Advanced Numerical Methods for Neutron Star Interfaces

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Acknowledgements

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- Ian Hawke
- Southampton General Relativity Group
- STFC
- IOP Gravity Group
- Classical and Quantum Gravity

Outline

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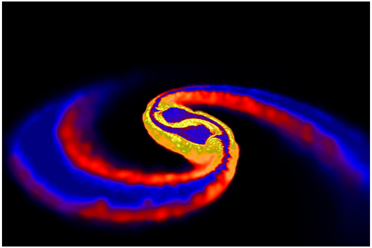
Motivation: Binary Mergers

- Neutron star binary mergers are strong candidates for the emission of detectable gravitational waves.
- Accurate gravitational wave templates are required to directly detect gravitational waves.
- Full non-linear numerical simulations are required for the plunge and merger phases.
- Therefore, accurate numerical simulations are needed.

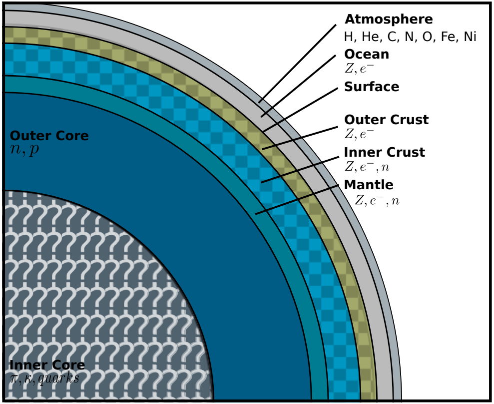


Credit: Daniel Price (U/Exeter) and Stephan Rosswog (Int. U/Bremen)

Outline

What features do we need to consider?

- Exterior - Vacuum/Plasma
- Atmosphere - Fluid
- Crust - Elastic Matter
- Core - Fluid



How do we model a neutron star numerically?

We want to solve the most realistic physical model possible using maximum resolution. Problem?

- Important length-scales vary over 12 orders of magnitude if including viscous boundary layers.
- Using adaptive mesh refinement cannot cover this range.

Solution:
Relativistic multimodel approach developed by Millmore & Hawke.
Approximate interfaces to be infinitely thin.

The multimodel approach

Simplify the most complex model by:

- Taking appropriate limits in different regions of the domain.
    - Force-free / Electro-vacuum / Multi-fluid MHD
    - Ideal MHD / Resistive MHD
    - Elasticity
- Combining the different approximations with sharp interfaces.
- Using evolution equations based on conservation laws.
    - This allows us to accurately capture the locations of shock waves
- Moving the complicated physics to the interfaces.
    - Boundary conditions to impose physics

This approach also allows a proper treatment of the surface.
Therefore, we no longer need a numerical atmosphere.

Conservation Laws

The evolution equations are a system of non-linear partial differential equations in conservation law form.

$$\frac{\partial \mathbf{q}(x,t)}{\partial x} + \frac{\partial \mathbf{F}(\mathbf{q}(x,t))}{\partial x} = 0, \tag{1}$$

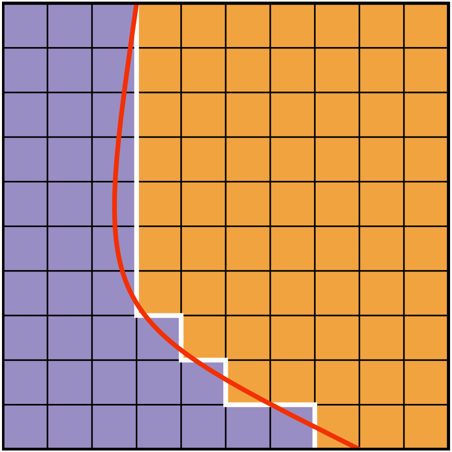
where $\mathbf{q}$ are the conserved variables and $\mathbf{F}$ are the fluxes. The numerical update is then given by.

$$q_i^{n+1} = q_i^n + \frac{\Delta t}{\Delta x}\left(F_{i-1/2}^n - F_{i+1/2}^n\right), \tag{2}$$

where n is the time index and i is the spatial index.

