Metal oxide FET transistors

MOSFET transistor DC analysis

MOSFET transistor : small signal analysis

application as an amplifier

application as an amplifier

MOSFET

金属氧化层半导体场效电晶体

(20) **Metal oxide semiconductor field-Effect-Transistor**

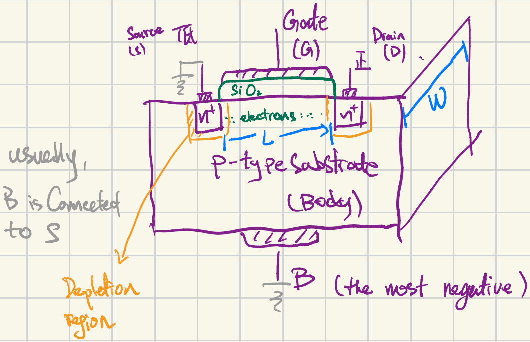
(small, simple) ⇒ larger-scale IC

§ n-channel enhancement-type MOSFET

SiO_2: 绝缘体 (no current go through, only voltage)

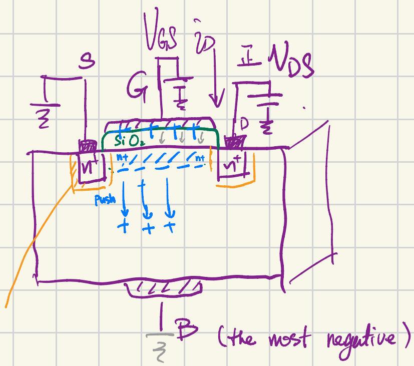
L: channel length

"A voltage is applied to Gate to Control the current flow between S and D" (How it work)



§ operation.

- no gate bias (zero bias)
  back-to-back diodes, $i_D = 0$ (V_{DS} is applied)
- apply a positive V_{GS}
  push holes down to the substrate ⇒ deplete the holes under SiO_2
  further increase V_{GS} ⇒ inverse P to n under SiO_2



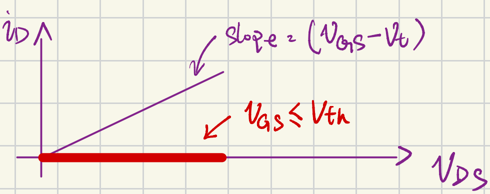
The Value of V_{GS} at which a just sufficient number of mobile electrons accumulate in the channel is called the threshold voltage

V_{th} or V_t = 0.2~1V

Drift Current: $i_D = j_n = q\,n\,\mu_n E$

electron: $n \propto (V_{GS} - V_{th})$ effective (overdrive)

[Electric Field $= \frac{V_{DS}}{L}$]

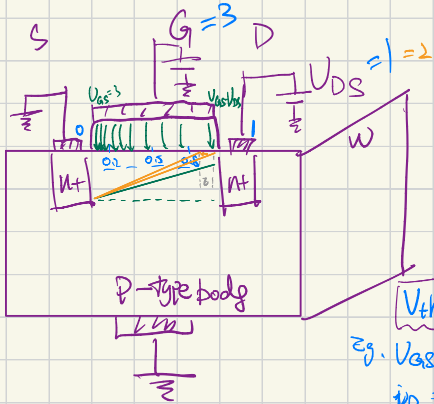


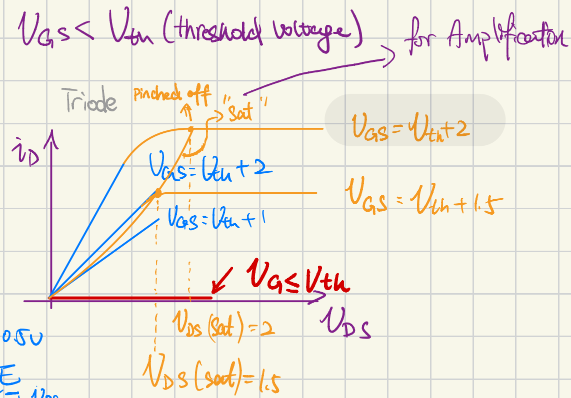
Tips

① G 中 ↕

② S $\xrightarrow{e^-}$ D

② § n-channel enhancement-type MOSFET





$\boxed{V_{th}=1V}$

Eg. $V_{GS}=V_{th}+0.5V$

$j_D = q n \mu_n E$, $E = \frac{V_{DS}}{L}$

$n \propto (V_{GS}-V_{th})$

Effective

The Voltage between G and points along the channel decrease from V_{GS} at S end to $V_{GS}-V_{DS}$ at D end

Eventually, when $V_{GS}-V_{DS}=V_t$. → $V_{DS(sat)} = V_{GS}-V_{th}$

the channel depth at 0 is almost zero ⇒ the channel is pinched off

Tips:
- Sat area: Amplification
- Triode ≈ resistor

工作在饱和区：$V_{DS} > [V_{DS(sat)} \doteq V_{GS}-V_{th}]$

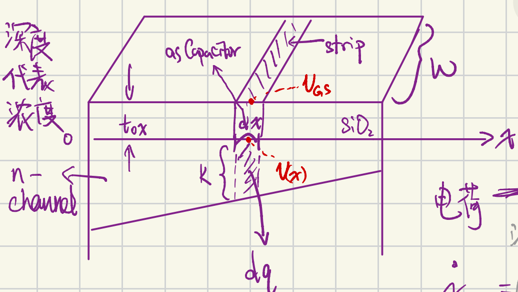
OR 看 $V_{GS}-V_{th} \gtrless V_{DS}$ (G高于D)

§ 4.1.6 Derivation of the $i_D - V_{DS}$ relationship

Consider "Triode" region

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \quad \epsilon_{ox} = 3.9\epsilon_0 = 3.45\times10^{-11}\ F/m$$

(per unit area)



for the strip: $C_{ox} \cdot W dx$

Tips: $C = \frac{dQ}{dV}$

电荷 → $dq = C_{ox} W dx \,(V_{GS} - V_s(x) - V_{th})$ → 特性

浓度 换算

$$j_n = q n \mu_n E, \quad \frac{dq}{W dx k}, \quad \mu_n\left(-\frac{dV}{dx}\right)$$

Electron current → $j_n = qn\mu_n E = \dfrac{dq}{W\,dx\,k}\,\mu_n\left(-\dfrac{dv}{dx}\right)$ ← 方向 ↓ unknown

$i = j_n kW = qn\mu_n E = \dfrac{dq\,kW}{W\,dx\,k}\,\mu_n\left(-\dfrac{dv}{dx}\right)$

$i_D = j_n kW = \dfrac{dq}{dx}\,\mu_n\left(\dfrac{dv}{dx}\right) = \mu_n C_{ox} W\left(V_{GS} - V(x) - V_{th}\right)\dfrac{dv}{dx}$

$\int_0^L \underset{\text{Const}}{i_D}\,dx = \mu_n C_{ox} W \int_0^{V_{DS}} \left[\underbrace{(V_{GS}-V_{th})}_{\text{const}} - V(x)\right] dv$

$i_D L = \mu_n C_{ox} W\left[(V_{GS}-V_{th})V_{DS} - \frac{1}{2}V_{DS}^2\right] \Rightarrow$

$i_D = \mu_n C_{ox}\left(\dfrac{W}{L}\right)\left[(V_{GS}-V_{th})V_{DS} - \frac{1}{2}V_{DS}^2\right]$

(~~当V_{DS}很小时~~) slope ϕ

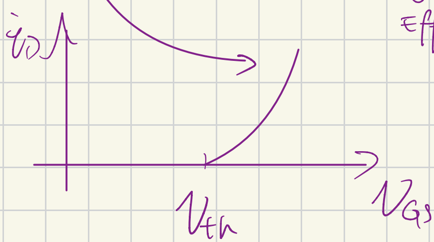
Consider "saturation"

$i_D = \mu C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS}-V_{th})V_{DS(sat)} - \frac{1}{2}V_{DS(sat)}^2\right]$

$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\underbrace{(V_{GS}-V_t)^2}_{\text{Effective}}$

$V_{DS(sat)} = (V_{GS}-V_t)$

$V_{GS}-V_t \equiv V_{ov}$ (overdrive)



Consider "Boundary" (把 Sat point 连起来)

$i_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)V_{DS}^2$

$\boxed{V_{ov} = V_{GS} - V_{th}}$

Eg. for $t_{ox} = 8nm$, $\mu_n = 450\ cm^2/V\cdot s$ $V_t = 0.7V$

a) find $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-11}}{8\times10^{-9}} = 4.32\times10^{-3}\ F/m^2$

$\mu_n C_{ox} = 194\ \mu A/V^2$

b) $\frac{W}{L} = 8\ \mu m / 0.8\ \mu m$, for "Sat" with $I_D = 100\ \mu A$, $V_{GS} = ?$ $V_{DS}(min) = ?$

