

From Equilibrium to Currents

- Doping (adding impurities) alters the number of free electrons (in n-type) or holes (in p-type), but without any external voltage, these carriers remain distributed evenly.
- 当施加电压或电场时，载流子（电子和空穴）不再处于平衡状态。它们开始响应电场运动，产生电流。这是当半导体从平衡状态过渡到有电流流动的状态。

Drift Currents

- 当电场在半导体上施加时，自由电荷载体（电子和空穴）由于电场而受到力，这使得它们移动。这种电荷的运动就是我们所说的漂移电流。**Drift Currents**

- **Electron Movement:**

As the electrons move, they collide with atoms and other electrons in the material. These collisions slow them down, so instead of accelerating infinitely, they reach a constant **drift velocity**.

- **Hole Movement:**

When an electron fills a hole, it leaves a new hole behind. This process looks like the hole itself is moving through the material.

drift velocity for either electrons or holes:

$$v_d = \mu E$$

Mobility depends on the material and the type of carrier (electrons or holes). It reflects how easily a carrier can move through the material.

E is the applied electric field.

current density J

$$J_n = -en v_d \quad J_p = ep v_d$$

- e is the charge of an electron,
- n and p are the number densities of electrons and holes, respectively,

Mobility:

What Affects Mobility?

Type of charge carrier (electron or hole): Electrons generally have higher mobility than holes because they are lighter and experience fewer interactions in the material.

Material: Different materials have different intrinsic mobilities for electrons and holes. For instance, silicon has a lower mobility compared to gallium arsenide.

Temperature: As temperature increases, carriers collide more frequently with atoms, decreasing mobility.

Currents in Terms of Mobility

- express the current density J using mobility. The current density is the amount of electric current per unit area. For drift currents, the current density depends on the number of charge carriers, their charge, drift velocity, and mobility.
 - $J_n = -en \mu_n E$
 - $J_p = ep \mu_p E$
 - $J = J_n + J_p = e(n \mu_n + p \mu_p)E$

Collisions and Mobility

迁移率受电荷载流子在半导体中移动时所经历的碰撞影响。这些碰撞发生在移动的电子或空穴与晶格原子或其他载流子之间。这些碰撞分散了载体，减慢了它们的速度，并有效地降低了它们的机动性。

- At **low temperatures**, there are fewer collisions, and *mobility is higher*.
- At **high temperatures**, the atoms vibrate more, leading to more collisions *and lower mobility*.

- **Doping Concentration:** When semiconductors are doped (with either donor or acceptor impurities), the additional atoms introduce more scattering sites, *which can reduce mobility*.
- **Material Quality:** The **crystal structure** and **defects** in the material also play a role. **High-quality crystals with fewer defects have higher mobility** because there are fewer disruptions to the motion of carriers.
- **Electric Field Strength:** At very **high electric fields**, **mobility can decrease** due to phenomena like **velocity saturation**, where the drift velocity reaches a maximum limit because carriers undergo so many collisions.

- On average, a charge carrier will reach the following maximum velocity between collisions:

$$v_{\max} = \left(\frac{q\tau_c}{m^*} \right) E$$

- We can now write the mobility of electrons and holes in terms of their effective mass and average time between collisions:

$$\mu_n = \frac{e\tau_{cn}}{m_n^*} \quad \mu_p = \frac{e\tau_{cp}}{m_p^*}$$

- • q is the charge of the carrier (for an electron, it's the elementary charge),
- • τ_c is the **average time between collisions** (often called the relaxation time),
- • $m^* m^* m^*$ is the **effective mass** of the carrier (it depends on whether the carrier is an electron or a hole)

Conductivity:

Conductivity σ is a material's ability to conduct electric current. It's the inverse of **resistivity** ρ and is related to the mobility and concentration of charge carriers (electrons and holes) in a semiconductor.

$$\sigma = q(n\mu_n + p\mu_p)$$

- σ is the conductivity,
- q is the elementary charge,
- n and p are the concentrations of electrons and holes, respectively,
- μ_n and μ_p are the mobilities of electrons and holes, respectively.

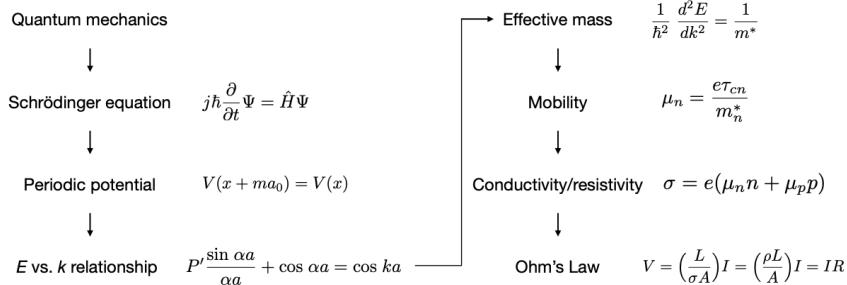
Ohm's Law

$$J = \sigma E$$

EXTRA:

Making some connections

- It's worth stopping here and reflecting on the path we have taken from crystallography and quantum mechanics to this point
- We have now connected an equation (Ohm's Law) that many of us learned in high school to another equation (the Schrödinger equation) that describes the most fundamental physics that many of us will ever study:



Velocity Saturation

(drift velocity v_{dv} is directly proportional to the electric field E)

saturation velocity v_s : As the electric field increases, carriers experience more frequent collisions, preventing them from accelerating indefinitely.