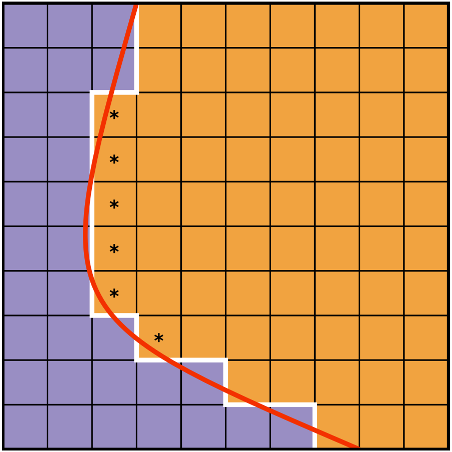
Numerical Grid

- We define a grid on which to evolve our models
- The red line indicates the true interface
- The white line indicates the numerical interface



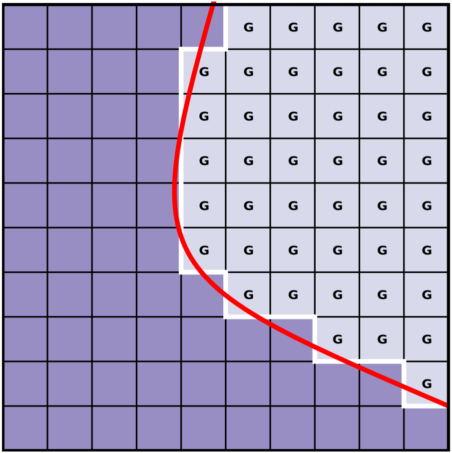
Moving Interface

- The interface is advected with the models
- The numerical interface tracks the physical interface
- Points can change model



Ghost Zones

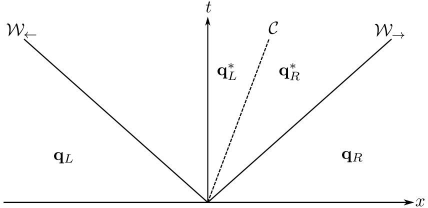
- Each model has a region of ghost zones
- As the update of each point is non-local, it is important to fill these cells correctly
- How do we impose the correct boundary conditions? By filling the ghost cells



Riemann Problem and Solution

- Each normal cell is updated by following the Godunov approch
- Each cell boundary is a Riemann problem
- A Riemann problem occurs when there is discontinuity
- We update each cell by calculating the flux through the boundaries

Hydro Riemann Fan



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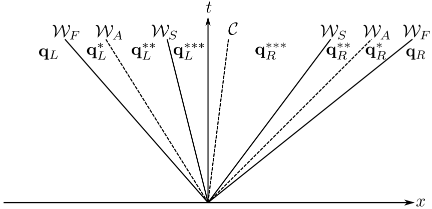
SRMHD Riemann Fan



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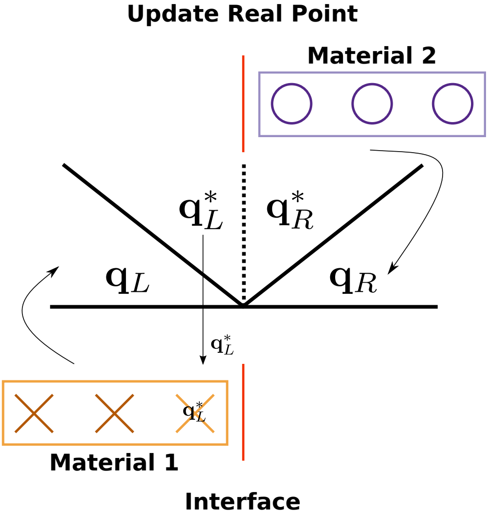
Boundary Conditions: Ghost Fluid Method



Follow the ghost fluid approach.

