.48	11.11	8.12	11.43	11.86	16.72
	10	14.43	13.59	14.44	9.57	9.64	10.07	11.73	12.99	11.67	0.00	6.29	5.64	12.80	11.88	10.29	9.54	11.01	16.90
Mathews	11	12.87	12.03	12.77	8.50	9.41	10.22	11.18	11.16	11.75	6.29	0.00	8.02	11.71	9.70	10.39	10.64	8.53	17.67
	12	13.23	12.65	14.13	9.67	9.21	10.44	10.76	12.45	9.77	5.64	8.02	0.00	11.64	12.01	9.02	9.87	10.81	16.39
	13	11.63	11.64	13.48	11.33	10.52	11.46	12.34	9.41	9.48	12.80	11.71	11.64	0.00	7.06	9.53	13.40	11.80	18.19
Merkel	14	12.76	12.00	13.42	9.69	9.82	10.82	12.56	8.82	11.11	11.88	9.70	12.01	7.06	0.00	9.97	12.15	8.99	18.21
	15	12.36	11.99	13.35	10.14	9.14	10.51	10.85	10.13	8.12	10.29	10.39	9.02	9.53	9.97	0.00	11.14	10.70	15.84
	16	15.35	14.41	15.57	8.84	7.55	7.98	11.57	12.47	11.43	9.54	10.64	9.87	13.40	12.15	11.14	0.00	8.95	15.69
Temba	17	13.70	12.69	13.68	7.64	8.40	9.18	11.51	10.29	11.86	11.01	8.53	10.81	11.80	8.99	10.70	8.95	0.00	16.93
	18	17.60	17.81	18.73	16.06	15.92	16.54	13.48	18.48	16.72	16.90	17.67	16.39	18.19	18.21	15.84	15.69	16.93	0.00

# Mutual Distance Averages

AVERAGES	Chancelor			Green			Gryc			Mathews			Merkel			Temba		
Chancelor		5.67		13.48	13.93	14.20	12.72	12.32	12.40	14.15	12.56	13.34	12.25	12.73	12.57	15.11	13.36	18.05
Green	14.12	13.40	14.09		6.73		10.79	10.73	10.24	9.76	9.38	9.77	11.10	10.11	9.93	8.12	8.41	16.18
Gryc	12.32	11.99	13.13	10.75	10.07	10.93		10.87		12.13	11.36	11.00	10.41	10.83	9.70	11.82	11.22	16.22
Mathews	13.51	12.76	13.78	9.25	9.42	10.24	11.22	12.20	11.07		6.65		12.05	11.20	9.90	10.02	10.12	16.98
Merkel	12.25	11.88	13.42	10.39	9.83	10.93	11.92	9.45	9.57	11.66	10.60	10.89		8.85		12.23	10.50	17.41
Temba	15.55	14.97	15.99	10.85	10.63	11.24	12.19	13.74	13.33	12.48	12.28	12.36	14.47	13.12	12.56		13.86	

# Out of Sample Prints

	Chancellor	Green	Gryc	Mathews	Merkel	Temba
Chancellor	4.77	13.75	12.99	13.48	12.42	13.49
	3.43	13.49	12.61	13.02	11.91	12.41
	6.06	14.27	13.29	13.99	12.80	12.91
Green	13.09	6.90	12.49	13.10	10.42	6.63
	13.38	7.42	10.53	13.75	10.32	6.32
	13.73	9.28	11.53	13.88	11.11	7.14
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	12.22	9.99	10.50	12.08	4.44	9.54
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Temba	14.97	11.31	14.98	14.22	13.39	9.93



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- All pictures need to be processed identically!
- This technique could be used in other settings

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