$$v_{d,n} = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}}$$

$$v_{d,p} = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)^2\right]^{1/2}}$$

$v_{d,n}$: Drift velocity of electrons

$v_{d,p}$: Drift velocity of holes

- For silicon at $T = 300$ K, we can use:

$v_s = 10^7$ cm/s :	Saturation velocity
$E_{on} = 7 \times 10^3$ V/cm	
$E_{op} = 2 \times 10^4$ V/cm	

27

Charge Carrier Diffusion

Diffusion is the process by which particles move from regions of high concentration to regions of low concentration, driven by random motion and the desire to achieve equilibrium

- **charge carrier diffusion** refers to movement of **electrons** and **holes** (charge carriers) from areas where their concentration is high to areas where their concentration is low. This process occurs naturally and spontaneously, without the need for an electric field.

It's a result of thermal energy, (and it satisfies the second law of thermodynamics by increasing entropy (disorder).)

- The movement of charge carriers due to diffusion creates a **diffusion current**.

Diffusion for Electrons and Holes:

- The **diffusion current density for electrons** is given by:

$$J_{n,\text{diff}} = eD_n \frac{dn}{dx}$$

- Here, D_n is the **electron diffusion coefficient**, n is the electron concentration, and $\frac{dn}{dx}$ represents the gradient (change in concentration) of electrons along the distance x .
- For **holes**, the diffusion current density is:

$$J_{p,\text{diff}} = -eD_p \frac{dp}{dx}$$

- Here, D_p is the **hole diffusion coefficient**, and $\frac{dp}{dx}$ is the gradient of hole concentration.

Total Current Density

The **total current density** in a semiconductor is the sum of two types of currents:

1. **Drift current**: Due to the movement of carriers in response to an electric field.
2. **Diffusion current**: Due to the movement of carriers from high to low concentration.

The total current density J is the sum of these contributions for both electrons and holes:

$$J = en\mu_n E_x + ep\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

Where:

- $en\mu_n E_x$ is the **drift current** for electrons,
- $ep\mu_p E_x$ is the **drift current** for holes,
- $eD_n \frac{dn}{dx}$ is the **diffusion current** for electrons,
- $-eD_p \frac{dp}{dx}$ is the **diffusion current** for holes.

Relation:

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_B T}{e}$$

Where:

- k_B is the **Boltzmann constant**,
- T is the **absolute temperature**,
- e is the **elementary charge**.

这个关系告诉我们，具有更高迁移率（即 carrier 载流子在电场作用下更容易移动）的材料也具有更高的扩散系数。mobility and diffusion measure how easily carriers move

Week 6:

Merging Drift, Diffusion, and Excess Carriers
Drift, diffusion, excess charge carriers

Complexity with Dynamic Concentrations:

- When you have **excess carriers** (extra electrons and holes introduced externally), the situation becomes more complex because the concentration of charge carriers changes with both **time** and **space**. These variations in time and space mean that we need to account for dynamic processes such as carrier generation, recombination, and movement.

Example of Merging Drift and Diffusion

Imagine we shine light on a small spot on a semiconductor using a **magnifying glass**. The light generates **electron-hole pairs (e-h pairs)** only in the region where the light hits.

- Initially, the electron-hole pairs are confined to this small region, but they begin to **diffuse** (spread out) into the surrounding areas because of the concentration gradients.
- At the same time, the e-h pairs will start to **recombine** (electrons and holes annihilate each other), reducing the number of free carriers over time.

If the intensity of the light is not constant but **time-varying**, it becomes even more complicated because the rate of carrier generation will change over time.

To account for these complexities, we need more advanced mathematical models, which lead to the **time-dependent diffusion equations**.

→ Time-Dependent Diffusion Equations

These equations describe how excess carrier **concentrations (electrons and holes) change with both time and space**. The equations are partial differential equations (PDEs), which account for:

- **Spatial variation** (how concentrations change with distance),
- **Temporal variation** (how concentrations change with time).

The general forms of the time-dependent diffusion equations are:

For **holes**:

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \left(E \frac{\partial(\delta p)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial(\delta p)}{\partial t}$$

For **electrons**:

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} - \mu_n \left(E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial(\delta n)}{\partial t}$$

Where:

- D_p and D_n are the diffusion coefficients for holes and electrons, respectively,
- μ_p and μ_n are the mobilities for holes and electrons, respectively,
- E is the electric field,
- g_p and g_n are the generation rates for holes and electrons,
- τ_{pt} and τ_{nt} are the **recombination lifetimes** for holes and electrons,
- $\delta p(x, t)$ and $\delta n(x, t)$ represent the **excess carrier concentrations** as functions of both space and time.

What These Equations Mean

1. **Diffusion Terms:** The first terms in each equation represent the diffusion of carriers (holes and electrons). The second derivatives with respect to x account for how the carrier concentrations change spatially.
2. **Drift Terms:** The terms involving mobility μ and the electric field E account for the **drift** component, describing how the electric field causes carriers to move.
3. **Generation:** The g_p and g_n terms represent the **generation** of excess carriers (such as through light shining on the semiconductor).
4. **Recombination:** The $\frac{p}{\tau_{pt}}$ and $\frac{n}{\tau_{nt}}$ terms represent **recombination**, where the excess carriers (electrons and holes) annihilate each other over time.

→What is Ambipolar Transport?