$i_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_t)^2$

$100\mu A = \frac{1}{2}\cdot\left(\frac{W}{L}\right) 194\frac{\mu A}{V^2}(V_{GS}-0.7)^2 \Rightarrow V_{GS} = 1.02V$ ✓

Sat $\rightarrow V_{DS} > \underbrace{V_{GS}-V_t}_{1.02-0.7} \Rightarrow V_{DS}(min) = 0.32V$ ✓

c) for a $1000\text{-}\Omega$ resistor, for very small V_{DS}

$1000 = r_{DS} = \frac{V_{DS}}{i_D} = 1/\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_t)$

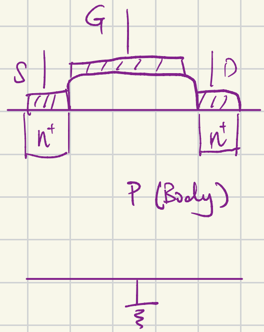
$\therefore V_{GS} = 1.22$

Tips: BY

$i_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS}-V_{tn})V_{DS} - \frac{1}{2}V_{DS}^2\right]$

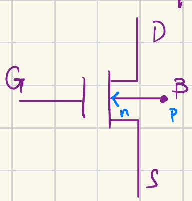
2次方舍掉 ∵ 很小

(22) § n-channel (enhancement-type) MOSFET

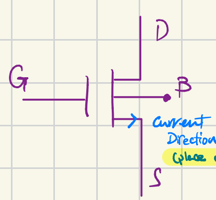


Circuit Model

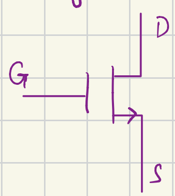
n → n-channel
P → Body



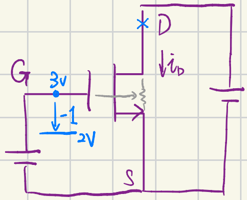
电子从S出发，∴电流从D出发



Usually S and B Connected

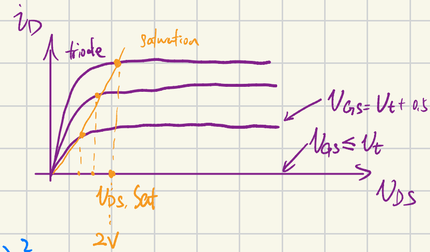


$V_{th} = 1V$



$3V = V_{GS}$

V_{DS} if $> 2V$ Sat



$$\begin{cases} Sat \Rightarrow i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS}-V_t)^2 \\ triode \Rightarrow i_D = \mu C\left(\frac{W}{L}\right)\left[(V_{GS}-V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right] \end{cases}$$

Sat: $V_{DS} > V_{GS} - V_{th}$ (V_{OV})

改变R性质

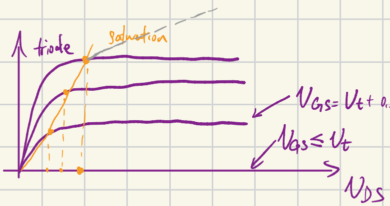
$V_{GS}\uparrow R\downarrow$

very small

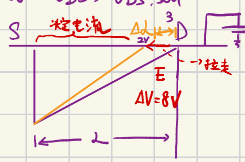
ignore

i_D与V_{DS}成linear关系

§ 4.2 Channel-length modulation effect (Early-Effect)



as $V_{DS} > V_{DS,sat}$



V_{DS}

$\left(\frac{\Delta L}{L} = \frac{V_{DS}}{V_A}\right)$ (1次方)

proportion to

$\Delta L \propto V_{DS}$

V_A : Early Voltage

$$i_D = \frac{1}{2}\mu C\left(\frac{W}{L-\Delta L}\right)(V_{GS}-V_t)^2$$

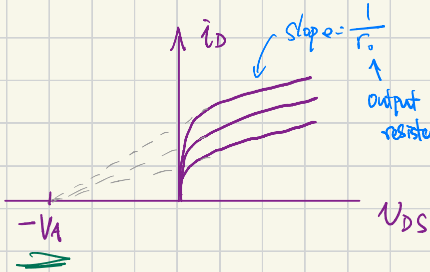
effective

$$i_D = \frac{1}{2}\mu C\frac{W}{L}\left(\frac{1}{1-\frac{\Delta L}{L}}\right)(V_{GS}-V_t)^2$$

$$i_D \doteq \frac{1}{2}\mu C\left(\frac{W}{L}\right)\left(1+\frac{\Delta L}{L}\right)(V_{GS}-V_t)^2$$

$$i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)\left(1+\frac{V_{DS}}{V_A}\right)(V_{GS}-V_t)^2$$

V_A : Constant



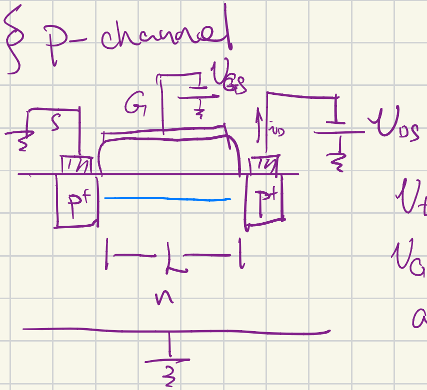
i_D (DC版)

$$r_o = \frac{1}{\left(\frac{\partial i_D}{\partial V_{DS}}\right)} = \frac{1}{\frac{I_D}{V_A}}$$

no early effect

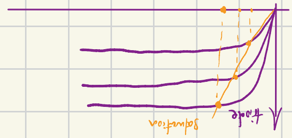
$$\boxed{r_o = \frac{V_A}{I_D}}$$

§ P-channel



$V_{GS} - V_{DS} = V_t$

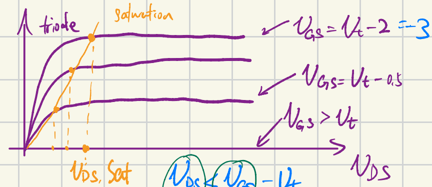
pinched off



$V_t = -1$

$V_t = -1\,V$

$V_{GS} = V_t$ just forms a P-channel

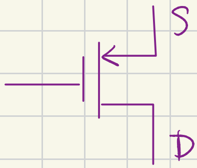


$V_{DS} < V_{GS} - V_t$

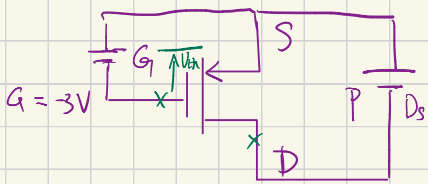
$\parallel$

$V_{GS} - V_t = -3 - (-1) = -2$

Sat $V_{DS} < V_{GS} - V_t$



$i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$



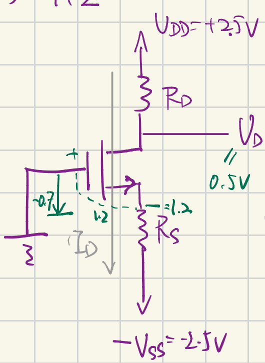
$= -3$ Sat

-4

-5

$\vdots$

§ 4.2



$R_D = ?$ $R_S = ?$

So that $I_D = 0.4\,mA$, $V_D = 0.5V$

$(V_{th} = 0.7V,\ \mu C = 100\ \mu A/V^2,\ L = 1\ \mu m,\ W = 32\ \mu m)$

n-channel

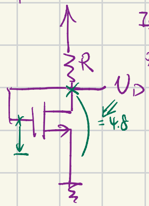
Sat $\Rightarrow$ $i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$

$400 = \frac{1}{2}\cdot 100\left(\frac{32}{1}\right)(V_{GS} - 0.7)^2 \Rightarrow V_{GS} = 1.2V$

$R_S = \frac{-1.2 - (-2.5)}{0.4}\,k\Omega = 3.25\,k\Omega$

$R_D = \frac{2.5 - 0.5}{0.4} = 5\,k\Omega$

Eg. 4.3



$I_D = 80\mu A$ ($V_t = 0.6V$, $\mu C = 200\mu A/V^2$, $L = 0.8\mu m$, $W = 4\mu m$)

$R = ?$ $V_D = ?$

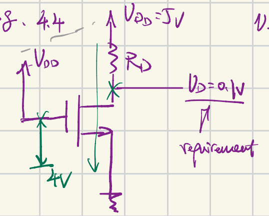
Check Sat $\because$ In Source wire $\therefore$ Sat

$i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$

$80 = \frac{1}{2} 200 \left(\frac{4}{0.8}\right)(V_{GS} - 0.6)^2 \Rightarrow V_{GS} = 1V,\ V_D = 4.8$ ✓

$R = \frac{3-1}{0.08} = 25k\Omega$ ✓

Eg. 4.4



$V_t = 1V$, $\mu C\left(\frac{W}{L}\right) = 1mA/V^2$, $R_D = ?$, $V_D = 0.1$

Check sat ?

