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Tarea 6

① Calcula la probabilidad de ruina $\psi(c_0)$ cuando $X \sim Exp(x \mid 1/\mu)$, con $E(X)=\mu$

$\Rightarrow F(x) = 1 - e^{-x/\mu}, \quad x \geq 0$

$\Rightarrow M_X(t) = (1-\mu t)^{-1}, \quad t < \mu^{-1}$

Sabemos por el coeficiente de Lundberg:

$1 + (1+\theta)\mu r = M_X(r)$

$\Rightarrow 1 + (1+\theta)\mu r = (1-\mu r)^{-1}$

$\Rightarrow (1-\mu r) + (1+\theta)\mu r(1-\mu r) = 1$

$\Rightarrow -\mu r + (1+\theta)\mu r - (1+\theta)\mu r^2 = 0$

$\Rightarrow \mu r(-1 + (1+\theta) - (1+\theta)\mu r) = 0$

$\Rightarrow -1 + (1+\theta) - (1+\theta)\mu r = 0$

$\Rightarrow \theta - (1+\theta)\mu r = 0$

$$\boxed{r = \frac{\theta}{(1+\theta)\mu}}$$

$$\Rightarrow \boxed{\psi(c_0) = \frac{\exp\left(-\frac{\theta}{(1+\theta)\mu}c_0\right)}{E_{F_{S^*}}\left[\exp\left(-\frac{\theta}{(1+\theta)\mu}(r)\right) \mid T<\infty\right]}} \leq \exp\left(-\frac{\theta}{(1+\theta)\mu}c_0\right)$$

como r existe $\Rightarrow$ $\underline{E_{F_{ST}}\left[\exp\left(-\frac{\theta}{(1+\theta)\mu}(T)\right) \mid T<\infty\right] > 1}$

② $F_X(x) = Pa(x|\alpha)$ $\quad f_X(x) = (1+x)^{-\alpha}\, \mathbb{I}_{[0,\infty)}(x)$ $\quad \alpha > 0$

Primero tenemos que obtener r

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f_X(x)\,dx = \int_0^{\infty} e^{tx}(1+x)^{-\alpha}\,dx$$

$$F_X(x) = \int_0^x f_X(t)\,dt = \int_0^x (1+t)^{-\alpha}\,dt = \frac{(1+x)^{1-\alpha}-1}{1-\alpha} \quad \alpha > 0$$

$$\Rightarrow F_X(x) = Pa(x|\alpha) = \frac{(1+x)^{1-\alpha}-1}{1-\alpha}$$

$$\int_0^{\infty} \exp\{rx\}\, F_X(x)\,dx = \int_0^{\infty} \exp\{rx\} \frac{(1+x)^{1-\alpha}-1}{1-\alpha}\,dx = *$$

Se tiene que cumplir:

$$\frac{1-F_X(x)}{\exp\{-rX\}} \leq E_{F_X}(\exp\{rX\})$$

$$\Rightarrow \frac{1-\frac{(1+x)^{1-\alpha}-1}{1-\alpha}}{\exp\{-rX\}} = \frac{1-\alpha-(1+x)^{1-\alpha}+1}{\exp\{-rX\}(1-\alpha)}$$

$$* = \left(\frac{1}{1-\alpha}\right)\Big[\underbrace{\int_0^{\infty} e^{rx}(1+x)^{1-\alpha}\,dx}_{①} - \underbrace{\int_0^{\infty} e^{rx}\,dx}_{②}\Big)$$

② $\left.\dfrac{e^{rx}}{r}\right|_0^{\infty} = \dfrac{\infty}{r} - \dfrac{1}{r} = \infty$

① $\int_0^{\infty} e^{rx}(1+x)^{1-\alpha}\,dx \to$ no converge; integral no definida