Ambipolar transport refers to the combined movement of **excess electrons** and **holes** (charge carriers) in a semiconductor **under the influence of an applied electric field**.

Understanding the Process for Ambipolar Transport

1. Applied Electric Field:

- Imagine we apply an electric field E_{app} to a semiconductor, and then impulsively introduce **excess electron-hole pairs** into a specific region of the semiconductor.

2. Initial Behavior:

- Immediately after introduction, the **excess electrons** will experience a force that drives them in one direction (toward the positive terminal), while the **excess holes** experience a force in the opposite direction (toward the negative terminal).

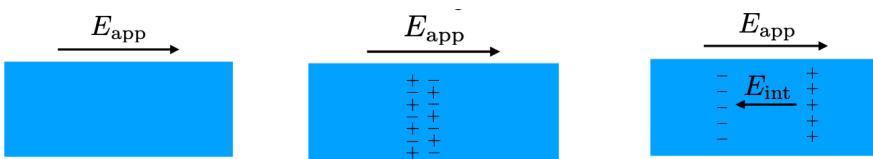
3. Separation of Charge:

- Because these excess charge carriers are introduced only in a specific region of the semiconductor, the initial drift of electrons and holes in opposite directions creates a **separation of positive and negative charges** in that region.
- **This separation generates an internal electric field E_{int}** , which opposes the movement of the electrons and holes.

4. Total Electric Field:

- The **total electric field** that affects the excess charge carriers: $E = E_{app} + E_{int}$

This internal field binds the movement of electrons and holes together, meaning that instead of moving in opposite directions, they begin to move **together in the same direction**.



→Ambipolar Transport and Diffusion

- The **internal electric field** forces the electrons and holes to "stick together" and move as a single entity, even though they have opposite charges. This movement of electrons and holes together is known as **ambipolar diffusion** or **ambipolar transport**.

Mathematical Approximation for Ambipolar Transport

In ambipolar transport, the concentrations of electrons $\delta n(x, t)$ and holes $\delta p(x, t)$ are approximately equal:

$$\delta n(x, t) \approx \delta p(x, t)$$

This approximation holds because the internal electric field keeps the charges bound together, preventing significant separation between the electron and hole distributions.

A key question is: **In which direction do the electrons and holes move?** depends on several factors:

- The **total concentration** of electrons and holes in the semiconductor,
- The **mobilities** of electrons and holes (how easily they move in response to the electric field).

For some conditions, the electrons and holes move in the same direction as the electric field (drift along with the field). For other conditions, they move in the opposite direction.

→Understanding the Basics

The **Ambipolar Transport Equation** describes how **excess electrons** and **holes** (generated by some external source like light) move together in a semiconductor under the influence of both an applied electric field and diffusion.

Derive: Ambipolar Transport Equation (detailssss)

1. Maxwell's Equation:

The equation starts from **one of Maxwell's equations** for the electric displacement field D , which describes the relationship between the electric field and charge density:

$$\nabla \cdot \mathbf{D} = \rho$$

- **D** is the **electric displacement field**, and it can be written as:

$$\mathbf{D} = \epsilon \mathbf{E}$$

where ϵ is the **permittivity** of the medium (in this case, the semiconductor), and **E** is the **electric field**.

- The charge density ρ is due to the difference between the concentrations of electrons and holes, δn and δp (excess electron and hole densities):

$$\rho = e(\delta p - \delta n)$$

2. Internal Electric Field:

The difference in charge densities creates an **internal electric field** E_{int} , which can be written as:

$$\nabla \cdot (\epsilon \mathbf{E}) = e(\delta p - \delta n)$$

In one-dimensional (1D) form, this becomes:

$$\frac{\partial E_{\text{int}}}{\partial x} = \frac{e(\delta p - \delta n)}{\epsilon_s}$$

where ϵ_s is the **permittivity of the semiconductor**.

3. Coupled Drift-Diffusion Equations:

Next, the **drift-diffusion equations** for both electrons and holes are written as:

- For **holes**:

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \left(E \frac{\partial(\delta p)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial(\delta p)}{\partial t}$$

- For **electrons**:

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} - \mu_n \left(E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial(\delta n)}{\partial t}$$

Where:

- D_n and D_p are the diffusion coefficients for electrons and holes.
- μ_n and μ_p are the mobilities for electrons and holes.
- g_n and g_p are the generation rates for electrons and holes.
- τ_{nt} and τ_{pt} are the recombination lifetimes for electrons and holes.

4. Assumptions for Ambipolar Transport:

To simplify the system, we assume the following:

- The internal electric field E_{int} is much smaller than the applied electric field E_{app} :

$$|E_{\text{int}}| \ll |E_{\text{app}}|$$

- The concentrations of excess electrons and holes are approximately equal:

$$\delta n(x, t) \approx \delta p(x, t)$$

This simplifies the system significantly and leads us to the **Ambipolar Transport Equation**.

Ambipolar transport equation

- This is the end result:

$$D' \frac{\partial^2(\delta n)}{\partial x^2} + \mu' E \frac{\partial(\delta n)}{\partial x} + g - R = \frac{\partial(\delta n)}{\partial t}$$

- Where $\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$ is the **ambipolar mobility**

- And $D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} = \frac{D_n D_p (n + p)}{D_n n + D_p p}$ is the **ambipolar diffusion coefficient**

→Low Injection

Low injection refers to a situation in a semiconductor where the concentration of **excess carriers** (electrons and holes generated by an external source, like light) is much smaller than the **majority carrier concentration**.