- Define left and right states.
- Calculate the star states using an appropriate method.
- Fill the points neighbouring the interface.
- Extrapolate into the ghost zones.

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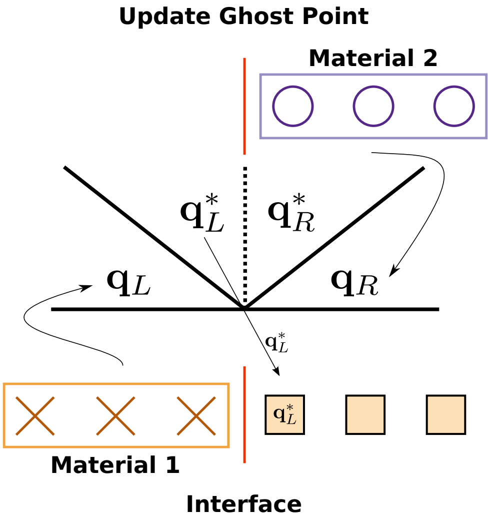

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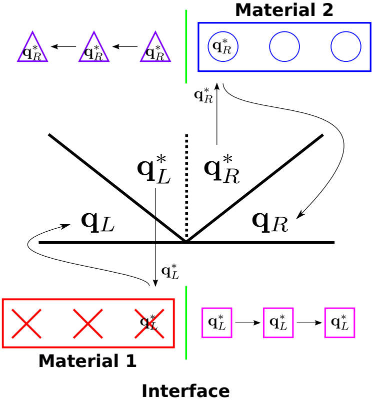

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How do we calculate the star states?

To calculate the star states we follow the Roe approximation method and linearise about an appropriate state $\mathbf{w}_X(\mathbf{q}_X)$ for the left and right states. Where $X = L/R$.

$$\partial_t \mathbf{w}_X + \hat{A}_X \partial_x \mathbf{w}_X = 0. \tag{3}$$

The star states are then given by the following equations.

$$\mathbf{w}_L^* = \mathbf{w}_L + \sum_{i=1}^{N_L} c_i^L \mathbf{r}_L^{(i)}, \tag{4}$$

$$\mathbf{w}_R^* = \mathbf{w}_R - \sum_{j=1}^{N_R} c_j^R \mathbf{r}_R^{(j)}. \tag{5}$$

Where $\mathbf{r}_X$ are the right eigenvectors of the matrix $\hat{A}_X$.

How do we impose the interface boundary conditions?

To calculate the coefficients c^X we need $N = N_L + N_R$ compatibility conditions at the interface. Where N_X is the number of waves between the initial and star state.
It is through these compatibility conditions that we impose the correct physical boundary conditions at the interface.
We assume that these compatibility conditions take the form:

$$\Delta w_j = w_j^{*R} - w_j^{*L} = 0, \quad j = 1, \dots, N. \tag{6}$$

This gives a linear system which we can easily solve,

$$\mathbf{w}_R - \mathbf{w}_L = \sum_{j=1}^{N_R} c_j^R \mathbf{r}_R^{(j)} + \sum_{i=1}^{N_L} c_i^L \mathbf{r}_L^{(i)}. \tag{7}$$

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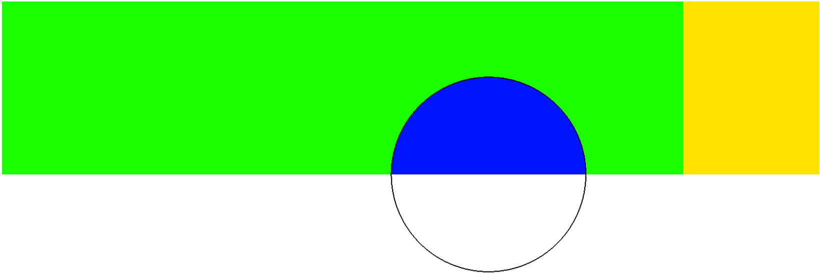
Vorticity Propagation - Low Magnetic Field $\beta = 1000$



