

Comp 310

for goal

X
is
Utility
what's

Lecture 2

o Rational Decisions

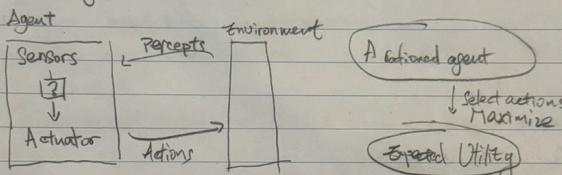
↳ Rational: maximally achieving pre-defined goals

↳ Rationality only concerns what decisions are made

↳ Goals: are expressed in terms of Utility of outcomes

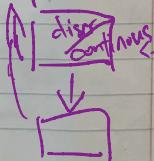
↳ Being rational means Maximizing your expected Utility

o Rational Agents



Percepts, Environment, action space's Characteristics → decide techniques for selecting rational action

Episodic



Environment properties

↳ Observability: Agent sensor full access (partial access)

↳ Multagent / Single Agent: Cooperation, decisions are influenced by other agent. One agent makes decisions & takes actions

↳ Deterministic / non Deterministic: Every action predictable. Current state \rightarrow Next state
Actions not certain, involve probabilistic reasoning to make decisions

↳ Stochastic: Deals with probability

↳ Episodic / Sequential: Agent's experience divided into discrete, current decision \rightarrow Subsequent decision outcome, long-term strategies

↳ static / dynamic: no change while agent deliberating. no environment might change when agent acts compared to when it began deliberation, agent needs to be responsive and adapt to changes

↳ Discrete / Continuous: time and percepts and actions are divided into separate intervals, true and percepts flow in uninterrupted manner

Discrete: Chess

Continuous:
Self-driving

Agent Design

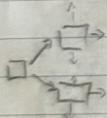
- o Environment type determines Agent Design
 - ↳ Partially observable → agent requires memory (internal state)
 - ↳ Stochastic → agent may have to prepare for contingencies
 - ↳ Multi-agent → an agent might behave randomly at times to be unpredictable to other agents
 - ↳ Static → If the environment no change while agent deliberating, agent has luxury time to compute sectional decision
 - ↳ Real-Time → In real-time environment, the agent operates as a continuously active controller
 - ↳ Unknown Physics: Unknown environment, the agent needs to explore to understand how to operate
 - ↳ Unknown Performance Measure: When criteria for success unclear, agent might observe and interact with humans

Search Problems

⇒ Search Problem consists

- 1 ↳ A state space
- 2 ↳ A successor function (with actions, cost)
- 3 ↳ start/goal state

o Solution: A sequence of actions (a plan) transforms Start → goal state



o State Space graph: ↳ [mathematical representation] of search problem

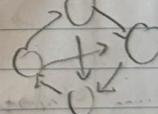
↳ Node: are (abstracted) world configuration

↳ Arc represent successors (action result)

↳ The goal Test is a set of goal node (maybe only one)

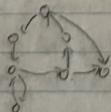
→ each state occurs only once

Tiny search graph for tiny
Search problem



State Graph / Search Trees

State Graph



Contract both on demand

Contract As Small As

Possible

Search Tree



Each Node is tree is
an [entire PATH] in
problem graph

Search Problem Concepts:

- ↳ Initial State
- ↳ Goal State: a set of target destination
- ↳ Action
- ↳ Transition model: describes what each action does
- ↳ Action cost function

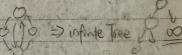
General Tree Search Algorithm

1. Initialization
2. Loop
 - Selection: Choose a node for expansion
 - ~~Expansion~~ Generate one child node
 - Goal Check
 - Queuing: Add the new node to a list of required nodes
3. Strategies
 - DFS
 - BFS
4. Termination

$\rightarrow \leftarrow$ IS Empty

General Graph Search Algorithm

1. Initialization
2. Loop
 - Selection: Select a node from the frontier
 - Goal Check
 - Expansion and Queuing
 - Mark Visited
3. Strategies
 - DFS
 - BFS
4. Termination

Tip:
Consider:


Tree:

① Initialize

② Loop

- ↳ IsEmpty
- ↳ Selection (stack or queue)
- ↳ Goal Check
- ↳ Expansion

Graph:

① Initialize (visited set)

② Loop

- ↳ IsEmpty
- ↳ Select
- ↳ Goal
- ↳ Mark Node
- ↳ Expansion

Search Properties

- ↳ Complete Planning Vs Replanning: a full plan or solution before EXECUTION
plan just enough to move next step. Entails planning increment
- ↳ Completeness: whether the algorithm is guaranteed to find a solution if exists (or no solution)
- ↳ Cost Optimality: Algorithm finds lowest cost
- ↳ Time Complexity
- ↳ Space Complexity
- ↳ Computation Complexity

Search Type:

① Uninformed Search Types

↳ BFS

↳ DFS

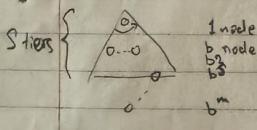
↳ Uniform Cost Search

② Informed Search Type

↳ A*

↳ Greedy Search

③ BFS



Characteristics:

Completeness / Optimality

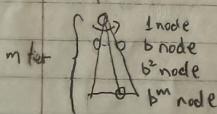
(some path cost)



time complexity / runtime memory $\mathcal{O}(b^s)$

children

④ DFS



Characteristics:

Completeness (not infinite) / may not optimality ↗?

time and space complexity $\mathcal{O}(b^m)$

Space fringe take (Only has siblings on path to root, so $\mathcal{O}(b^m)$)

DFS: → Memory Efficiency → Solutions are Deep → Graph Many Node and Few Solution
→ Detecting Cycles → Safe for loop

BFS → Shortest path → Shallow Solution → Tree Level Order Traversal → Find All Solutions
→ limit memory

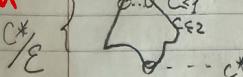
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Uniform Cost Search (UCS)

- Strategy: expand a cheapest node first
- Fringe is a priority queue (Priority: Cumulative Cost)

can be implemented
Priority Queue

Effective Depth



Characteristic:

- Processes all nodes with cost less than Cheapest Solution
- Solution Cost C^* , arc (action) cost at least ϵ
- then "effective depth" is C^*/ϵ
- Time $O(f(C^*/\epsilon))$

Tips:

- fringe take how much space, \rightarrow last tier $O(b^{C^*/\epsilon})$
- complete, if finite cost, minimum arc cost positive
- good: Complete / optimal bad: explores options in every "direction"
no info about goal location.

Informed Search (has Compass Version ①)

- ↳ Use a heuristic function $h(s)$ \rightarrow estimate how "far" the agent from goal
- ↳ $h(s)$ is optimistic (no overestimate) \rightarrow called admissible heuristics
- ↳ UCS try to minimize the Cost function $g(s)$
- ↳ Greedy search try to minimize heuristic $h(s)$
- ↳ A* try minimize $f(s)$ (Combination function $\Rightarrow f(s) = g(s) + h(s)$)

\rightarrow A heuristic is:

- ↳ A function, estimates how close a state is to a goal
- ↳ for a particular search problem
- ↳ Manhattan distance, Euclidean distance for pathing (for example)

Greedy Search local optimality

- ↳ Expand the node seems closest

- ↳ not optimal (not accounting for cost). (Best-first takes you straight to (wrong) state)
- ↳ Worst-Case: like a badly-guided DFS

only care next step

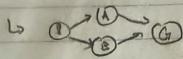
A* Search

$\hookrightarrow A^* \text{ Search : } f(n) = g(n) + h(n)$

orders by the sum = Uniform-Cost $g(n)$ + Greedy $h(n)$

Cost

goal proximity



Q: Stop when enqueue a goal?