For example, in a **p-type semiconductor**

$$n_0 \ll p_0 \text{ and } \delta n \ll p_0$$

Ambipolar Diffusion Coefficient in Low Injection:

In low injection, the **ambipolar diffusion coefficient** simplifies:

- For a **p-type semiconductor**, where the majority carriers are holes, the ambipolar diffusion coefficient D' becomes equal to the **electron diffusion coefficient** D_n :

$$D' = D_n$$

Ambipolar Mobility in Low Injection:

Similarly, the **ambipolar mobility** μ' simplifies to the **electron mobility** μ_n in a p-type semiconductor:

$$\mu' = \mu_n$$

same to n-type

g - R (Generation and Recombination)

Generation (g) and **Recombination (R)** describe how carriers are created and annihilated, respectively.

- **Recombination** refers to the process by which electrons and holes recombine, reducing the number of free charge carriers. For electrons, the recombination rate R is:

$$R = R_n = \frac{n}{\tau_{nt}}$$

where τ_{nt} is the **electron lifetime**, and n is the electron concentration.

- Similarly, for holes:

$$R = R_p = \frac{p}{\tau_{pt}}$$

where τ_{pt} is the **hole lifetime**.

- **Generation (g)** represents the creation of electron-hole pairs, often through external sources like light. The generation rate is denoted as g .

Simplified Generation-Recombination Terms:

In **thermal equilibrium**, the generation and recombination rates balance each other out. Under **low injection** conditions, the difference between the generation and recombination rates for electrons and holes simplifies to:

- For electrons:

$$g - R = g'_n - \frac{\delta n}{\tau_n}$$

- For holes:

$$g - R = g'_p - \frac{\delta p}{\tau_p}$$

Here, g'_n and g'_p are the excess generation rates for electrons and holes, and δn and δp are the excess carrier concentrations.

Final ambipolar equations

- We will define one final parameter: $g' = g'_n = g'_p$
- Now we can write the ambipolar equations for electrons and holes under low injection conditions
- **p-type:**
$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t}$$
- **n-type:**
$$D_p \frac{\partial^2(\delta p)}{\partial x^2} + \mu_p E \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t}$$

A Few Notes About Recombination Rates

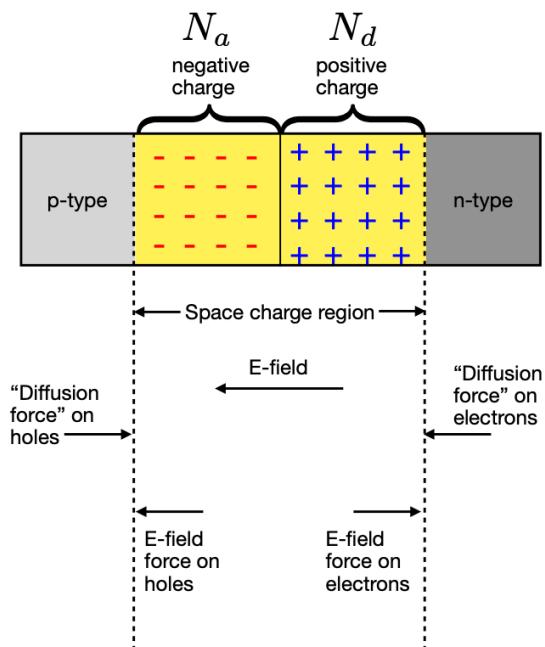
- **Traps:** Recombination rates can be significantly affected by energy levels in the **forbidden band gap**, referred to as **traps**. These traps can be introduced by **dopants** or **surface defects**, and they increase the likelihood of recombination.
- **Surface Recombination:** Recombination tends to occur more rapidly near the **surface** of a semiconductor, where more traps exist due to defects in the periodic crystal structure.
- **Diffusion Toward Surface:** As charge carriers diffuse, they will encounter higher trap densities near the surface, **where recombination will occur more quickly**.

Week8

Introduction to pn Junctions

- **pn junctions** consist of two regions in a semiconductor: a **p-type region** (doped with acceptors) and an **n-type region** (doped with donors).
 - **p-type region:** Doped with acceptors, meaning it has a higher concentration of holes (positive charge carriers).
 - **n-type region:** Doped with donors, meaning it has a higher concentration of electrons (negative charge carriers).
- **Diffusion** is the process where charge carriers (holes and electrons) move from regions of high concentration to regions of low concentration due to a concentration gradient.

As diffusion occurs, a **net negative charge** builds up on the **p-type side** (due to the immobile acceptor ions), and a **net positive charge** builds up on the **n-type side** (due to the immobile donor ions).



the **positive charges** in the **n-type region** of the depletion zone are **positive ions** (specifically, **immobile donor ions**) and not free positive charges.

Zero Bias (Thermal Equilibrium)

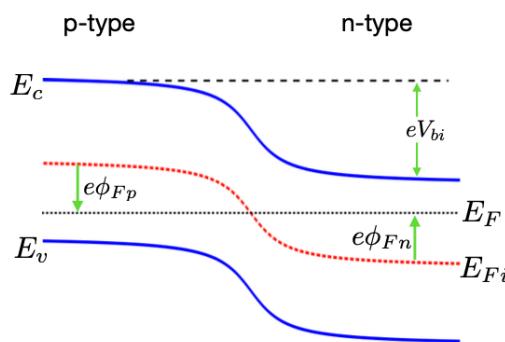
This situation represents **thermal equilibrium**, where there is no net current flowing across the junction because diffusion and drift currents balance each other out
FOCUSES:

- **Width:** How wide is the space charge (depletion) region?
- **Electric Field Strength:** How strong is the internal electric field in this region?
- **Potential Difference:** What is the potential difference across the depletion region?

