IMPACT OF TRANSMISSION ERRORS IN THE PERFORMANCE OF A WIRELESS NETWORK LINK.

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Task 1.

Consider a wireless link used by two stations for data communications. The link can be either in a normal state with a probability of p or in an interference state with a probability of 1 – p. Consider the following probability values: p = 99%, p = 99.9%, p = 99.99% and p = 99.999%. The bit error rate is 10^{-7} when the link is in the normal state and 10^{-3} when the link is in the interference state.
The two stations exchange from time to time a set of n consecutive control frames of size 64 Bytes each to decide if the link is in interference state. Both stations determine with a 100% probability if the control frames have been received with errors. The stations decide that the link is in interference state when the n consecutive control frames are received with errors.
Consider the following definitions:

- *a false positive is when a station decides wrongly that the link is in interference state (i.e., it receives n consecutive control frames with error and the link is in the normal state)*
- *a false negative is when a station decides wrongly that the link is in normal state (i.e., at least one of the n consecutive control frames is received without errors and the link is in the interference state)*

1.a. *For each value of p, determine the probability of the link being in the interference state and in the normal state when one control frame is received with errors (fulfil the following table). What do you conclude?*

To solve this problem first we used the **probability function of a binomial random variable**, defining n as the number of bits(**n=64*8**). For obtaining the probability in normal state, we set p as the bit error rate of normal state (**prob=10^{-7}**) and for the probability of interference state, we used the bit error rate for interference state(**prob2=10^{-3}**).

- ⇨ *pNormal=1-(1-prob)^n*
- ⇨ *pInterference=1-(1-prob2)^n*

Once we have obtained the probability for each of the states, we use the **Bayes' Law** to obtain the probability of the link in normal state (**pNwError**) and one minus this probability to obtain the probability of the link to be in interference state(**pIwError**).

*pNwError = (pNormal * p)/ ((pNormal*p)+(pInterference)*(1-p))*
pIwError = 1-pNwError

	P(normal)	P(interference)
P=99%	0.0125	0.9875
P=99.9%	0.1132	0.8868
P=99.99%	0.5608	0.4392
P=99.999%	0.9274	0.0726

1.b. *For each value of p and for n = 2, 3, 4 and 5, determine the probability of false positives. Fulfil the follow table:*

In this case we reuse the result obtained in part a for *pNormal* and *pInterference* we use **Bayes' Law** to calculate the probability of false positives, which is, as said in the statement, "when a station decides wrongly that the link is in interference state".

*pfalsePositive = (pNormal^n * p)/ ((pNormal^n*p)+(pInterference^n)*(1-p))*

Probability of false positives				
	n=2	n=3	n=4	n=5
P=99%	1.6150e-06	2.0627e-10	2.6346e-14	3.3649e-18
P=99.9%	1.6297e-05	2.0815e-09	2.6585e-13	3.3955e-17
P=99.99%	1.6309e-04	2.0834e-08	2.6609e-12	3.3986e-16
P=99.999%	0.0016	2.0835e-07	2.6612e-11	3.3989e-15

1.c. *For each value of p and for n = 2, 3, 4 and 5, determine the probability of false negatives. Fulfil the follow table:*

As done in part b, we reuse the results of *pNormal* and *pInterference* and the we apply **Bayes' Law** to calculate the probability of false negative, which is, as said in the statement, "when a station decides wrongly that the link is in normal state".

In order to calculate the probability of false positive, we have to put 1 minus the probability, to see if at least one of the frames has been received without errors, for example, if we put *pInterference^3* we are calculating the probability of receiving 3 wrong frames, but when we put *1-pInterference^3* we are calculating the probability of receiving 0, 1 or 2 wrong frames from a total of 3 frames, so we will have 3 right frames, 2 right frames or 1 right frame, respectively.

pfalseNegative = (1-pInterference^n)(1-p)/((1-pNormal^n)*p+(1-pInterference^n)*(1-p))*

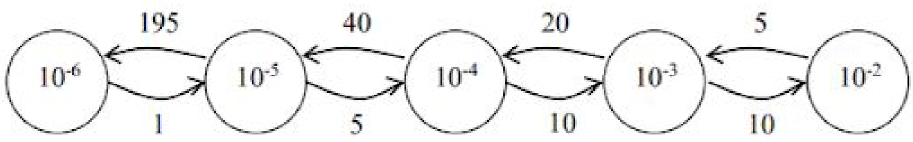
Probability of false negatives				
	n=2	n=3	n=4	n=5
P=99%	0.0084	0.0094	0.0097	0.0099
P=99.9%	8.3945e-04	9.3565e-04	9.7420e-04	9.8966e-04
P=99.99%	8.3933e-05	9.3559e-05	9.7418e-05	9.8965e-05
P=99.999%	8.3931e-06	9.3559e-06	9.7418e-06	9.8965e-06

1.d. *Describe and justify the influence of the values of p and n observed in the results obtained in 1.b and 1.c.*

We observe that *n* has influence in the value of probability obtained (the higher the n, the higher the probability), meanwhile *p* just influence in the number of decimal sour the results will have.