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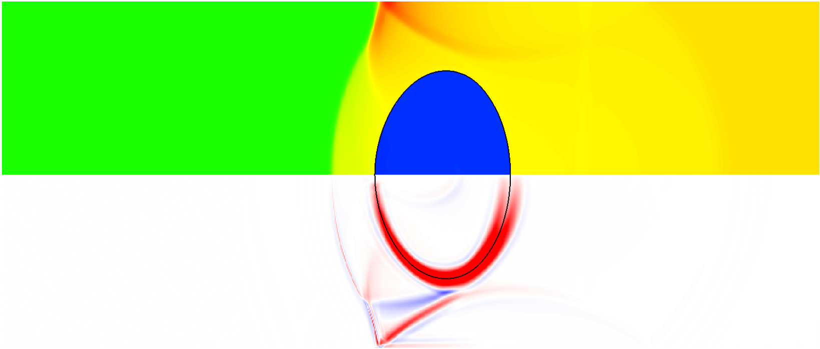

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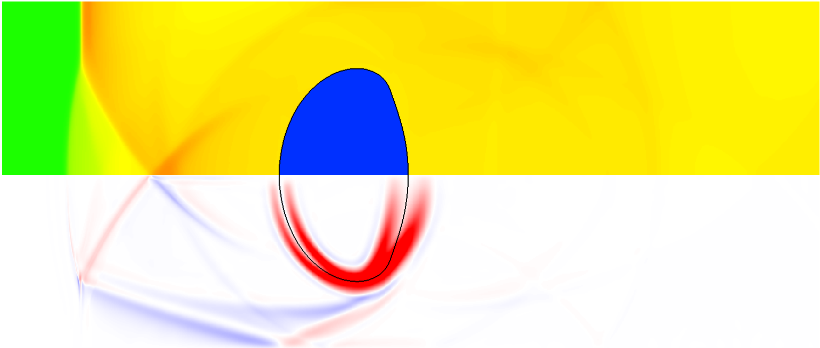

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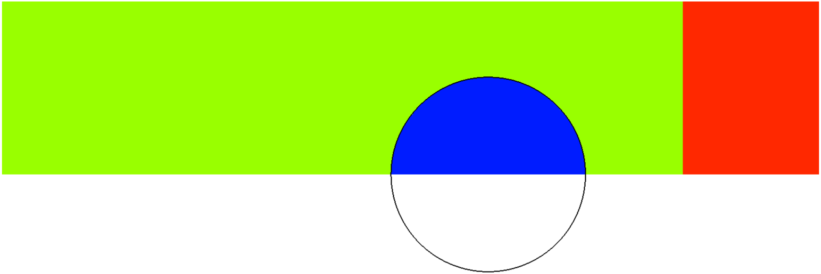
Vorticity Propagation - Low Magnetic Field $\beta = 0.01$



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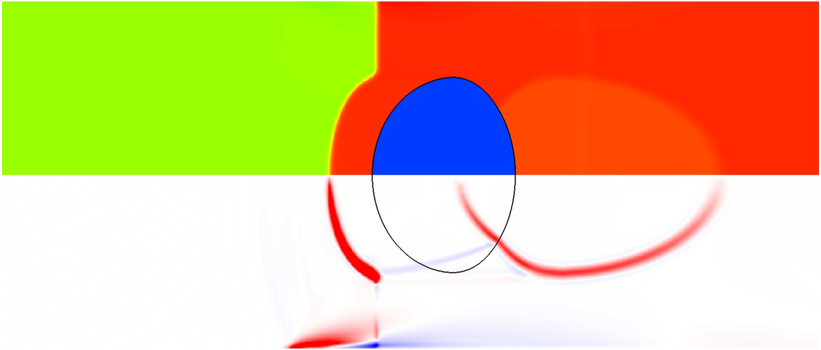