Built-in Potential Barrier

The **built-in potential barrier** is an intrinsic electric **potential** that forms across the pn junction in **thermal equilibrium**.

RECALL



- For n-type:

$$E_c - E_F = k_B T \ln \left(\frac{N_c}{n_0} \right)$$

- For p-type:

$$E_F - E_v = k_B T \ln \left(\frac{N_v}{p_0} \right)$$

This potential barrier, V_{bi} , prevents electrons in the n-type region from diffusing freely into the p-type region and vice versa for holes. (0.7V)

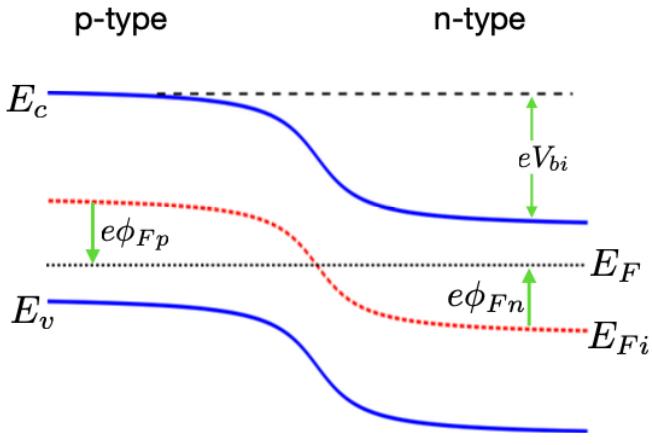
Built-in potential barrier

Intrinsic Carrier Concentration n_i

Hole Concentration p_i in Terms of n_i

$$n_i = N_c \exp \left[\frac{-(E_c - E_{Fi})}{k_B T} \right]$$

$$p_i = n_i = N_v \exp \left[\frac{-(E_{Fi} - E_v)}{k_B T} \right]$$



$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

$$e\phi_{Fn} = E_{Fi} - E_F$$

$$e\phi_{Fp} = E_{Fi} - E_F$$

Φ Fn electrostatic potential difference (in volts) between the Fermi level EF and the intrinsic Fermi level Efi:

Expressing ϕ_{Fn} for the n-Type Side:

- For the n-type side, we use the relationship:

$$n_0 = N_c \exp \left(\frac{-(E_c - E_F)}{k_B T} \right)$$

- Rearranging, we get:

$$\phi_{Fn} = -\frac{k_B T}{e} \ln \left(\frac{N_d}{n_i} \right)$$

Expressing ϕ_{Fp} for the p-Type Side:

- For the p-type side, we similarly get:

$$\phi_{Fp} = \frac{k_B T}{e} \ln \left(\frac{N_a}{n_i} \right)$$

AND

The thermal voltage is defined as:

$$V_t = \frac{k_B T}{e}$$

the built-in potential barrier in terms of the basic properties of the p- and n-type semiconductors:

$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| = \frac{k_B T}{e} \ln\left(\frac{N_d}{n_i}\right) + \frac{k_B T}{e} \ln\left(\frac{N_a}{n_i}\right)$$

$$V_{bi} = \frac{k_B T}{e} \left[\ln\left(\frac{N_d}{n_i}\right) + \ln\left(\frac{N_a}{n_i}\right) \right] = \frac{k_B T}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

(normally 0.7V)

Electric Field in the Space Charge Region

These immobile ions create a **space charge region**, resulting in an electric field that opposes further diffusion of carriers across the junction.

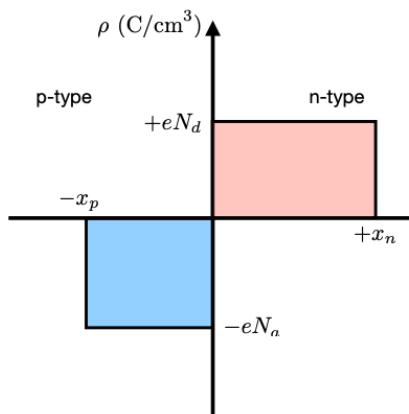
Poisson's Equation for the Electric Field:

- The electric field $E(x)$ within the depletion region can be determined using **Poisson's equation**:

$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\epsilon_s}$$

where:

- $\rho(x)$ is the charge density, which is positive on the n-type side (due to donor ions) and negative on the p-type side (due to acceptor ions).
- ϵ_s is the permittivity of the semiconductor material.



