149878

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Tarea 4

(2) $N_t \mid N_t \leq 15 \sim Po(n \mid 30)$

y $N_t \mid N_t > 15 \sim Bin(n \mid 100, 1/3)$

Consideremos la distribución Poisson como base i.e.

$$q_n = \widehat{\mathbb{P}}(N_t = n) = \frac{e^{-\lambda}\lambda^n}{n!}\,\mathbb{I}(n \leq 15)$$

$\Rightarrow \left(1 - \sum_{k=0}^{15} e^{-\lambda}\lambda^k/k!\right)$ corresponde a la masa de probabilidad que redistribuiremos a la parte Binomial.

•) Condicionamos la binomial a $(15 < n \leq 100)$

$$\frac{\mathbb{P}(N_t=n)}{\mathbb{P}(15<N_t\leq 100)} = \frac{\binom{100}{n}\theta^n(1-\theta)^{100-n}\,\mathbb{I}(15<n\leq 100)}{1-\sum_{k=0}^{15}\binom{100}{k}\theta^k(1-\theta)^{100-k}}$$

••) Reescalamos

$$q_n = \widehat{\mathbb{P}}(N_t \leq 15)\,\frac{\mathbb{P}(N_t=n)}{\mathbb{P}(15<N_t\leq 100)} = *$$

$$* = \left(1-\sum_{k=0}^{15} e^{-\lambda}\lambda^k/k!\right)\left(\frac{\binom{100}{n}\theta^n(1-\theta)^{100-n}}{1-\sum_{k=0}^{15}\binom{100}{k}\theta^k(1-\theta)^{100-k}}\right)\mathbb{I}(15<n\leq 100)$$

$\blacksquare$

① a) Identificamos los puntos cuyas masas de prob. se modifican

$Q(N_t=0)=0$; $Q(N_t=1)=1/3$ $\{0,1\}$

ii) Consideramos las $\mathbb{P}(N_t=n)$ originales en todos los puntos distintos a $\{0,1\}$.

$$\frac{\mathbb{P}(N_t=n)}{1-\sum_{i=0}^{1}\mathbb{P}(N_t=i)}\mathbb{I}(n\geq 2) = \frac{e^{-\lambda}\lambda^n/n!}{1-e^{-\lambda}-e^{-\lambda}\lambda}\mathbb{I}(n\geq 2)$$

iii) Reescalamos

$$(1-Q(N_t=0)-Q(N_t=1)) = 1-0-1/3 = 2/3$$

$$\Rightarrow Q(N_t=n) = 2/3\left(\frac{e^{-\lambda}\lambda^n/n!}{1-e^{-\lambda}-e^{-\lambda}\lambda}\right)\mathbb{I}(n\geq 2)$$