No, Triode

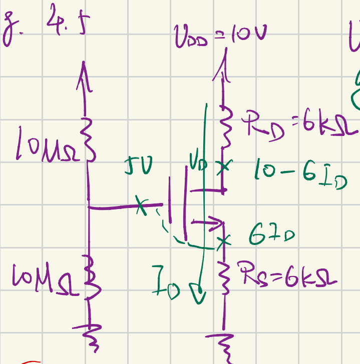
$\therefore i_D = \mu C\left(\frac{W}{L}\right)[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2]$

$= 1 \cdot [(5-1)(0.1) - \frac{1}{2}(0.1)^2]$

$= 0.395 mA$

$R_D = \frac{5-0.1}{0.395} = 12.4k\Omega$

Eg. 4.5



无电流：同分压

$V_t = 1V$, $\mu C\left(\frac{W}{L}\right) = 1mA/V^2$

guess, Sat

$i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$

$I_D = 0.5(5 - 6I_D - 1)^2$

$I_D = 0.89mA$ / $0.5mA$

or

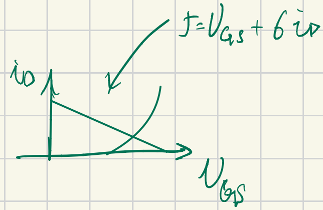
$6I_D = 6 \times 0.89 = 5.34$ ✗

or $6I_D = 6 \times 0.5 = 3V$ ✓

$V_D = 10 - 6 \times 0.5 = 7$

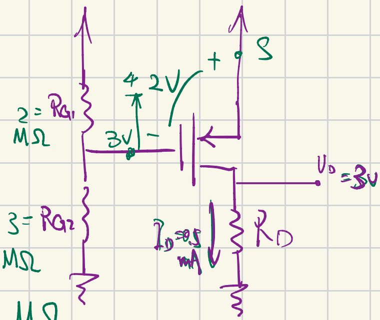
MΩ可为持信号(AC信号)

$7 > (5-1)$ ✓



23

Eg. 4.5



大电阻防漏(小AC)

Sat? Gate与Drain的大小 $\quad k'=\mu C$

Design the circuit

So that the MosFET is in "sat" with $I_D=0.5mA$ & $V_D=3V$

$\left(V_{th}=-1V,\ k_p'\left(\frac{W}{L}\right)=1\ mA/V^2\right)$

P-channel

$i_D=\frac{1}{2}k_p'\left(\frac{W}{L}\right)(V_{GS}-V_t)^2 \qquad V_{GS}-V_t=\pm 1$

$0.5=\frac{1}{2}1\cdot(V_{GS}+1)^2 \Rightarrow V_{GS}=-2V,\ 0$

✓ ✗

$R_D=\frac{3}{0.5}=6k\Omega$

What is the largest R_D that can have, while maintaining "Sat"

$R_D\uparrow$?

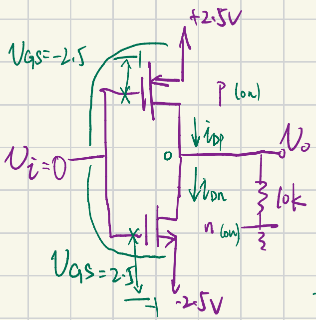
$R_D=\frac{4}{0.5}=8k\Omega$

Eg. 4.7

$k_n'\left(\frac{W}{L}\right)_n=k_p'\left(\frac{W}{L}\right)_p=1mA/V^2 \quad V_{tn}=-V_{tp}=1V$

find i_{DP}, i_{DN} for $V_i=0,\ 2.5,\ -2.5V$

CMOS



(CMOS)

$-2.5-(-1)$
$(V_{GS}-V_t)$
$2.5-1$

$\Rightarrow i_{DP}=i_{DN} \Rightarrow V_o=0$

by

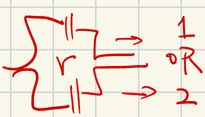
(Sol): Sat

$i_{DN}=\frac{1}{2}k_n'\left(\frac{W}{L}\right)(V_{GS}-V_t)^2$

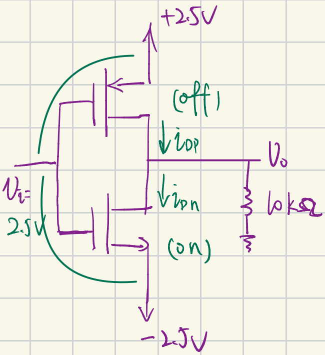
$=\frac{1}{2}\cdot 1\cdot(2.5-1)^2 \Rightarrow i_{DN}=1.125mA$

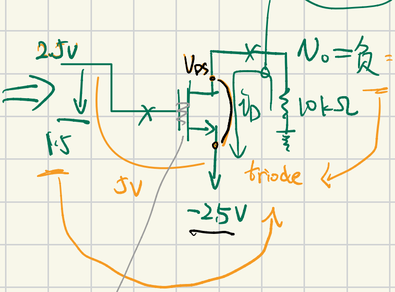
$i_{DP}=1.125mA$

Transmission Gate (信号选择)



$V_{GS}>V_{th}$ 导通





V_o = 负 $= -10\,i_D$

(ignore) Assume V_{DS} very small

$i_D = k_n'\left(\frac{W}{L}\right)\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$

$V_{GS} = 5V$, $V_t = 1V$

$V_{DS} = V_o - (-2.5) = 2.5 - 10\,i_D$

$\Rightarrow i_D = 0.244\,mA$

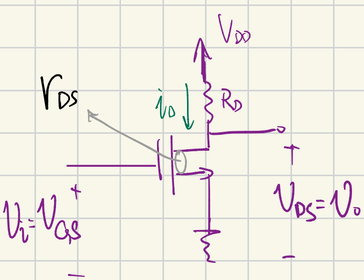
$\therefore V_o = -2.44V$, $V_{DS} = -2.44 + 2.5 = 0.06V$ (Assume check ✓)

$r_{DS} = \dfrac{1}{k'\left(\frac{W}{L}\right)(V_{GS} - V_t)}$ ⇓ large

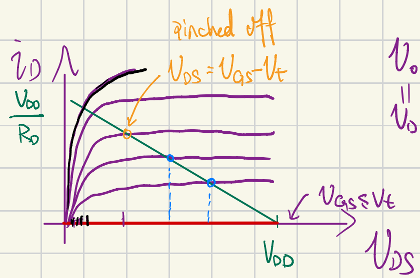
r_{DS} very small (大电压→小电阻)

$r_o = \dfrac{V_{DS}}{i_D}$ MOS 变 r

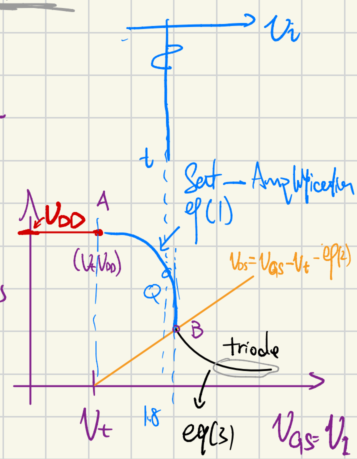
§ 4.4 Large-Signal operation – the transfer characteristic



Common Source (CS)



$V_{DD} = i_D R_D + V_{DS}$ (load line)



$V_o = V_{DD} - i_D R_D$

(eq1) $= V_{DD} - \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)^2 R_D$, $V_{GS} = V_i$

triode:

Eq(3) $V_o = V_{DD} - R_D \mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)\left[V_{DS} - \frac{V_{DS}^2}{2}\right]$

$V_o = V_{DD} - R_D \mu C\left(\frac{W}{L}\right)\left[(V_i - V_t)V_o - \frac{V_o^2}{2}\right]$ (ignore)

$\left[1 + R_D \frac{1}{r_{DS}}\right] = V_{DD}$, $\frac{1}{r_{DS}}$

$V_o = V_{DD} \dfrac{r_{DS}}{R_D + r_{DS}}$

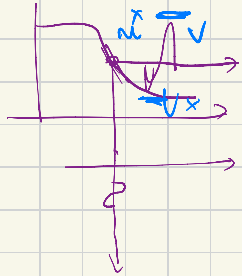
(B point can be solved from Eq(1) and eq(2))

Amplification.