The charge density is determined by the doping conditions:

$$\rho(x) = \begin{cases} -eN_a & \text{if } -x_p < x < 0 \\ eN_d & \text{if } 0 < x < x_n \end{cases}$$

Electric field:

- We can determine the electric field by integrating the charge density:

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_s} = -\frac{dE(x)}{dx}$$

- We can evaluate this first for the p-type side:

$$E = \int \frac{\rho(x)}{\epsilon_s} dx = - \int \frac{eN_a}{\epsilon_s} dx = \left[\frac{-eN_a}{\epsilon_s} x + c_1 \right]$$

- To determine c_1 , we assume that $E = 0$ at $x = -x_p$

- We get: $c_1 = \frac{-eN_a x_p}{\epsilon_s}$

- And:

$$E(x) = \frac{-eN_a}{\epsilon_s} (x + x_p), \quad -x_p \leq x \leq 0$$

We can do the same thing for the n-type side and get:

$$E(x) = \frac{-eN_d}{\epsilon_s} (x_n - x), \quad 0 \leq x \leq x_n$$

We also require the electric field to be continuous at $x = 0$:

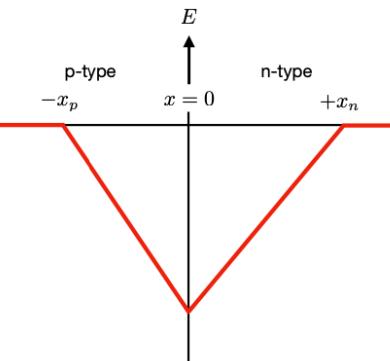
$$\frac{-eN_a x_p}{\epsilon_s} = \frac{-eN_d x_n}{\epsilon_s}$$

$$N_a x_p = N_d x_n$$

Each side of this equation has units of cm^{-2} and it states that the surface density of negative charges in the p-type region is the same as that of positive charges in the n-type region

$E(x)$ is plotted on the right

Within the depletion region, the electric field strength varies linearly with the spatial coordinate. This field is always present, even without an applied voltage.



Electric Potential and Work

1. Definition:

- The electric potential $\phi(x)$ at a point relates to the work W needed to move a charge q in an electric field E from one point to another.
- The potential difference between two points P_1 and P_2 in an electric field is:

$$V_2 - V_1 = - \int_{P_1}^{P_2} \vec{E} \cdot d\vec{\ell}$$

- If the potential difference is positive, work must be done to move a positive charge from P_1 to P_2 .

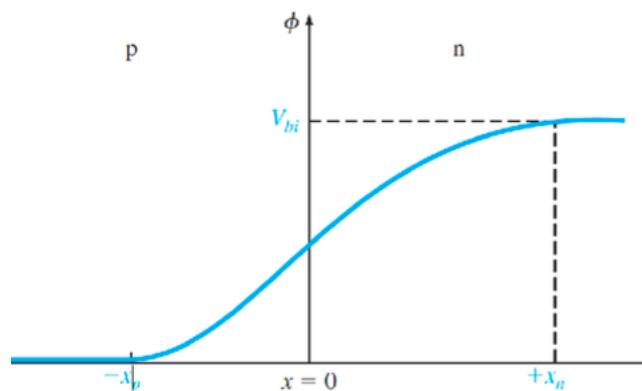
2. Application to the pn Junction:

- As we move across the pn junction from the p-type to the n-type region, the electric potential increases due to the built-in electric field in the depletion region.
- This increase in electric potential corresponds to the **built-in potential barrier** V_{bi} , which prevents charge carriers from freely crossing the junction.

The electric potential of a pn junction

$$\phi(x) = \frac{eN_a}{\varepsilon_s} \left(\frac{x^2}{2} + x_p \cdot x + \frac{x_p^2}{2} \right) = \frac{eN_a}{2\varepsilon_s} (x + x_p)^2 \quad -x_p \leq x \leq 0$$

$$\phi(x) = \frac{eN_d}{\varepsilon_s} \left(x_n \cdot x - \frac{x^2}{2} \right) + \frac{eN_a}{2\varepsilon_s} x_p^2, \quad 0 \leq x \leq x_n$$



Total Built-in Potential Barrier V_{bi}

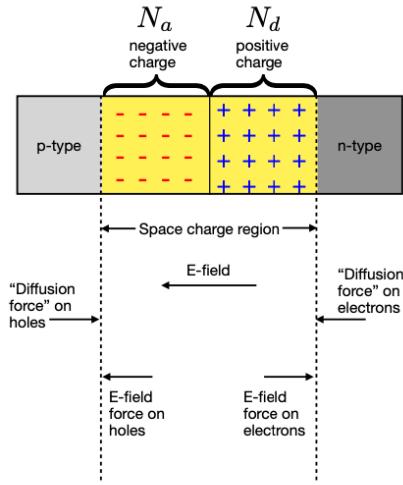
- The total potential difference (or built-in potential V_{bi}) across the junction from $x = -x_p$ to $x = x_n$ is:

$$V_{bi} = \phi(x_n) - \phi(-x_p)$$

- Substituting the expressions, we find:

$$V_{bi} = \frac{e}{2\epsilon_s} (N_d x_n^2 + N_a x_p^2)$$

Space charge width:



$$x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right]}$$

$$x_p = \sqrt{\frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_d}{N_a} \right] \left[\frac{1}{N_a + N_d} \right]}$$

The total width of the space charge region is

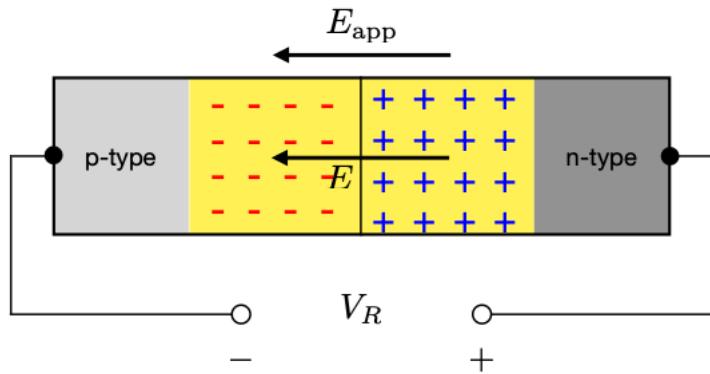
$$W = \sqrt{\frac{2\epsilon_s V_{bi}}{e}} \sqrt{\frac{N_a + N_d}{N_a N_d}}$$