A: No, when dequeue

not optimal



Admissible Heuristics

\hookrightarrow A heuristic h is admissible (Optimistic), Only if

original components $0 \leq h(n) \leq h^*(n)$ \rightarrow true cost $\geq 0 \rightarrow$ goal
 (from Original)

$$\text{Eg. } 0 \xrightarrow{-5-2} 15$$



= Optimal A*



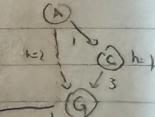
Consistency of Heuristics (slightly stricter)

\rightarrow main idea: estimated heuristic Cost \leq actual Cost

Consider
optimal

\hookrightarrow Admissibility: heuristic cost \leq actual cost to goal

$$h(A) \leq \text{actual cost } A \text{ to } G$$



\hookrightarrow Consistency: heuristic "arc" cost \leq actual cost for each arc

$$h(A) - h(C) \leq \text{cost } (A \text{ to } C)$$

\rightarrow Consequences of Consistency

$h(A), g(A) \xrightarrow{\dots}$ The f value along a path never increase

$h(A)$ faster than $g(A)$

$$f(c) = h(c) + g(c)$$

$$= h(c) + g(A) + \text{cost } (A \text{ to } C)$$

$$h(A) \leq \text{cost } (A \text{ to } C) + h(c)$$

\hookrightarrow A* graph search is optimal

(by one of two)

$$f(c) = h(c) + g(A) + h(A) - h(c)$$

Consistent Heuristic ensures heuristic estimate is always increasing

Optimal

decrease / some

$$h(A)$$

to goal

$$\leq \text{cost } (A, c) + h(c)$$

actions $O \rightarrow O$

by us

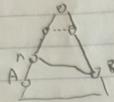
$$h(A) \leq g(A)$$

Heuristic Design often use relaxed problem

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Optimality of A* Tree Search

A → optimal
B → Suboptimal



Claim: A will be expanded before B

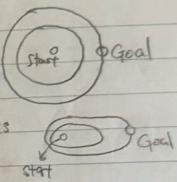
1. $f(n)$ is less or equal to $f(A)$
2. $f(A)$ less than $f(B)$
3. n expands before B

All ancestors of A expand before B

UCS vs A* Contour

→ Uniform-Cost expands equally in all "directions"

→ A* expands mainly toward the goal but does hedge its to sure optimality



Constraints Satisfaction Problems (CSP)

Conditions 1 → N variables 2 → domain D 3 → constraints

agent

Search for: [Planning], [Identification]

↪ planning: sequences of actions → path to the goal (important)

CSP → Identification: assignment to variables → goal itself (important), not path
(all path same depth)

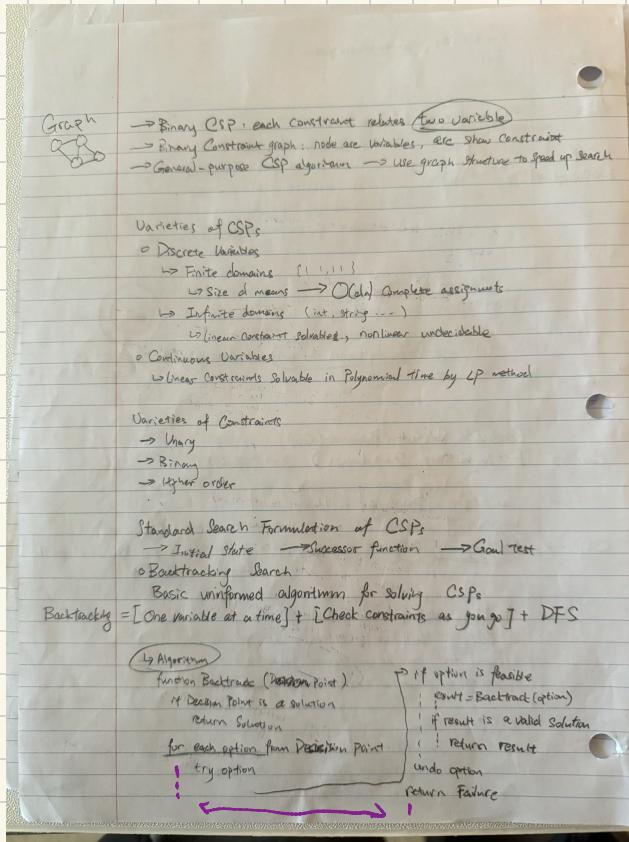
Standard Search problems:

- [state] is "black box": arbitrary data structure
- [Goal test] can be any function over states
- [Successor function] can be anything

CSP:

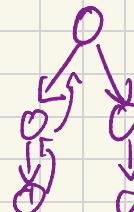
- A special (Subset) of search problem
- State defined by Variables X_i with value from domain D
- Goal test is a set of constraints