Task 2

Consider a wireless link between multiple stations for data communications. The bit error rate introduced by the wireless link due to the variation along with time of the propagation and interference factors is approximately given by the following Markov chain:



where the state transition rates are in number of transitions per hour. Consider that the link is in interference state when its bit error rate is 10^{-3} or higher.

2.a. *What is the average percentage of time the link is on each of the five possible states?*

The first thing we have to do is to calculate the probability of each state.

For the first state there is the "Birth-dead Markov chain" formula:

$$\pi_0 = \frac{1}{1+\sum_{i=1}^{\infty} \frac{\lambda_0 \lambda_1 \cdots \lambda_{i-1}}{\mu_1 \mu_2 \cdots \mu_i}}$$

*ps0 = 1/(1 + (1/195) + ((1*5)/(195*40)) + ((1*5*10)/(195*40*20)) + ((1*5*10*10)/(195*40*20*5)));*

Every other state is using a slightly different formula:

$$\pi_n = \frac{\lambda_0 \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n} \cdot \pi_0$$

```
ps1 = ps0*((1/195))*100
ps2 = ps0*(((1*5)/(195*40)))*100
ps3 = ps0*(((1*5*10)/(195*40*20)))*100
ps4 = ps0*(((1*5*10*10)/(195*40*20*5)))*100
```

After calculating all the formulas, we just have to multiply every probability by 100 to get the average percentage.These are the results:

State 0: 99.3314
State 1: 50.9392
State 2: 6.3674
State 3: 3.1837
State 4: 6.3674

2.b. *What is the average time duration (in minutes) that the link is on each of the five possible states?*

To calculate the average time duration of each possible state, we can use the equation T=1/qi (where as qi is the rate at which the process makes a transition when it is state i) and multiply by 60 to get the results in minutes.

```
t0 = (1/1)*60
t1 = (1/(195+5))*60
t2 = (1/(40+10))*60
t3 = (1/(20+10))*60
t4 = (1/5)*60
```

t0 = 60 minutes

t1 = 0.3000 minutes

t2 = 1.2000 minutes

t3 = 2 minutes

t4 = 12 minutes

2.c. *What is the probability of the link being in interference state?*

For a link to be in interference state, its bit error rate has to be 10^-3 or higher so it's only on the states 3 and 4. We have to calculate the probabilites of these two states and sum them to obtain the probability of the link being in interference state.

The probability of being in state 0 was already calculated as ps0.

```
ps3 = ps0*(((1*5*10)/(195*40*20)));
ps4 = ps0*(((1*5*10*10)/(195*40*20*5)));

p_interference = ps3 + ps4
```

p_interference = 9.5511e-04

2.d. *What is the average bit error rate of the link when it is in the interference state?*

To calculate the average bit error rate of the link we have to multiply the bit error rate of each state with its probability and divide with the sum of the probabilities of the two states so it can give a value in between the two values of bit error rate.

```
average = (ps3*10^(-3) + ps4*10^(-2))/(ps3+ps4)
```

average = 0.0070

2.e. *What is the average time duration (in minutes) of the interference state?*

To answer this question, we have to calculate the time between the point the link enters in the interference state (states 3 and 4) until it leaves.
To do this, first we calculate the time (in minutes) it takes to leave of states 3 and 4 and the respective probabilities.

```
t3 = (1/(20+10))*60
t4 = (1/5)*60

p32 = 20/30;
p34 = 10/30;
p43 = 1;
```

The next step is to calculate through the sum of the multiplication of probability of going from the state 3 to the 2, probability of going from the state 3 to the 4 to the power of i and the sum of t4*i and t3*(i+1) over the iteration of 10 times (since practically, after 10 times the value tends to stay the same) as shown below.

```
t=0

for i = 0:10
    t = t + p32*(p34^i) * (t4*i + t3*(i+1))
end
```

t=8.9991 minutes

This equation represents the time and the probability of being in the interference state because it shows when a link enters the state 3, and what is going to happen next (whether is going to leave the interference state or keep going to state 4 and so on).