- If we know N_a , N_d , and T , we can determine V_{bi}
- This gives us everything required to determine W

23

$$V_{bi} = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

Reverse Bias in a pn Junction:



相对于 pn 结的 p 型侧，在 n 型侧施加一个正电压 V_R 。

施加的电压增加了整个结的电位差，加强了内置的势垒 built-in potential barrier。

Widen:

施加反向偏置电压将 p 型区域的空穴拉向负极，将 n 型区域的电子拉向电压源的正极。

随着载流子从结处被抽离，更多的固定离子（p 型为负离子，n 型为正离子）暴露出来，有效地扩大了耗尽区。

Then:

With the depletion region widened, there is a greater separation of charges, which results in a stronger electric field in the space charge region.

The **electric field E in the depletion region increases** because there are more ions contributing to it over a wider area.

Total Potential Difference

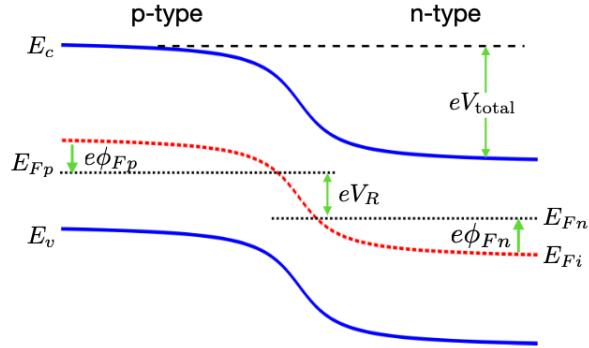
$$V_{\text{total}} = V_{bi} + V_R$$

这种增加的势垒进一步抑制了大多数载流子（p 型空穴和 n 型电子）穿过结的运动。

THEN:

当施加恒定的反向偏置时，最终达到新的平衡。这种平衡的特点是耗尽区变宽，电场增大，整个结的电压增加。

能带图中的能带弯曲变得更加明显，因为导带 E_c 和价带 E_v 随着电位的增加而移动。



NEW

The new space charge width

- We can repeat the analysis that was performed for the zero-bias case to determine x_n and x_p :

$$x_n = \sqrt{\frac{2\varepsilon_s(V_{bi} + V_R)}{e}} \sqrt{\frac{N_a}{N_d} \cdot \frac{1}{N_a + N_d}}$$

$$x_p = \sqrt{\frac{2\varepsilon_s(V_{bi} + V_R)}{e}} \sqrt{\frac{N_d}{N_a} \cdot \frac{1}{N_a + N_d}}$$

- We simply replace V_{bi} with V_{total} in the equations we came up with before:
- And the new width of the space charge region:

$$W = \sqrt{\frac{2\varepsilon_s(V_{bi} + V_R)}{e}} \sqrt{\frac{N_a + N_d}{N_a N_d}}$$

The new peak electric field

- As well, the electric field is still given by the equations that we came up with earlier

$$E(x) = \frac{-eN_a}{\varepsilon_s}(x + x_p), \quad -x_p \leq x \leq 0$$

$$E(x) = \frac{-eN_d}{\varepsilon_s}(x_n - x), \quad 0 \leq x \leq x_n$$

- The electric field will have the same value at $x = 0$:

$$E_{max} = \frac{-eN_d x_n}{\varepsilon_s} = \frac{-eN_a x_p}{\varepsilon_s}$$

- We usually want to express these “internal” parameters in terms of the things we can control externally. Substituting our expressions for x_n or x_p , we will get:

$$E_{max} = -\sqrt{\frac{2e(V_{bi} + V_R)}{\varepsilon_s} \cdot \left(\frac{N_a N_d}{N_a + N_d} \right)}$$

The peak electric field occurs at the junction $x=0$ $x=0$

The new peak electric field

- Compare W and E_{max} :

$$W = \sqrt{\frac{2\epsilon_s(V_{bi} + V_R)}{e}} \sqrt{\frac{N_a + N_d}{N_a N_d}} \quad E_{max} = -\sqrt{\frac{2e(V_{bi} + V_R)}{\epsilon_s} \cdot \left(\frac{N_a N_d}{N_a + N_d} \right)}$$

- If we already know W , we can easily calculate E_{max} :

$$E_{max} = \frac{-2(V_{bi} + V_R)}{W}$$

- This equation is also valid for the zero-bias case
- So, all of the equations we developed for the zero-bias case are still applicable to the reverse-bias case. We just need use $V_{total} = V_{bi} + V_R$ instead of V_{bi} .