AC gain

$$A_v = \frac{\partial v_o}{\partial v_i}\Big|_Q = -\mu C\left(\frac{W}{L}\right)(V_{GS}-V_t)R_D$$

DC (1.8V)



$$\left(\sqrt{i_D} = \sqrt{\tfrac{1}{2}\mu C\left(\tfrac{W}{L}\right)}\,(V_{GS}-V_t)\right)$$

$$= -R_D\sqrt{2 I_D\, \mu C\left(\tfrac{W}{L}\right)}$$

DC

$$\left(A_v \propto \sqrt{I_D}\right)$$

ac dc

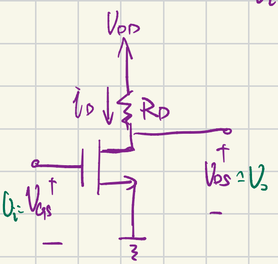
Ex

§ 4.4

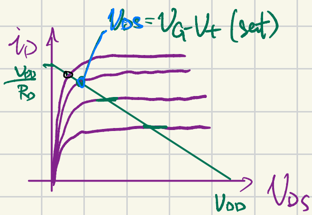
Eg. 4.8 Consider a CS circuit

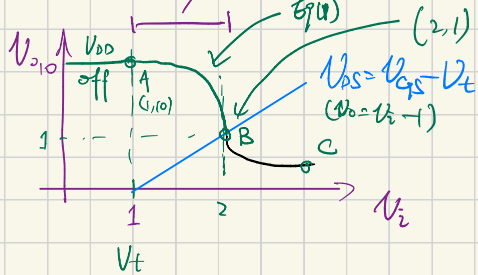
$V_t = 1V$, $\mu C\left(\frac{W}{L}\right) = 1mA/V^2$, $R_D = 18k\Omega$, $V_{DD} = 10V$

Amplification.



CS





$V_{DS} = V_{DD} - i_D R_D$ (load line)

(Sol) $v_o = V_{DD} - \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$ — Eq (1)

$\quad = 10 - \frac{1}{2}(v_i - 1)^2 \cdot 18$

$v_o = v_i - 1$

$\Rightarrow v_o = 1,\ v_i = 2$

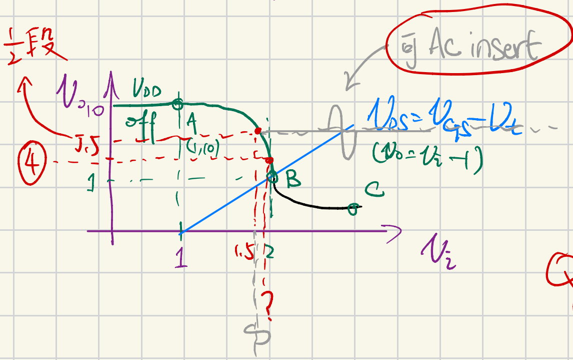
B (2,1)

$V_o = V_{DD} - \mu C\left(\frac{W}{L}\right)\left[(v_i - V_t)v_o - \frac{1}{2}v_o^2\right]R_D$ (ignore)

$V_o = 10 - [(v_i - 1)v_o] \times 18$

$V_o = 0.061\,V$

求放大? (Amplification)



Q

① Design $V_o = 4V \Rightarrow V_i$?

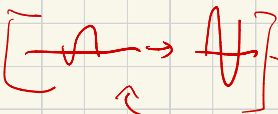
$$V_o = V_{DD} - \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_i - V_t)^2 R_D$$

$$4 = 10 - \frac{1}{2}(V_i - 1)^2 \cdot 18$$

$$V_i = 1.816V$$

Q

② How to know A_v?



$$A_v = \frac{\partial V_o}{\partial V_i} = \underbrace{-\mu C\left(\frac{W}{L}\right)}_{=1}(1.816 - 1)\underbrace{R_D}_{=18}$$

$= -14.7 V/V$

∵静 ∴ 概

§ 4.5 MOSFET DC Bias

bias stability

∴ $\Rightarrow A_v = -\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)R_D$ (sat) $\Rightarrow -R_D\sqrt{2\mu C\left(\frac{W}{L}\right)I_D}$

dc

$$i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$$

$$\sqrt{\frac{2 i_D}{\mu C\left(\frac{W}{L}\right)}} = (V_{GS} - V_t)$$

∴ $A_v \propto \sqrt{I_D}$

Stability? $\Rightarrow$ ΔT 会 affect ΔI_D, A_v 会改变 (μC 与 T 有关)

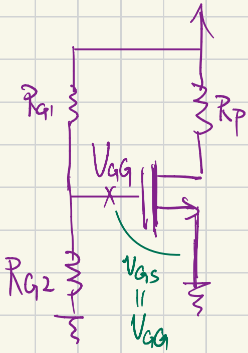
$\Downarrow$

$\Delta K'$ in a batch $\Rightarrow \frac{\Delta I_D}{I_D}$ ↓ as small as possible is good $\Rightarrow$ Stability ↑

§4.5

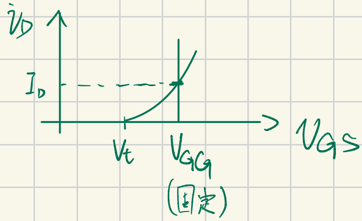
fixed V_{GS}

加入 R_S → Stability ↑

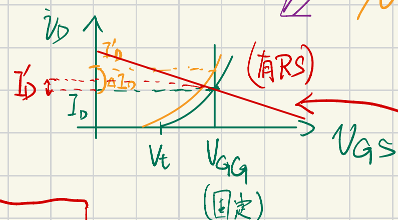


$$V_{GG} = V_{DD} \cdot \frac{R_{G2}}{R_{G1}+R_{G2}}$$

Sat. $i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS}-V_t)^2$ ⟹ REPLACE MOS

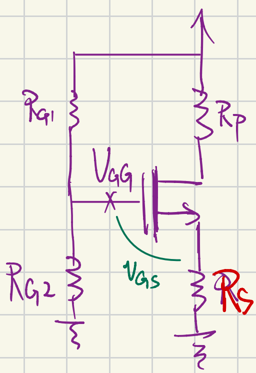


$\mu C - K'$ changed



(有RS)

$I_D' < \Delta I_D$

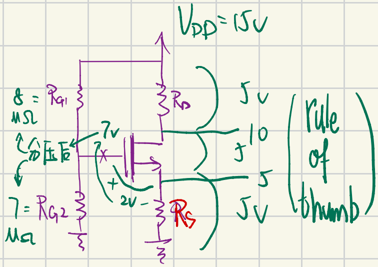


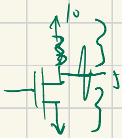
$$\boxed{V_{GG} = V_{GS} + i_D R_S}$$

∴ R_S 加入 ⟹ Increase Bias Stability

$K' \uparrow \Rightarrow i_D \uparrow \Rightarrow V_{GS} \downarrow \left(\xRightarrow{+R_S} \Delta i_D \downarrow \right)$

Eg. 4.9 Design to have $I_D = 0.5\,mA$ ($V_t = 1V$, $K'(\frac{W}{L}) = 1\,mA/V^2$, $V_{DD} = 15V$)



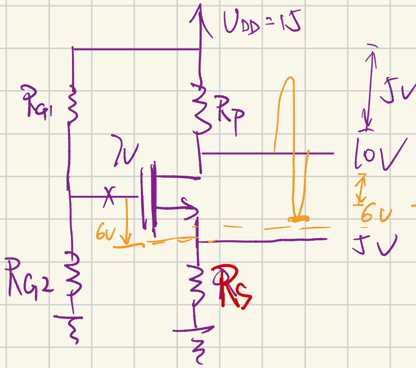


(Sol):

$R_D = 10k\Omega = R_S$

Sat $0.5 = \frac{1}{2} 1 \cdot (V_{GS}-1)^2$

$V_{GS} = 2$



Voltage Swing:

15→10 → positive swing 5V

10→6 → negative Swing 4V

} max

§ Replace by another, Same $k'(\frac{W}{L})$ but $V_t = 1.5V$

$i_D = \frac{1}{2}\cdot 1\,(V_{GS}-1.5)^2$ ①

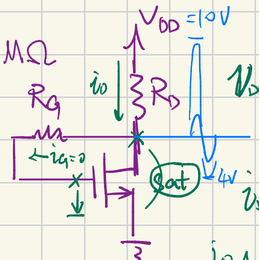
$V_{GG} = V_{GS} + i_D R_S$

$7 = V_{GS} + i_D \cdot 10$ ②

$I_D = 0.455\,mA$

$\frac{\Delta I_D}{I_D} = -9\%$

§ 4.5.3

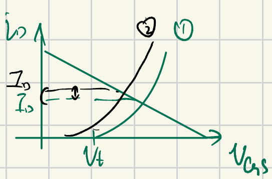


$V_{th}=1$

$V_{DD} = i_D R_D + V_{DS}$

$V_{DS} = V_{GS}$

$i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS}-V_t)^2$



∴ RG increase bias stability (negative feedback)

$\boxed{\text{if } k'\uparrow \Rightarrow i_D\nearrow = V_{GS}\downarrow \Rightarrow \Delta i_D\downarrow}$