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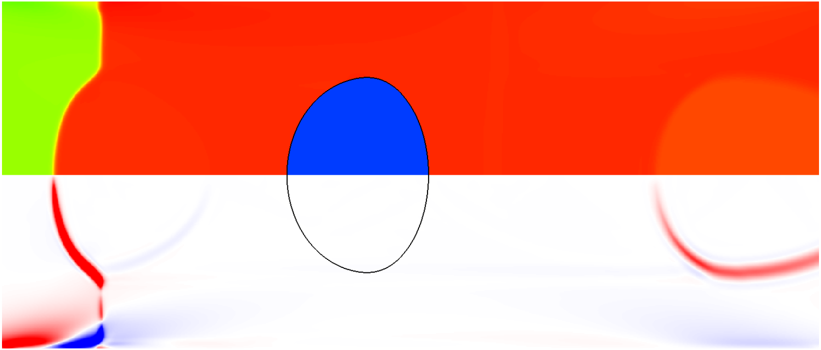

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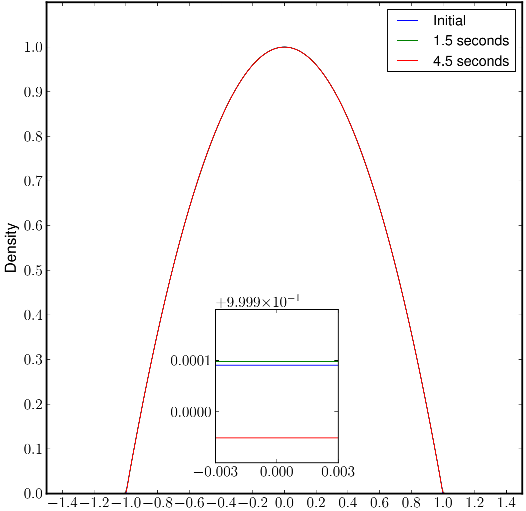
Toy Star: Vacuum - Hydro



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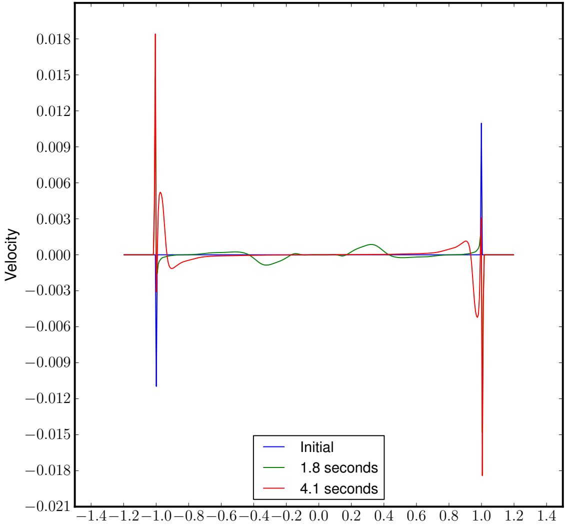

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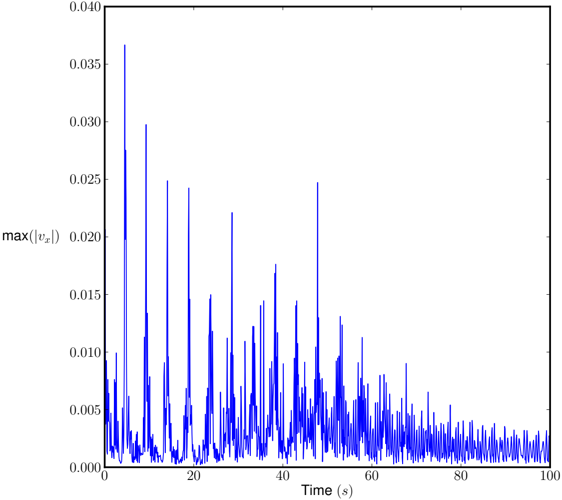

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Conclusion

Thank you