Junction capacitance / depletion layer capacitance:

Analogy to a pn Junction:

- In a reverse-biased pn junction, the depletion region acts similarly to a capacitor.
- As the reverse-bias voltage V_R increases:
 - Charges (electrons and holes) are pulled away from the junction, widening the depletion region (space-charge region).
 - This widened depletion region stores charge, similar to the plates of a capacitor.
- The **junction capacitance C'** is defined as:

$$C' = \frac{dQ'}{dV_R}$$

where:

- Q' : The charge per unit area in the space charge region (SCR).
- V_R : The reverse bias voltage.
- C' is typically measured in **farads per square centimeter (F/cm^2)**, reflecting capacitance per unit area.

junction capacitance formula and explore **one-sided junctions**

Junction capacitance / depletion layer capacitance

- From earlier, we found x_n to be:

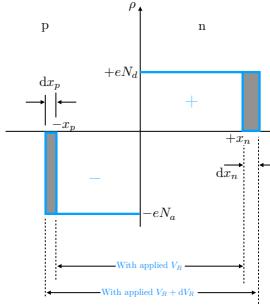
$$x_n = \sqrt{\frac{2\epsilon_s(V_{bi} + V_R)}{e}} \sqrt{\frac{N_a}{N_d} \cdot \frac{1}{N_a + N_d}}$$

- Take the derivative wrt V_R :

$$\frac{dx_n}{dV_R} = \sqrt{\frac{\epsilon_s}{2e(V_{bi} + V_R)}} \cdot \sqrt{\frac{N_a}{N_d} \cdot \frac{1}{N_a + N_d}}$$

- Now, we can determine the junction capacitance:

$$C' = eN_d \frac{dx_n}{dV_R} = \sqrt{\frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}}$$



33

- Alternatively, if we know the width W of the depletion region, we can express the junction capacitance as:

$$C' = \frac{\epsilon_s}{W}$$

One-Sided Junctions

One-sided junctions have highly asymmetric doping, where one side is much more heavily doped than the other. This simplifies the capacitance and depletion width calculations.

1. p⁺-n Junction (p-type heavily doped):

- In this case, $N_a \gg N_d$.
- The depletion region extends mostly into the n-type material.
- The expression for W simplifies to:

$$W \approx \sqrt{\frac{2\epsilon_s(V_{bi} + V_R)}{eN_d}}$$

- Since W depends primarily on the doping of the lighter-doped side (n-type here), the junction capacitance C' can be approximated by considering only the properties of the n-type side.

2. n⁺-p Junction (n-type heavily doped):

- Here, $N_d \gg N_a$.
- The depletion region extends primarily into the p-type material.
- The expression for W becomes:

$$W \approx \sqrt{\frac{2\epsilon_s(V_{bi} + V_R)}{eN_a}}$$

- The junction capacitance C' can thus be approximated using only the p-type side's parameters.

(e.g., (p⁺)-n, x_n >> x_p) the depletion region mostly extends into the n-type material

Depletion Width Approximation:

- The width W of the depletion region can be approximated by the expression for the lightly doped side:

$$W \approx x_n = \sqrt{\frac{2\epsilon_s(V_{bi} + V_R)}{eN_d}}$$

- Junction capacitance C' for a one-sided junction can then be approximated as:

$$C' \approx \sqrt{\frac{e\epsilon_s N_d}{2(V_{bi} + V_R)}}$$

- This approximation simplifies capacitance calculations in high-asymmetry doping scenarios.

In a one-sided junction, where one side is heavily doped and the other is lightly doped, the depletion region W extends mostly into the lightly doped side. This is because the lightly doped side has fewer charge carriers, so it requires a larger width to balance the same charge as the heavily doped side. Therefore, W can be approximated using parameters (like doping concentration) of the lightly doped side.

Junction Breakdown:

When a reverse-biased pn junction experiences a **high electric field**, it can enter a **breakdown state**, allowing significant reverse current to flow. Breakdown can happen in two ways:

1. Avalanche Breakdown:

在轻掺杂的 pn 结中，随着反向电压的增加，电场变得足够强以加速自由载流子。这些载流子可以通过碰撞电离将电子从原子键中撞出，产生更多的电子空穴对，并导致链式反应。

这一过程被称为雪崩击穿，发生在具有较宽损耗区的连接处。

Avalanche Breakdown

1. Definition:

- Avalanche breakdown occurs in **lightly-doped pn junctions** under high reverse bias.
- Here, **Zener tunneling is unlikely** because the energy bands are too far apart.

2. Mechanism:

- When the reverse-bias voltage is high, free electrons in the p-type region are accelerated to high speeds in the electric field.
- These high-energy electrons can collide with the lattice atoms, creating additional electron-hole pairs in a process called **impact ionization**.
- This process can trigger a chain reaction, where newly created electrons are also accelerated, causing more impact ionization.

2. Zener Breakdown (Zener Tunneling):

齐纳击穿发生在高掺杂的 pn 结中，损耗宽度 W 非常小。

在相对较低的反向偏置下，n 型侧的导带与 p 型侧的价带紧密对齐。

量子隧穿允许电子穿过狭窄的耗尽区，从 p 侧价带移动到 n 侧导带，尽管存在能量障碍。

这种隧穿效应就是齐纳效应，它会产生反向偏置电流。

- The Zener effect occurs when the depletion region is narrow enough for quantum tunneling to happen.
- The heavily doped pn junction reduces the built-in potential barrier, making it easier for electrons to tunnel under low reverse bias conditions.

The depletion width W in heavily doped pn junctions is very small because high doping concentrations lead to a high density of charge carriers. To balance the charges across the junction, only a small width is needed to accommodate the required opposite charges from each side.

BREAKDOWN VOLTAGE: eg. N-type

- The maximum voltage we can apply, or the **breakdown voltage**, can be expressed as:

$$V_B = \frac{\varepsilon_s E_{\text{crit}}^2}{2eN_d}$$