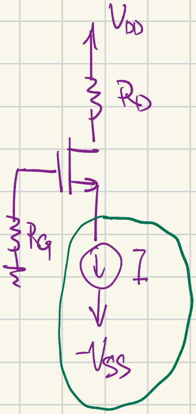
但是

Swing不平衡

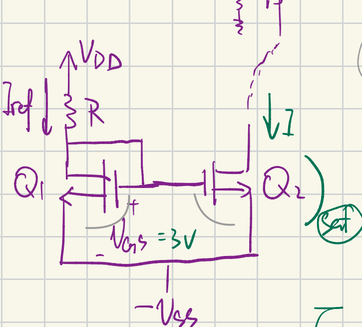
negative Swing 太小，只有 V_{th}

§ 4.5.4

Biasing using a Constant-current source



§ Current Mirror

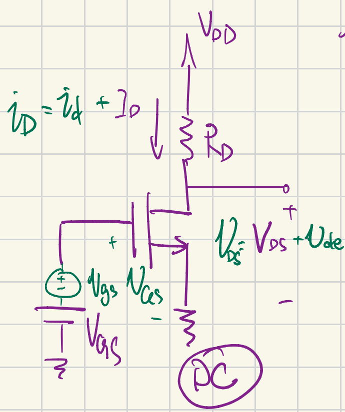


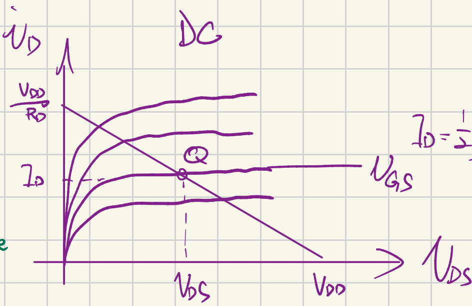
$$\left.\begin{aligned} I_{ref} &= \tfrac{1}{2}K'\left(\tfrac{W}{L}\right)(V_{GS}-V_t)^2 \\ \frac{V_{DD}-V_{GS}-(-V_{SS})}{R} &= I_{ref} \end{aligned}\right\} \text{Solve}$$

$\Rightarrow$ Solve $I_{ref} = f(R)$

$$\left[\begin{aligned} I &= \tfrac{1}{2}K_2'\left(\tfrac{W}{L}\right)_2(V_{GS}-V_t)^2 \\ \frac{I}{I_{ref}} &= \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} = 1 \quad \text{(matched)} \end{aligned}\right]$$

§ 4.6 Small signal operation





$I_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS}-V_t)^2$

$V_{DD} = I_D R_D + V_{DS}$

include DC
AC

Ac analysis $\Rightarrow i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(\underset{V_{GS}+V_{gs}}{V_{GS}} - V_t)^2$

$= \frac{1}{2}\mu C\left(\frac{W}{L}\right)\left[(V_{GS}-V_t)^2 + 2(V_{GS}-V_t)V_{gs} + V_{gs}^2\right]$

$\downarrow$

Assume

$V_{gs} \ll 2(V_{GS}-V_t) \Rightarrow$ Small signal Approximation

$\downarrow$

微分後剩AC信号

$$= \underbrace{I_D + \mu C\left(\frac{W}{L}\right)(V_{GS}-V_t)v_{gs}}_{i_d} \qquad v_{gs} = v_{ac}\sin$$

Define: transconductance

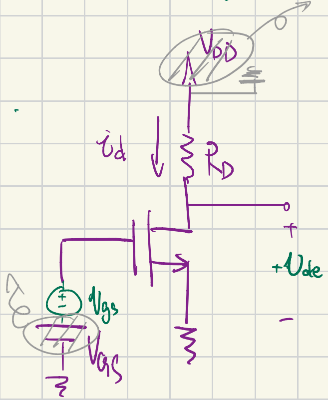
(typically $g_m = 1mA/V$) $\quad g_m \equiv \frac{i_d}{v_{gs}} = \mu C\left(\frac{W}{L}\right)(V_{GS}-V_t) = \frac{\partial i_D}{\partial v_{GS}}$

DC

OR

$$\sqrt{I_D} = \sqrt{\tfrac{1}{2}\mu C\left(\tfrac{W}{L}\right)}(V_{GS}-V_t) \quad = \sqrt{2\mu C\left(\tfrac{W}{L}\right) I_D}$$

$$i_d = g_m v_{gs}$$



$V_{DS} = V_{DD} - i_D R_D$

$V_{DS} + v_{ds} = V_{DD} - (I_D + i_d)R_D = V_{DD} - I_D R_D - i_d R_D$

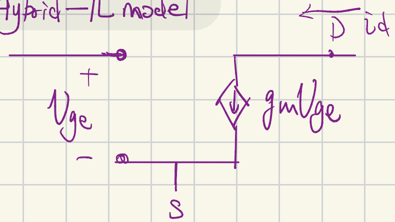
ignore DC $\qquad$ ignore DC

$\Rightarrow \underbrace{v_{ds} = -i_d R_D}_{AC}$

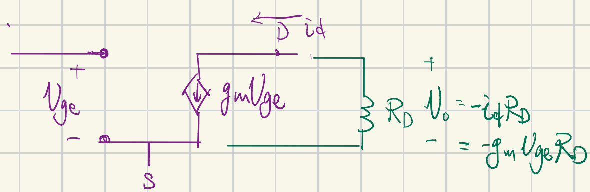
AC : Short all DC Voltage

§ Small-signal equivalent circuit Model — $i_d = g_m v_{gs}$

Hybrid-π model



Eg.

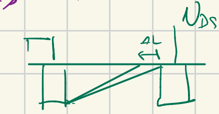


$V_o = -i_d R_D$
$= -g_m v_{gs} R_D$

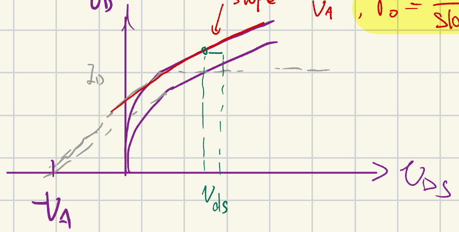
§ channel-length Modulation Effect (Early Effect) $v_{DS}\uparrow$

$$i_D=\frac{1}{2}\mu C\left(\frac{W}{\underset{L-\Delta L}{L}}\right)(V_{gs}-V_t)^2=\frac{1}{2}\mu C\left(\frac{W}{L}\right)\left(\frac{1}{1-\frac{\Delta L}{L}}\right)(V_{gs}-V_t)^2$$

$$i_D=\frac{1}{2}\mu C\left(\frac{W}{L}\right)\left(1+\frac{V_{DS}}{V_A}\right)(V_{gs}-V_t)^2$$



$\Delta L \propto V_{DS}$

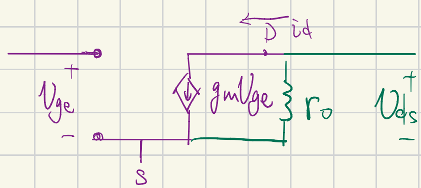


slope $=\frac{I_D}{V_A}$, $r_o=\frac{1}{\text{slope}}=\frac{V_A}{I_D}$

$$i_D=i_D(V_{GS},V_{DS})$$

$$di_D=\underbrace{\frac{\partial i_D}{\partial V_{GS}}}_{g_m}\cdot \underbrace{dV_{GS}}+\frac{\partial i_D}{\partial V_{DS}}\underbrace{dV_{DS}}$$

$$i_d=g_m v_{ge}+\frac{1}{V_A}I_D$$



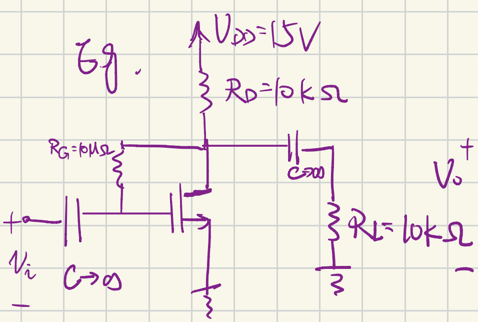
$$i_d=g_m v_{ge}+\frac{v_{ds}}{r_o}$$

r_o

Step: DC analysis → g_m (AC放大倍數) → Early Effect equation 求 r_o 修正
(r_o, Early Effect consider)

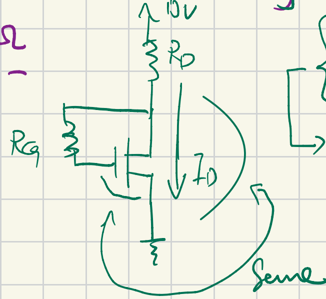
§ 4-6

Eg.