Backtracking



```

function Backtrack(DecisionPoint):
    if DecisionPoint is a solution:
        return Solution
    for each option from DecisionPoint:
        try option
        if option is feasible:
            result = Backtrack(option)
            if result is a valid solution:
                return result
            undo option
    return Failure
    
```



Algorithm BackTrack (Sol)

If isSol (solution)

Output (solution)

return

for choice in GetChoice (sol)

If isValid (choice, sol)

ApplyChoice (sol, choice)

BackTrack (sol)

UndoChoice (sol, choice)

Improving Backtracking

General-purpose ideas give gains in speed

- ordering
- Filtering
- Structure

Filtering: Forward Checking

track of domains for Unassigned Variable & Cross of bad option

→ Forward checking propagates information but no early detection for failures

↳ Cross of violated constraint, when added to assignment

↳ Consistency w.r.t a single Arc: An arc $X \rightarrow Y$ is consistent iff every x in tail there is some y in the head which could assign without violating

↳ Arc Consistency detects failure earlier than forward checking

↳ if X loses a value, neighbors of X need to be recheck

↳ Can be run as a preprocessor or after each assignment

Exact Algorithm

function AC-3 (CSP)

queue = all arcs in CSP

while queue is not Empty

: $(X, Y) = \text{queue.pop}()$

: If remove inconsistent value (X, Y)

: for each Z in neighbors of X except Y ,

: queue.add $((Z, X))$

function remove-inconsistent-value (X, Y)

removed = false

for each x in domain of X

: if no value y in domain of Y allows (x, y) to satisfy the constraint in $X \rightarrow Y$

: domain of $X = \text{domain of } X - \{x\}$

: removed = true

Return removed

CSP → not path, but where is Goal
 → pass Constraints → Goal

↳ Ordering
 Pick 2 ↳ Minimum Remaining Value (Variable Ordering)
 {A,B} {A} • Choose the variable with the fewest legal left values in domain
 Pick 2 • Most constrained Variable
 {1,2,3} {2,3} ↳ Least Constraining Value (Value Ordering)

Eg. puzzle → select MRV → Select LCV → Recursion with Backtracking
 → Repeat

↳ Iterative Algorithms for CSPs

↳ Local Search Algorithm ↳ Take an assignment with unsatisfied constraints
 ↳ Any Conflicts ↳ Operators: realign variable values.
 [Min-Conflicts] → fewest constraint algorithm:
 $R = \frac{\# \text{ of conflicts}}{\# \text{ of variables}}$ ↳ Variable selection: any conflicted variable
 ↳ (min-conflicts heuristic)
 (choose a value violates the fewest constraint)

↳ Local Search (inherently an Iterative Algorithm)
 locate smaller ↳ (Vs. tree search keeps unexplored alternatives on the fringe (Completeness))
 not whole ↳ improve a single option until you can't make it better
 ↳ NEW successor function: local changes
 ↳ Generally much faster and more memory efficient

↳ Hill Climbing Algorithm (a type of Local Search)
 ↳ idea: start wherever → Repeat: move to best neighboring state
 ↳ If no neighbors better, quit
 ↳ uncertain Complete/Optimal

global max is optimum point
 (best possible solution)

Iterative algorithm

迭代算法伪代码（使用最小冲突启发式）：

1. 开始时，随机为CSP中的每个变量分配一个值，即使这个初始分配可能违反一些约束。
2. while (当前分配未能解决所有约束) do
 - a. 选择一个冲突变量。冲突变量是指参与了至少一个约束冲突的变量。
可以随机选择，或者选择冲突最多的变量。
 - b. 为选定的变量选择一个新值。这个新值应该是最小化该变量当前约束冲突的值。
即，选择一个新值，使得改变后违反的约束数量最小。
 - c. 如果找不到能减少冲突的新值（意味着当前变量在所有可能值中冲突都一样多），
可以选择保持当前值，或随机选择一个新值尝试打破僵局。
 - d. 更新变量的值为步骤b中选择的新值。
3. 当所有变量的分配不再产生任何冲突时，算法结束，当前的分配被认为是CSP的解。