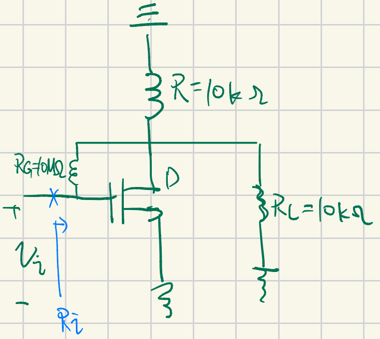
$V_{tn}=1.5V$, $\mu C(\frac{W}{L})=0.25mA/V^2$, $V_A=50V$
find V_o/V_i & R_i

(Sol) DC analysis:



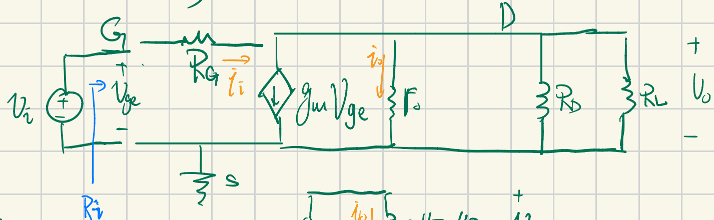
$$\begin{cases} I_D=\frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS}-V_t)^2 \\ 15=I_D R_D+V_{GS} \quad (\because V_{DS}=V_{GS}) \end{cases}$$

$\to I_D=1.06mA$, $V_{GS}=4.4V$

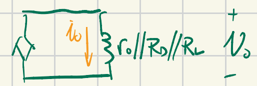


$$g_m = \mu C\left(\frac{W}{L}\right)(V_{GS}-V_t) = 0.725\,mA/V$$

AC analysis:



$$r_o = \frac{V_A}{I_D} = \frac{50}{1.06\,mA} = 47\,k\Omega$$



$$i_o = i_i - g_m V_{gs}$$

Assume $i_i \ll g_m V_{gs}$ ($\because R_G = 10M\Omega$)

$$i_o = -g_m V_{gs}$$

$$\frac{V_o}{V_i} = -g_m(r_o//R_D//R_L) \Leftarrow V_o = -g_m \underbrace{V_{gs}}_{=V_i}(r_o//R_D//R_L)$$

$$= -3.3\,V/V$$

$$R_i = \frac{V_i}{i_i} = \frac{V_i R_G}{4.3\,V_i} = \frac{R_G}{4.3} = 2.33\,M\Omega$$

$$i_i = \frac{V_i - V_o}{R_G} \quad (V_o = 4.3)$$

(R_i取较大, why?) ⇒ ∵ 电源也有内电阻,

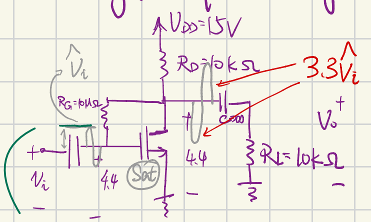
∴ R_s与R_i分压, R_i分大电压 正常 circuit 工作

Proof:

$$\frac{1}{g_m} \ll \frac{V_{gs}}{i_i} \quad (V_{gs}) = V_i$$

$$1.3\,k\Omega \ll (R_i = 2.3\,M\Omega)$$

The largest input Voltage Swing? Keep Sat



$$4.4 + \hat{V}_i$$

$$4.4 - 3.3\hat{V}_i$$

$$4.4 - 3.3\hat{V}_i \geq 4.4 + \hat{V}_i - V_t$$

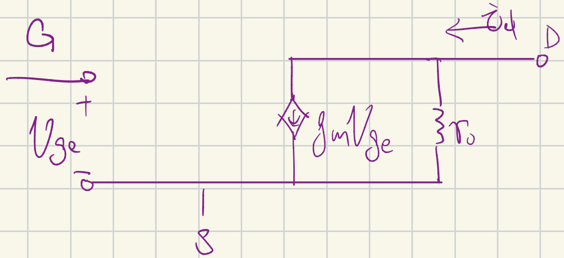
$$\hat{V}_i \leq 0.34V$$

Sat条件

26

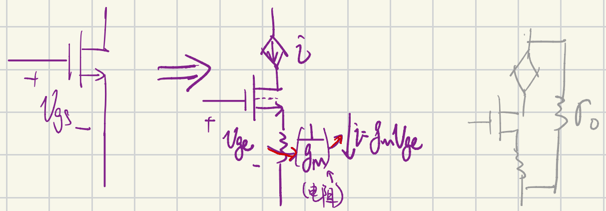
Small-Signal operation

⇒ Hybrid-π model

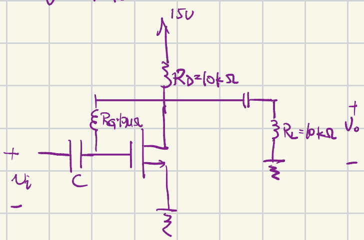


$$g_m = \mu C\left(\frac{W}{L}\right)(V_{gs} - V_{th})$$
$$= \sqrt{2\mu C\left(\frac{W}{L}\right) I_D}$$

⇒ T-model



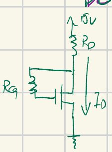
Eg. 4.10



$V_{th} = 1.5V$

$\mu C\left(\frac{W}{L}\right) = 0.25\ mA/V^2$, $V_A = 50V$

DC analysis:

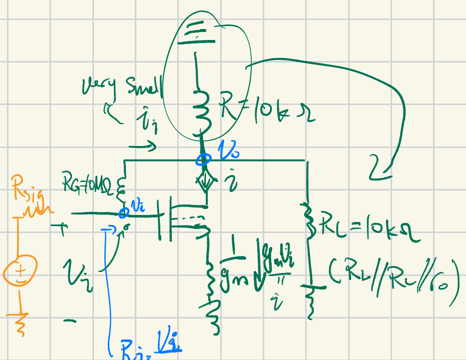


$$\begin{cases} I_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS} - V_t)^2 \\ 15 = I_D R_D + V_{GS} \quad (\because V_{DS} = V_{GS}) \end{cases}$$
$\to I_D = 1.06mA$, $V_{GS} = 4.4V$

$$g_m = \mu C\left(\frac{W}{L}\right)(V_{GS} - V_t) = 0.725\ mA/V$$

$$r_o = \frac{V_A}{I_D} = \frac{50}{1.06} = 47k\Omega$$

AC analysis



$$V_o = -g_m V_i (r_o // R_D // R_L)$$

$$A_v = \frac{V_o}{V_i} = -3.3\ V/V$$

$$R_i = \frac{V_i}{i_i} = \frac{V_i}{\frac{V_i - V_o}{R_G}} = \frac{R_G}{4.3} = 2.33M\Omega$$

Case 2

$$V_i = V_s \times \frac{R_i}{R_{sig} + R_i}$$

$$\frac{V_o}{V_s} = -3.3 \times \frac{R_i}{R_{sig} + R_i}, \text{ if } R_i \gg R_{sig} \Rightarrow \text{no further attenuation}$$

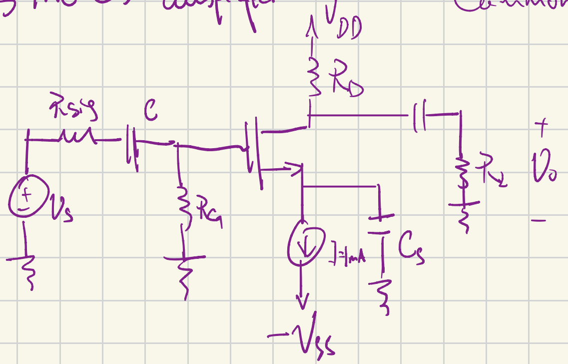
Consider Voltage Swing (Back to DC)

DC电压源 short
DC电流源 open

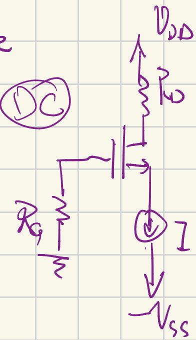
$\hat{V}_o = V_{tn} = 1.5V$ $\quad \frac{1.5}{3.3} = \hat{V}_i \doteq 0.4V$

§ 4.7 Single-Stage MOS amplifier

§ The CS amplifier — Common Source



DC



→ Current Mirror

By Current Source – Mirror:

$$\begin{cases} I_{ref} = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS}-V_t)^2 & ① \\ V_{DD} = I_{ref}R + V_{GS} - V_{SS} & ② \end{cases}$$

$\Rightarrow I_{ref} \Rightarrow Q_1 = Q_2 \Rightarrow I = I_{ref}$

$V_D = V_{DD} - IR_D$

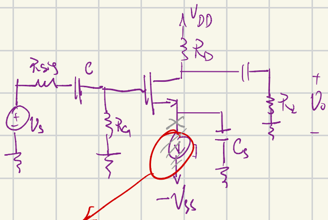
$I = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS}-V_{tn})^2$

AC g_m

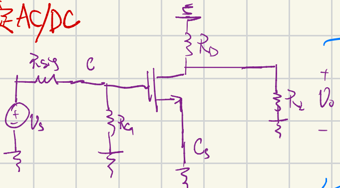
$I_c \Rightarrow g_m = \sqrt{2\mu C\left(\frac{W}{L}\right) I}$

$g_m = 1mA/V$

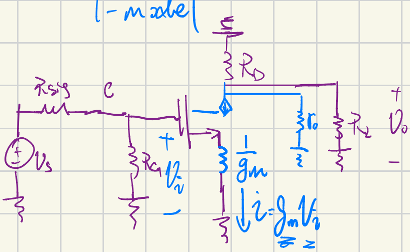
(2)

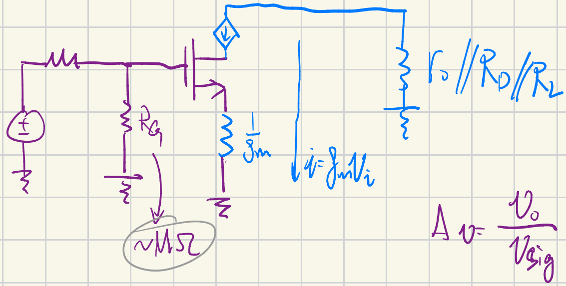


会固定AC/DC



T-model





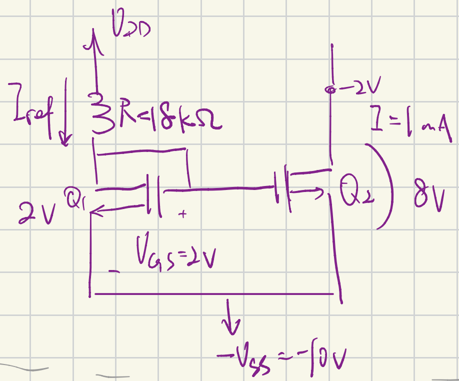
$v_o = -g_m v_i (r_o // R_D // R_L)$

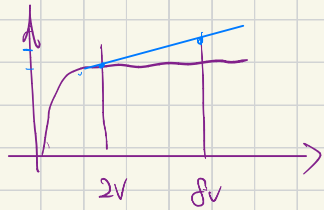
$v_i = v_{sig} \frac{R_G}{R_{sig}+R_G}$

$$A_v = \frac{v_o}{v_{sig}} = -\frac{R_G}{R_{sig}+R_G} \cdot g_m (r_o // R_D // R_L)$$

very big

(27) Consider Early Effect

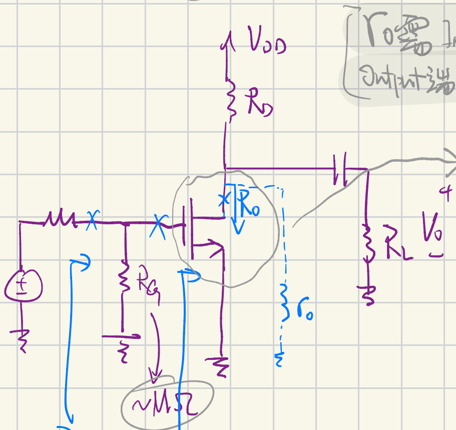




Early Effect: $I > I_{ref}$

信号

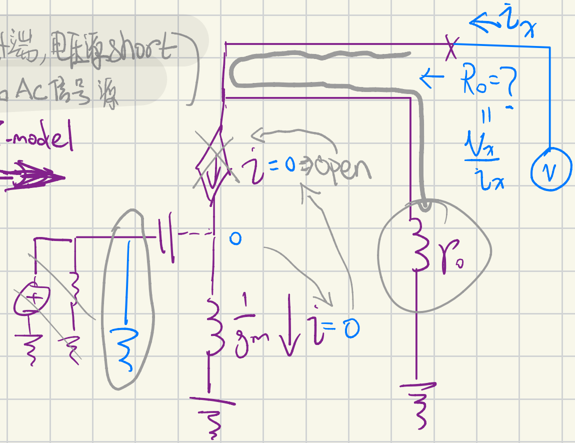
ro需 Input端，电压源short
Output端加AC信号源

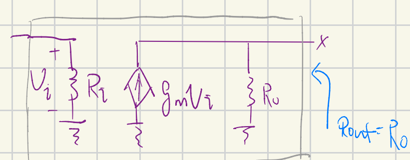


$R_{in} = R_G$ ∞

R_i very big

T-model

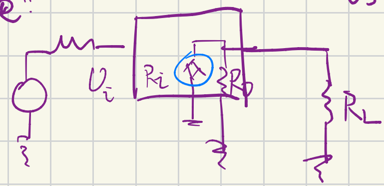




$\begin{cases} R_i \text{ very large} \\ R_o \text{ very large} \end{cases} \Rightarrow$ close to ideal transconductance amp

↳ reference:

$i_o \cong v_s \times g_m$



所放大信号会被各种电阻影响

input (电压角度看)

output (电流角度看)

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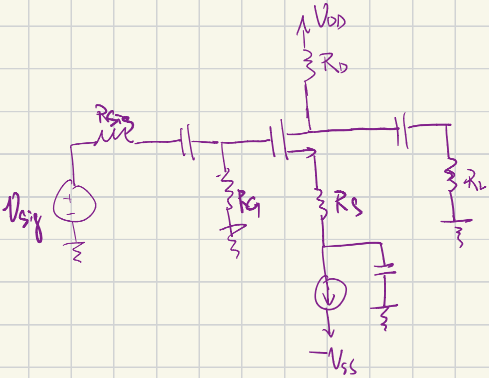
§ 4.7 Single Stage MOS amplifier

§ The CS amplifier with R_s

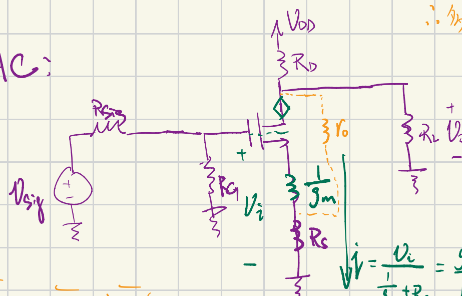
(increase bias stability)

∴ 多 R_s, $I\downarrow$, Gain$\downarrow$



DC similar to above

AC:



(neglect r_o)

$r_o \to \infty$

∴ 更复杂

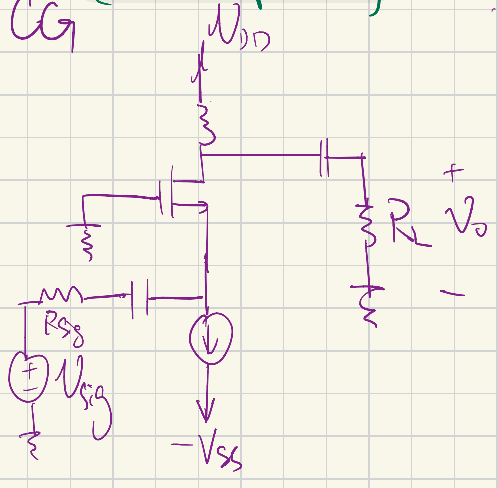
$$i = \frac{v_i}{\frac{1}{g_m} + R_s} = \frac{g_m v_i}{1 + g_m R_s}$$

(current i is reduced by a factor of $1+g_m R_s$)

$$\therefore v_o = -i\,(R_D // R_L) = -\frac{g_m}{1+g_m R_s} v_i\,(R_D // R_L)$$

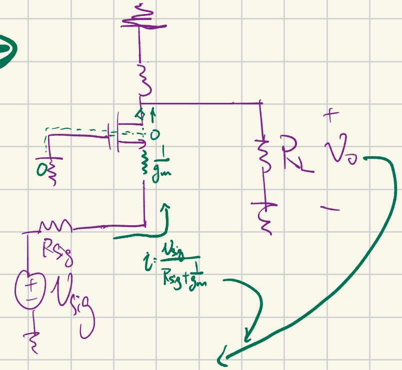
$$v_i = v_s \frac{R_G}{R_{sig} + R_G}$$

§ CG (current follower)



(DC: Similar to above

AC:



$$V_o = i\,(R_D /\!/ R_L)$$

$$V_o = V_{sig}\,\frac{g_m\,(R_D/\!/R_L)}{1+g_m R_{sig}}$$

$$\text{CS} \quad A_v = \frac{V_o}{V_{sig}} = \frac{g_m(R_D /\!/ R_L)}{(1+g_m R_{sig})}$$

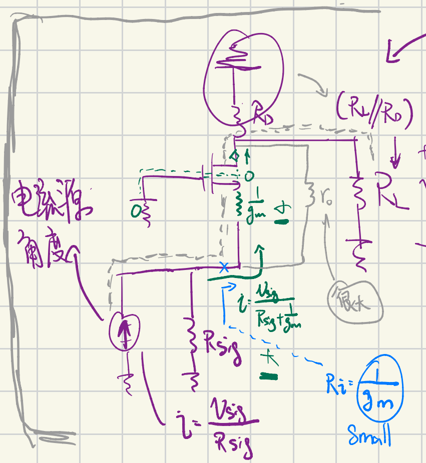
A_v Reduced by a factor of $(1+g_m R_{sig})$