→ Hill Climbing Algorithm (most basic local search)

function H-C (problem) returns a state if local max

Current = Make-Node (problem, InitialState)

loop

neighbor = a highest-valued successor of current

If neighbor. Value ≤ current. Value

then return current, state

current = neighbor

→ Lecture 5 Adversarial Search

→ Types of Game (many kinds of game)

- o Game = task environment > 1 agent

- o Axes: Deterministic, perfect information, multiple player, turn-taking, Zero-Sum
(Want Algorithm for calculating a strategy (policy)) not plan

Policy = Strategy

→ "standard" Games (Solution is policy not plan)

- o "Standard" → Deterministic, observable, 2 player, turn-taking, Zero-Sum

- o Game formulation:

- o Initial State $S(s_0)$

- o Player $P = \{1 \dots N\}$

- o Actions $A = \{\text{depends}\}$

- o Transition Function: $S \times A \Rightarrow S$

- o Terminal / goal Test: $S \rightarrow \{t, f\}$

- o Terminal Utilities (value): $S \times P \rightarrow \mathbb{R}$

- o Solution for player is policy: $S \rightarrow A$

A_1 's Utility + B_1 's Utility

= 0

→ zero-Sum Games

- o Agents have opposite utilities

- o Pure Competition:

A maximizes, B minimizes it

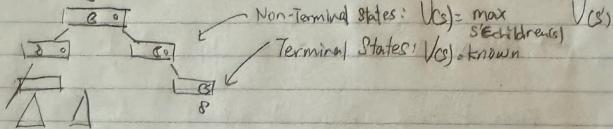
Vice Versa

- General Sum Games
- Agent have independent utilities
 - Cooperation, indifference, competition, shifting alliances, etc
- Team games
- Common payoff for all team member

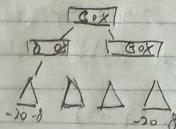
→ Adversarial Search

A $\xrightarrow{\text{Goal}} \beta$
optimum get

◦ Single-Agent Trees



◦ Adversarial Game Trees



→ Minimax Values

State under Agent's Control

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

under Opponent's

$$V(s) = \min_{s' \in \text{successors}(s')} V(s)$$

→ Minimax

◦ Deterministic, Zero-Sum games

Tic-Tac-Toe → (one maximizes result, one minimizes result)

◦ Minimax Search

A State-Space Search tree

Player alternates turns

Compute each node's Minimax value (max myself, min opponent)

o Algorithm

def value (state)

If the state is terminal state : return state's utility

If the next agent is MAX : return max-value (state)

If the next agent is Min : return min-value (state)

Traversal



def max-value (state)

initialize $V = -\infty$

for each successor of state:

$V = \max(V, \text{value}(\text{successor}))$

return V

def min-value (state)

initialize $V = +\infty$

for each successor of state

$V = \min(V, \text{value}(\text{successor}))$

return V

[DFS, Time: $O(b^m)$ Space: $O(bm)$]

SM

o Generalized minimax for not zero-sum but multiple players

Terminal pay utility tuples $\square\square\square\square\square\square$ (Node value also utility tuple)

Each player maximizes its own component

Can Cooperate and Compete for its own goal

o Resource limits (realistic, won't search to leaves)

→ Depth-limited search (use evaluation function)

(no guarantee of optimal play)

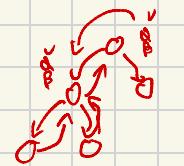
↳ use iterative deepening for an anytime algorithm

o Evaluation function

$$\text{Eval}(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

(weight + feature function)

Traversal |



~~2 > P~~

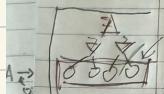
Alpha-Beta Pruning Tree Pruning

→ Alpha-Beta Pruning

o = Max player 目标为上得高 lowest score

◦ The order of generation matters: more pruning is possible if good moves come first

B = Min player 目标为上得低 highest score



↳ 1. initialize

2. for Search, Check Max or Mini to upload α or β

3. pruning, If a node $\alpha \geq \beta$ don't have to keep search (Max player) ^{for}
has a biggest point. Same

(for min player) If a node $\beta \leq \alpha$, don't have to search them

(With perfect ordering: $O(b^{D/2})$ Time, Doubles depth solvable)