∵ CS gain is largest $A_v = g_m(R_D/\!/R_L)$

if g_m is very large $\Rightarrow g_m R_{sig} \gg 1$

$$V_o = V_{sig}\,\frac{\cancel{g_m}\,(R_D/\!/R_L)}{\cancel{1+g_m}R_{sig}} \Longrightarrow \frac{V_o}{V_i} = \frac{R_D/\!/R_L}{R_{sig}}$$

如何去理解



∵ If g_m is very large $\Rightarrow g_m R_{sig} \gg 1 \Rightarrow R_{sig} = \frac{1}{g_m}$

大 　 小

∴ Current 不走大电阻，走小电阻

Current 流入 MOS

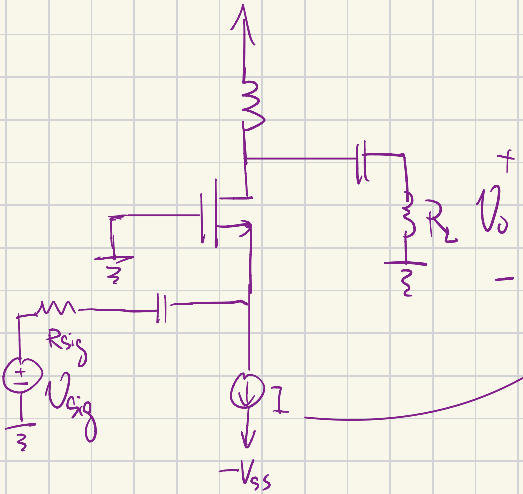
$$\therefore \frac{V_o}{V_{sig}} = \frac{i_s(R_D/\!/R_L)}{i_s(R_{sig})}$$

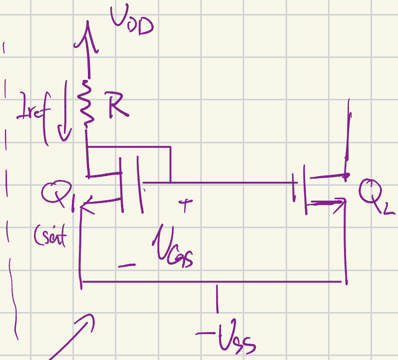
(Current 被拉过来)

Close to an ideal current amp (gain=1)

28

§ The CG





$I_{ref} = i_D = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(V_{GS}-V_{th})^2$

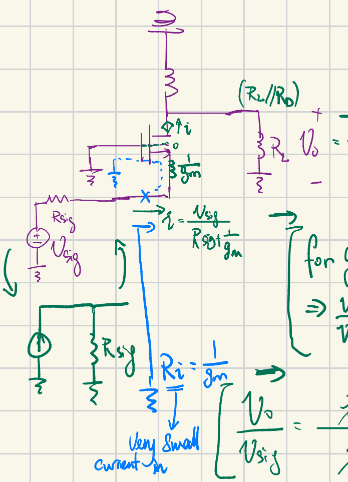
$I_{ref} = \dfrac{V_{DD}-V_{GS}+V_{SS}}{R}$

solve I_{ref}, V_{GS}

Perfect match $Q_1 = Q_2$, $I = I_{ref}$

$g_m = \sqrt{2\mu C\left(\frac{W}{L}\right) I_D}$

AC (ignore r_o now)



$V_o = i(R_D // R_L) = \dfrac{V_{sig}\, g_m}{1 + g_m R_{sig}}(R_D // R_L)$

$i = \dfrac{V_{sig}}{R_{sig} + \frac{1}{g_m}}$

$\left[\text{for } g_m R_{sig} \gg 1 \Rightarrow \dfrac{V_o}{V_{sig}} = \dfrac{R_o // R_L}{R_{sig}}\right]$ output input

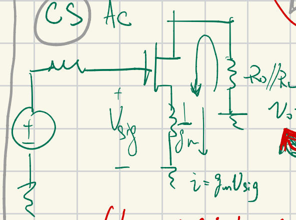
$R_{sig} \gg \frac{1}{g_m}$

$\left[\dfrac{V_o}{V_{sig}} = \dfrac{\cancel{i_s}(R_L // R_D)}{\cancel{i_s} R_{sig}}\right]$

[R_i very small, R_{out} very large] $\Rightarrow$ close to an ideal current amp ($A_i = 1$)

Tips: CS AC

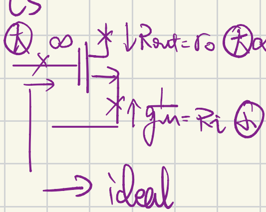
largest gain



$V_o = -g_m V_{sig}(R_D // R_L)$

(largest $\cong A_v = g_m \times (R_L)$)

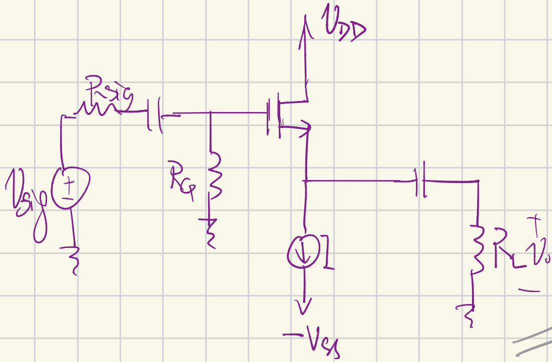
CS



$\rightarrow$ ideal

[如果有r_o，CS largest Gain $g_m \times (r_o)$ (intrinsic gain)]

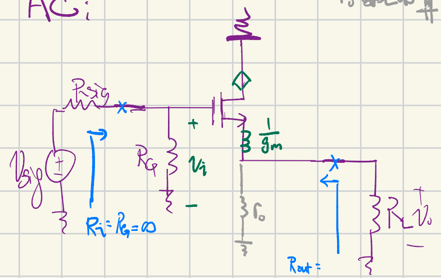
§ CD (Voltage follower)



DC similar to above

AC:

r_o 接地好算



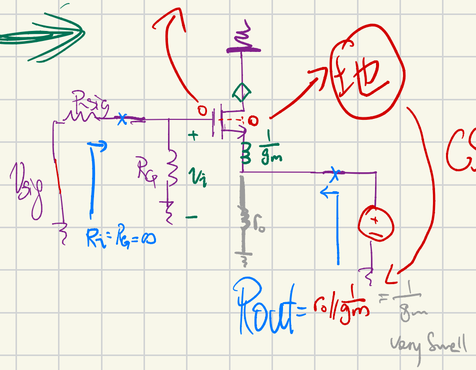
$$v_o = v_i \frac{(r_o // R_L)}{\frac{1}{g_m} + (r_o // R_L)} \doteq 1$$

$\frac{1}{g_m}$: very small for $(r_o // R_L) \gg \frac{1}{g_m}$

$$v_{sig} \frac{R_G}{R_{sig} + R_G} \doteq 1$$

$$\frac{v_o}{v_{sig}} \doteq 1$$

(AC信号 short 电压源)



$R_{out} = r_o // \frac{1}{g_m} = \frac{1}{g_m}$ very small

CS 放大

$$g_m \times r_o \gg 1$$

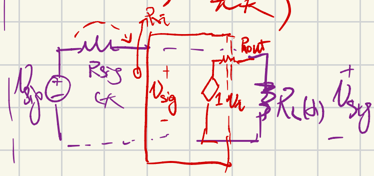
$$\frac{1}{g_m} \ll r_o$$

$R_i \to \infty$

$R_o \to 0$

$A_v \to 1$

ideal Voltage amp (与内电阻/负载 无关)

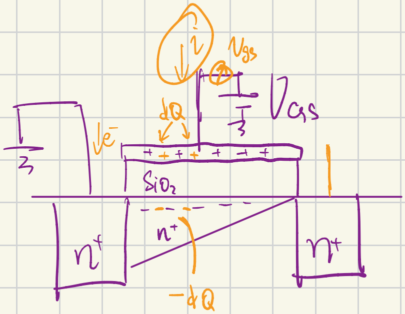


§ 4.8 The Mos internal Capacitance and High-f model

$$\underbrace{i_D}_{I_D + i_d} = \frac{1}{2}\mu C\left(\frac{W}{L}\right)(\underbrace{V_{GS}}_{V_{GS} + v_{gs}} - V_t)^2 \Rightarrow i_d = g_m v_{gs}$$

indep ~~of~~ f (linear relation)

$\bigcirc \sin 2\pi f t$ $\quad \bigcirc \sin 2\pi f t$



高频AC会有G电流

$dQ = i\,dt$

$= C\,dv$

(Time)